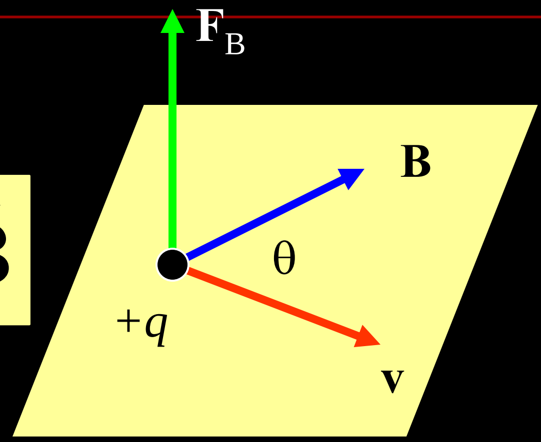
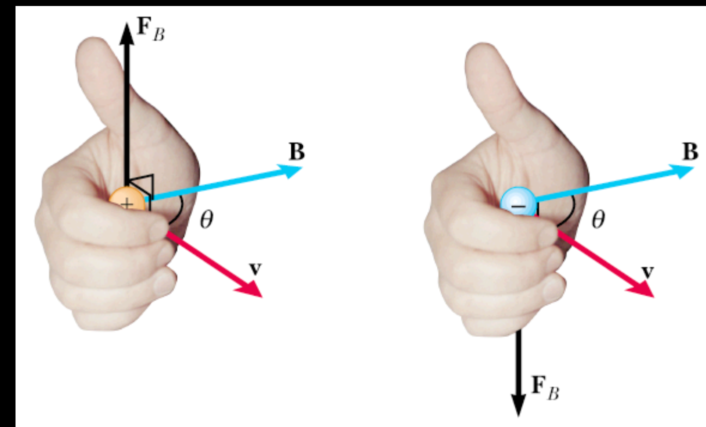
# Overall Story

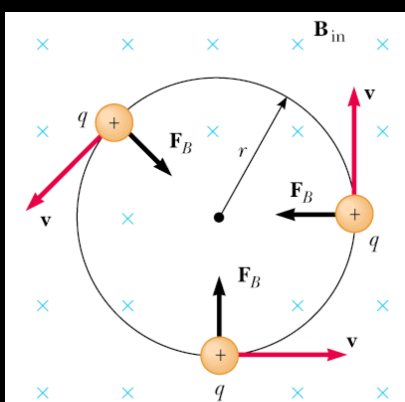
- Magnetic fields are generated by moving charges
- These fields can exert a magnetic force on other moving charged particles and objects.
- Changing magnetic fields can induce currents and vice versa.

# Magnetic Force and Charges

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$



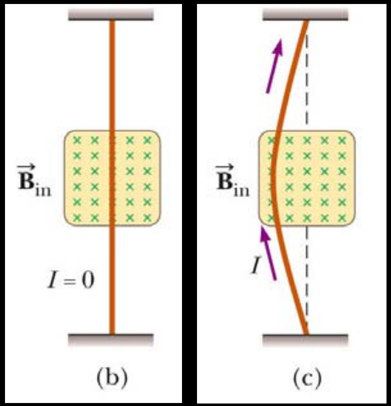

$$F_B = qvB = m\frac{v^2}{r}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$ → Cyclotron frequency

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

# Magnetic Force and Currents

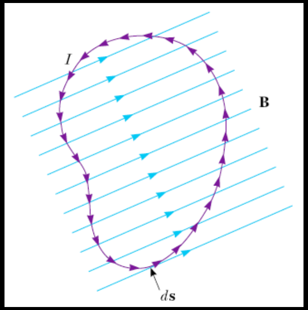


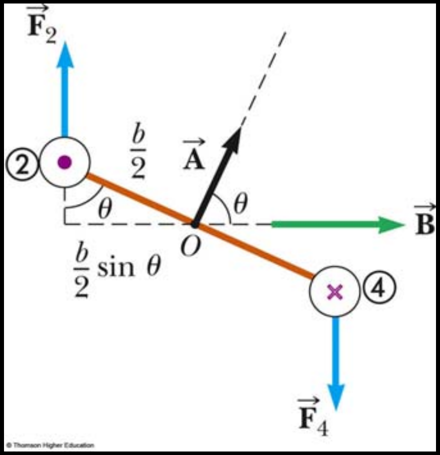

$$\vec{F}_B = I\vec{L}\times\vec{B}$$

For a straight wire in a uniform field

$$F_B = 0$$

For a loop of current in a uniform field



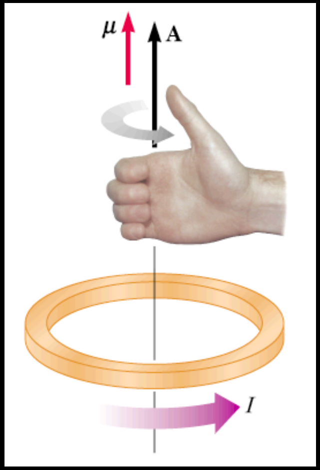




$$\vec{\tau} = I\vec{A}\times\vec{B}$$

$$\vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

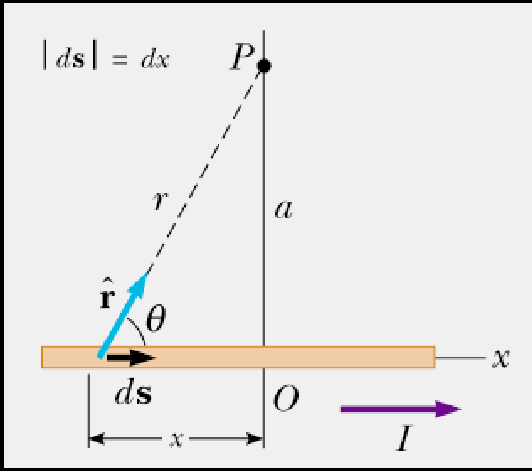
$$U = -\vec{\mu}\cdot\vec{B}$$



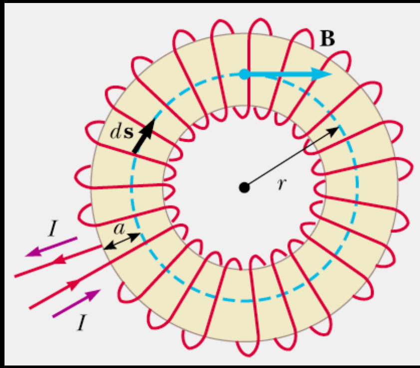
# Sources of Magnetic Field

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

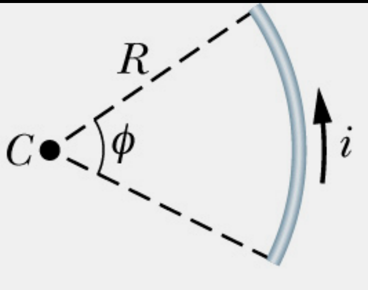
$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{s}} = \mu_0 I$$



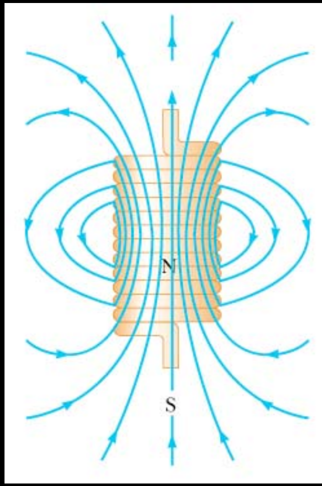

$$B = \frac{\mu_0 I}{2\pi a}$$




$$B = \frac{\mu_0 N I}{2\pi r}$$



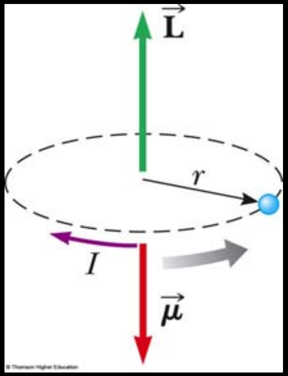

$$B = \frac{\mu_0 i \phi}{4\pi R}$$



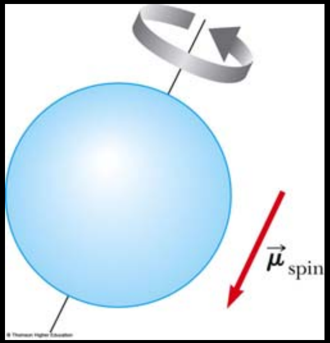

$$B = \mu_0 n I$$

# Natural Sources of Magnetism

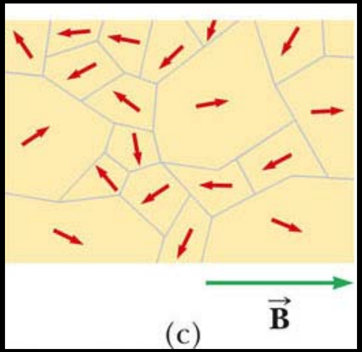
Orbital Magnetic Moment
of an Electron




Spin Magnetic Moment
of an Electron




Ferromagnetism




Paramagnetism

Induced orbital
magnetic moment

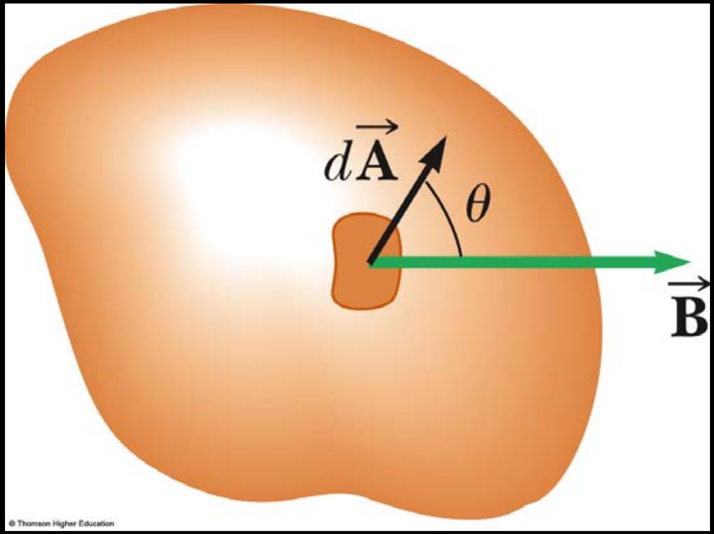
Diamagnetism

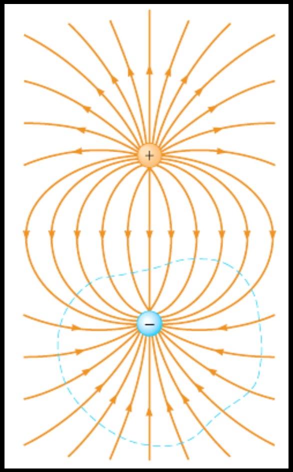
Induced internal field

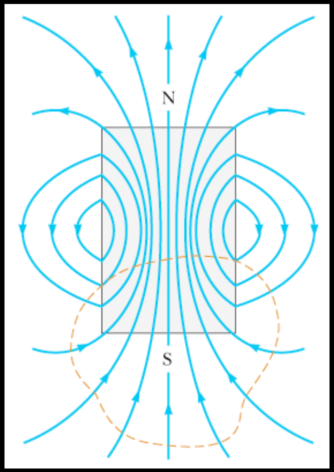
# Magnetic Flux

$$\Phi_B = \int \vec{\mathbf{B}}.d\vec{\mathbf{A}}$$

$$\oint \vec{\mathbf{B}}.d\vec{\mathbf{A}} = 0$$



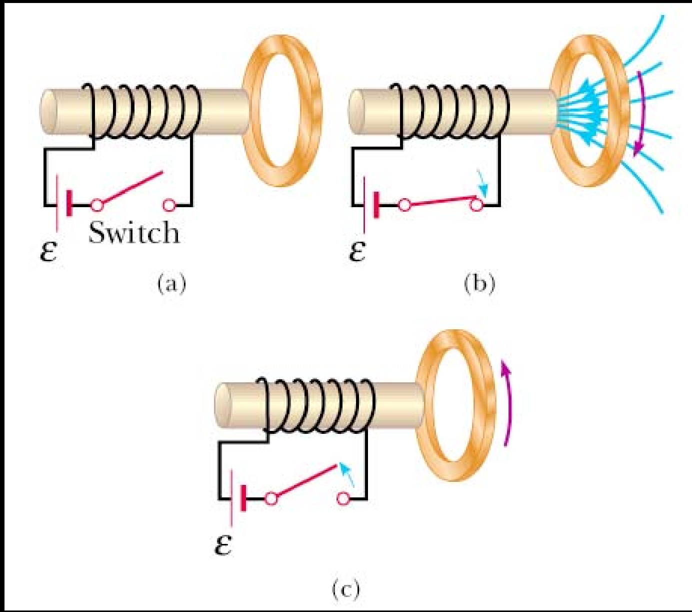

Electric Field Lines    Magnetic Field Lines

# Faraday's and Lenz's Laws

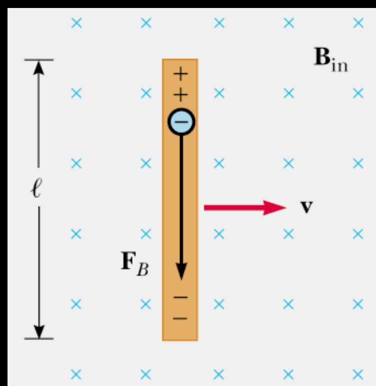
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{E}.d\mathbf{s} = -N\frac{d\Phi_B}{dt}$$

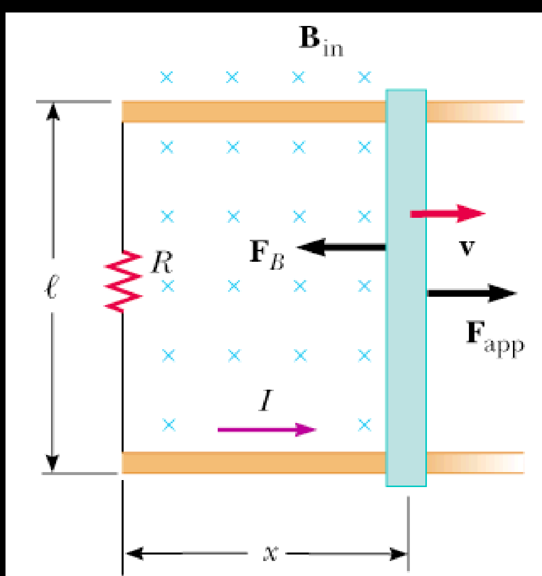
The polarity of the induced emf is such that it tends to produce a current that creates a magnetic flux to oppose the change in magnetic flux through the area enclosed by the current loop.



# Motional EMF



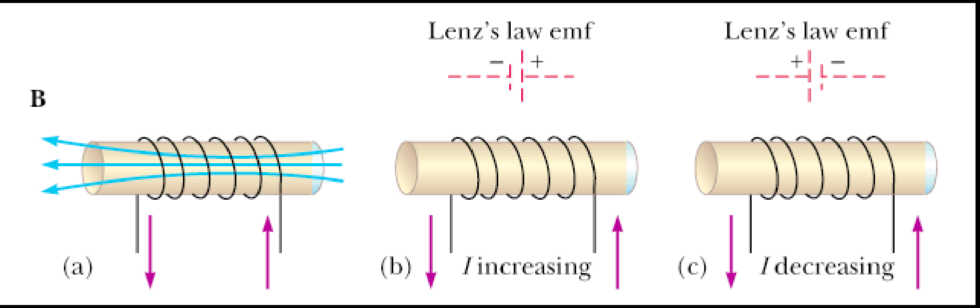

$$\Delta V = El = Blv$$




$$F_{app} = F_B = IlB$$

$$\mathscr{P} = F_{app} v = (IlB) v = \frac{B^2 l^2 v^2}{R} = \frac{\mathcal{E}^2}{R}$$

# Inductance



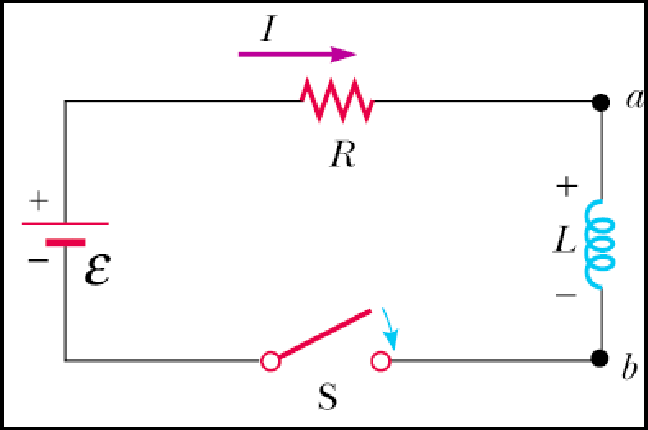

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

$$L = \frac{N\Phi_B}{I}$$

$$U = \frac{1}{2}LI^2$$

$$u_B = \frac{U}{Al} = \frac{B^2}{2\mu_0}$$

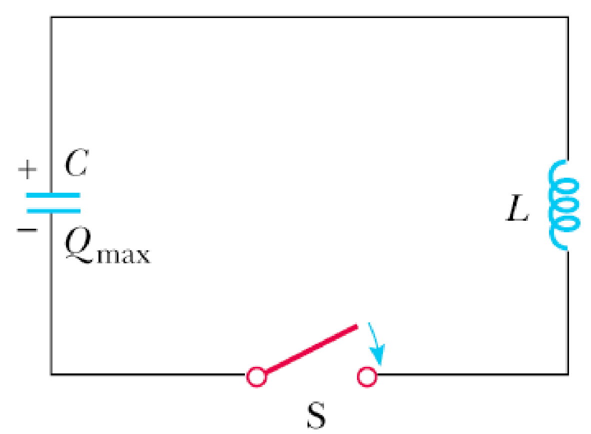
$$M_{12} = \frac{N_2\Phi_{12}}{I_1}$$




$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

# LC Oscillators



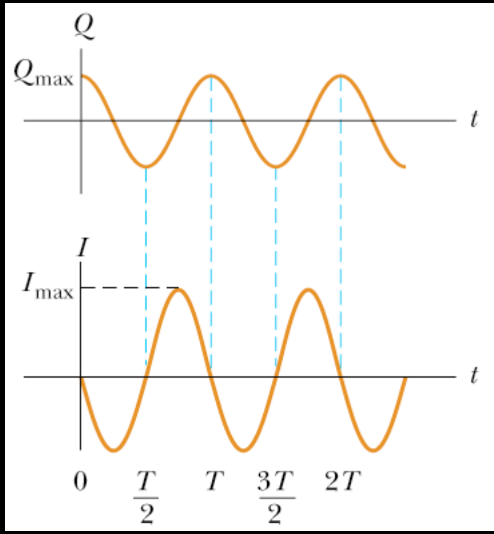

$$\omega = \frac{1}{\sqrt{LC}}$$

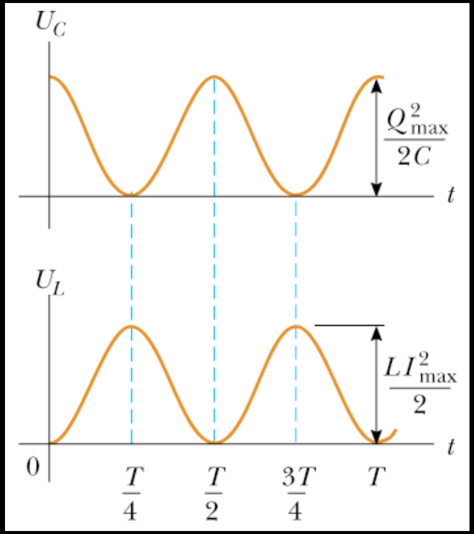
Natural frequency of oscillation

$$Q = Q_{\max} \cos \omega t$$

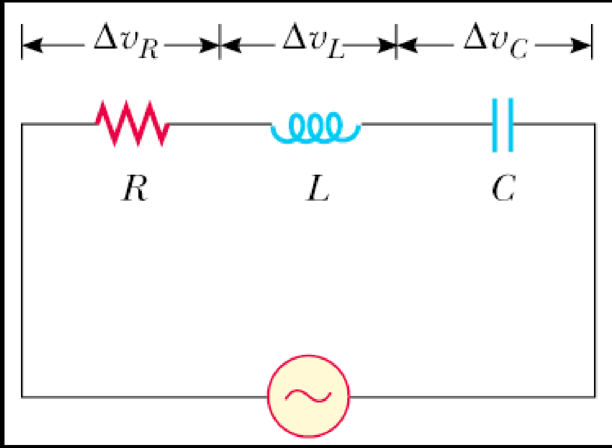
$$I = -\omega Q_{\max} \sin \omega t = -I_{\max} \sin \omega t$$

$$U = \frac{Q_{\max}{}^{2}}{2C}$$



# RLC Circuits




$$\Delta v = \Delta V_{\max} \sin \omega t$$

$$i = I_{\max} \sin(\omega t - \phi)$$

$$\Delta V_{\max} = I_{\max} Z$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$ Impedance

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

$$\mathcal{P}_{av} = \frac{1}{2} I_{\max} \Delta V_{\max} \cos\phi = I_{rms}{}^2 R$$

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

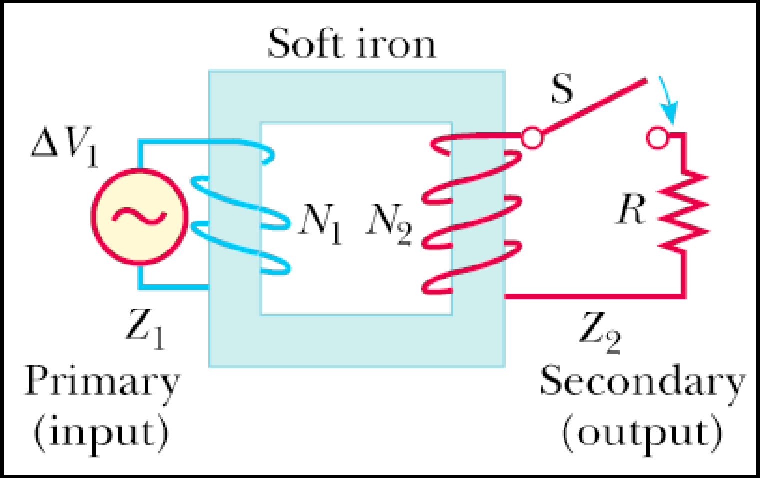
$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right)$$

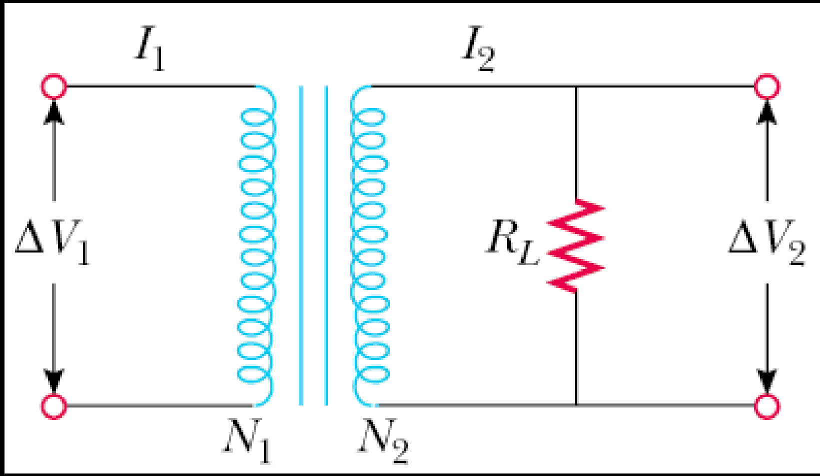
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\mathcal{Q} = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R}$$

# Transformers




$$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$$




$$R_{eq} = \left( \frac{N_1}{N_2} \right)^2 R_L$$