PHY 114 – Summer 2009 - Midterm 2 Solutions

Conceptual Question 1:

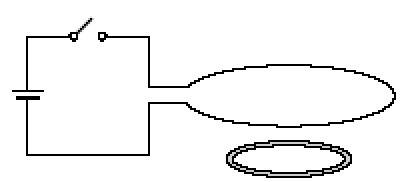
Can an electric or a magnetic field, *each constant in space and time*, be used to accomplish the actions described below? Explain your answers. Indicate if the answer is valid for any orientation of the field(s). Must any other condition be satisfied?

(a) move a charged particle in a circle;
(b) exert a force on a piece of dielectric;
(c) increase the speed of a charged particle;
(d) accelerate a moving charged particle;
(e) exert a force on an electron initially at rest.

(a) A magnetic field can move a charged particle in a circle if the particle is already in motion and moving perpendicular to the magnetic field. The force on the particle is always perpendicular to its velocity. A constant electric field can not move a particle in a circle since the electric force is either parallel or anti-parallel to the field.
(b) Since dielectrics are insulators, they do not have moving charges. Therefore, no magnetic force is exerted. An electric field can exert a force on the dielectric by polarizing its atoms.
(c) A magnetic field can only change the direction of the velocity not its speed since the magnetic force is always perpendicular to the magnetic field and velocity. An electric field can change the speed because it exerts a force along itself.
(d) Both can accelerate a moving charged particle. The acceleration provided by the magnetic field is a rotational acceleration, affecting only the direction of the velocity, whereas the electric field can change both the direction and the speed.
(e) Only an electric field can exert a force on a charge at rest. Once the charge is set in motion by the electric field, it can experience a magnetic force.

Conceptual Question 2:

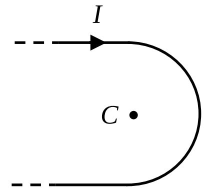
In the following figure, a coil connected to a battery and a switch lies above a thin metallic ring. The switch is initially closed. When the switch is opened, is the ring repelled by or attracted to the coil? (Hint: remember the jumping ring demo)



When the switch is closed a clockwise current starts to run through the coil and creates a field directed downwards. There will be an induced emf in the ring that opposes this field and therefore a counterclockwise current will flow through the ring. Since currents in opposite directions repel each other, the coil and the ring will repel each other.
When the switch is opened, the current in the coil and the magnetic field it creates is decreasing rapidly. The induced current will try to keep it constant and reinforce the field. Therefore, to induce a downward field, the current has to be clockwise. The coil and ring will attract each other.

Problem 1:

A long straight wire is bent as shown to form two parallel straight wires and a semicircle (radius = 2.0 m). A current of 40 A is directed as shown. What is the magnitude of the magnetic field at point C, the center of the circle along which the semicircle lies.



Divide the wire into three regions: the top straight line, the semi-circle and the bottom line. The magnetic field of the top line is going to point into the page, $B_{top} = \frac{\mu_0 I}{4\pi r}$ (the extra factor of ½ is because it is a semi-infinite line). The magnetic field of the bottom line is also going to point into the page, $B_{bot} = \frac{\mu_0 I}{4\pi r}$

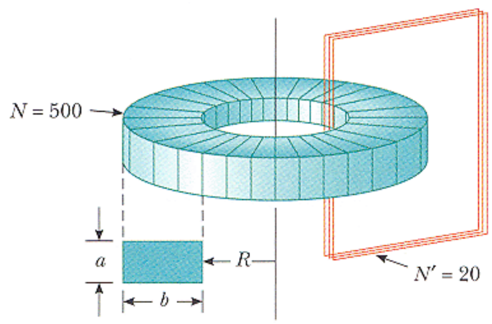
The magnetic field of the semi-circle also points into the page, $B_{s-c} = \frac{\mu_0 I \pi}{4\pi r} = \frac{\mu_0 I}{4r}$

So the total field will be the sum of all three and will point into the page:

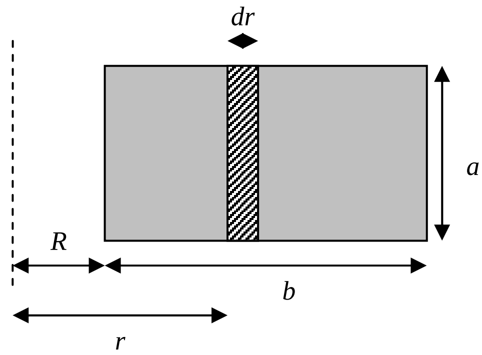
$$B_T = \frac{\mu_0 I}{4r}\left(\frac{2}{\pi}+1\right) = \frac{(4\pi\times10^{-7})(40)}{4(2)}\left(\frac{2}{\pi}+1\right) = 1.03\times10^{-5}T$$

Problem 2:

A toroid having a rectangular cross section (a = 2.00 cm by b = 3.00 cm) and inner radius R = 4.45 cm consists of 500 turns of wire that carries a sinusoidal current $I = I_{max}\sin\omega t$, with I_{max} = 51.0 A and a frequency $f = \omega/2\pi = 60.0 Hz$. A coil that consists of 20 turns of wire links with the toroid, as shown in figure. Determine the emf induced in the coil as a function of time.



The toroid has a magnetic field that is confined to the core, circular and perpendicular to its cross-section (the rectangle with sides a and b). Therefore, when calculating the flux through the coil, we only need to include the area of the toroid. This field is a function of radius (meaning the field is higher at $r=R$ than at $r=R+b$). Drawing the cross-section of the toroid, we can divide the area into thin strips of width dr and length a. The figure shows the cross-sectional area of the toroid. The dashed line is the center axis of the toroid. The field is directed either out of the screen or in to it (depending on the direction of the current in the



toroid coils) and is confined to the toroid. The area is divided into thin strips of width dr and length a. The total flux is calculated as the integral of the flux through these strips over the whole width of the cross-section.

$$\Phi_{B,coil} = \int B_{toroid} dA_{toroid} = \int_R^{R+b} \frac{\mu_0 N_{toroid} I}{2\pi r} a dr$$

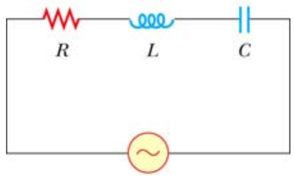
$$\Phi_{B,coil} = \frac{\mu_0 N_{toroid} I a}{2\pi} \int_R^{R+b} \frac{dr}{r} = \frac{\mu_0 N_{toroid} I a}{2\pi} \ln\left(\frac{R+b}{R}\right) = \left(5.26\times 10^{-5} Wb\right)\sin(\omega t)$$

Then the induced emf is: $\boxed{\mathcal{E} = -N_{coil}\frac{d\Phi_{B,coil}}{dt} = -1.05\times 10^{-3}\frac{d\sin(\omega t)}{dt} = -(0.4V)\cos(120\pi t)}$

Problem 3:

In a series RLC circuit, $\Delta V = 200\text{V} \sin(100t)$, $R = 500\Omega$, $L = 0.1\text{H}$ and $C = 2\mu\text{F}$

(a) Determine the rms current for the circuit.
(b) Determine the instantaneous voltage drop across the inductor in the circuit.
(c) What is the power factor?



Bonus: Which change would increase the amount of power delivered to circuit more: doubling the operating frequency or doubling the capacitance?

$$X_L = \omega L = 100(0.1) = 10\Omega \text{ and } X_C = \frac{1}{\omega C} = \frac{1}{100\left(2\times 10^{-6}\right)} = 5000\Omega$$

Then, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{500^2 + (10-5000)^2} = 5010\Omega$ and

$$\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{10-5000}{500}\right) = -84.3°$$

(a) $V_{rms} = \frac{V_{\max}}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141V$ then, $\boxed{I_{rms} = \frac{V_{rms}}{Z} = \frac{141}{5010} = 28.2mA}$

(b) The current through the circuit will have a phase difference of ϕ with the voltage across it. The voltage across the inductor will lead the current by $\pi/2$ rad.

$i(t) = I_{\max}\sin(\omega t - \phi) = \left(\sqrt{2}I_{rms}\right)\sin(\omega t - \phi)$, $v_L(t) = X_L\left(\sqrt{2}I_{rms}\right)\sin(\omega t - \phi + \pi/2)$

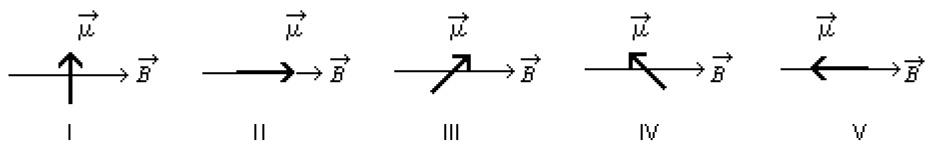
Then, $\boxed{v_L(t) = (10)\left(\sqrt{2}\right)\left(28.2\times 10^{-3}\right)\sin(100t + 3.04) = (0.4V)\sin(100t + 3.04)}$

(c) $\boxed{\cos\phi = \cos(-84.3°) = 0.0993}$

Bonus: In its original form, the circuit is highly capacitive ($X_C >> I_L$) resulting in low power delivery. Doubling the operating frequency increases X_L and decreases X_C and improves the efficiency. Doubling the capacitance only affects the capacitance and the improvement is less.

Multiple Choice Questions:

1. The diagrams show five possible orientations of a magnetic dipole $\vec{\mu}$ in a uniform magnetic field $\vec{B}$. For which of these does the magnetic torque on the dipole have the greatest magnitude?



a) I
b) II
c) III
d) IV
e) V

Since $\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \vec{\mu} \times \vec{B}$, the torque is maximum when μ and B are perpendicular.

2. Of the three chief kinds of magnetic materials (diamagnetic, paramagnetic, and ferromagnetic) which are used to make permanent magnets?
a) only diamagnetic
b) only ferromagnetic
c) only paramagnetic
d) only paramagnetic and ferromagnetic
e) all three

Only ferromagnetism involves a permanent effect.

3. Gauss' law for magnetism tells us:
a) the net charge in any given volume
b) that the line integral of a magnetic field around any closed loop must vanish
c) the magnetic field of a current element
d) that magnetic monopoles do not exist
e) charges must be moving to produce magnetic fields

Since the law states that the net magnetic flux through a closed surface should be zero, this implies that there are no magnetic "charges" or monopoles.

4. A charged capacitor and an inductor are connected in series. At time $t = 0$ the current is zero, but the capacitor is charged. If T is the period of the resulting oscillations, the next time, after $t = 0$ that the voltage across the inductor is a maximum is:
   a) T
   b) $T/4$
   c) $T/2$
   d) T
   e) $2T$

   The capacitor will fully discharge in a quarter of the period and then recharge in the opposite polarity by T/2. The inductor voltage is essentially equal to the capacitor voltage in magnitude, so when the capacitor voltage is at a maximum, so is the inductor voltage. Since the capacitor voltage is proportional to the charge on the capacitor, it will be at a maximum at T/2.

5. The primary of an ideal transformer has 100 turns and the secondary has 600 turns. Then:
   a) the power in the primary circuit is less than that in the secondary circuit
   b) the currents in the two circuits are the same
   c) the voltages in the two circuits are the same
   d) the primary current is six times the secondary current
   e) the frequency in the secondary circuit is six times that in the primary circuit

   An ideal transformer changes the voltage and current across its primary and secondary coils but the power and frequency remain the same.