

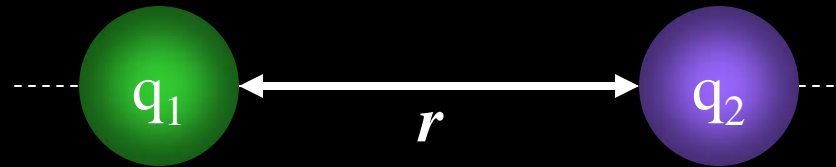
Overall Story

- Protons and electron carry **charge**
- Presence of charge creates an **electric field**
- The field creates **potential differences** between points within the space it exists
- Other charges react to this field and potential and move about (**current**)

Electrical Charges

- Charge is carried by **electrons** and **protons**.
- Can be **positive** or **negative**.
- **Like** charges **repel**, **opposite** charges **attract**.
- Total charge in a system is **conserved**.
- Charges come in **discrete** quantities.
- Charges are measured in **Coulombs** (C).
- Usually denoted by **q**.

Electric Force and Field



$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

$$k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$$

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_0} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

E-Field

$$\vec{\mathbf{E}}$$

Force

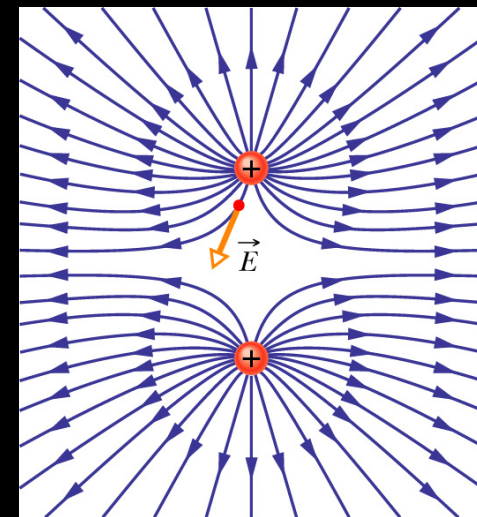
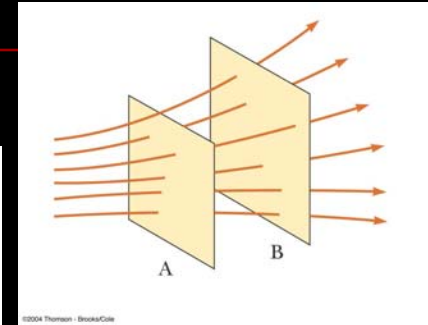
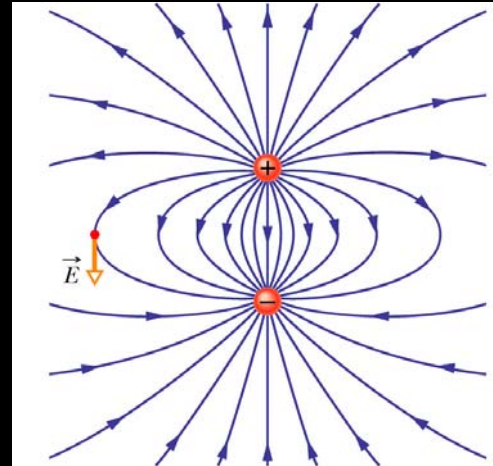
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

Acceleration

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} = \frac{q\vec{\mathbf{E}}}{m}$$

Electric Field Lines

- A way to visualize field patterns over space.
- The e-field is tangent to the field lines at each point and along the direction of the field arrow.
- The density of the lines is proportional to the magnitude of the e-field.
- Field lines start from positive charges and end at negative ones.
- Field lines can not cross.



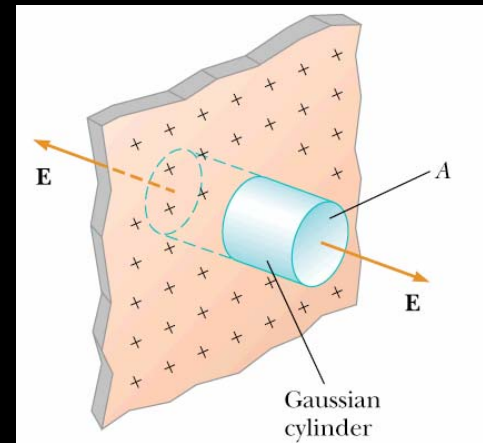
Gauss' Law

Electric Flux

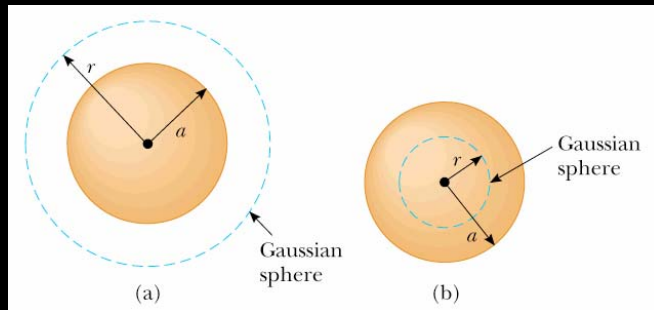
$$\Phi_E = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$



$$E = \frac{\sigma}{2\epsilon_0}$$

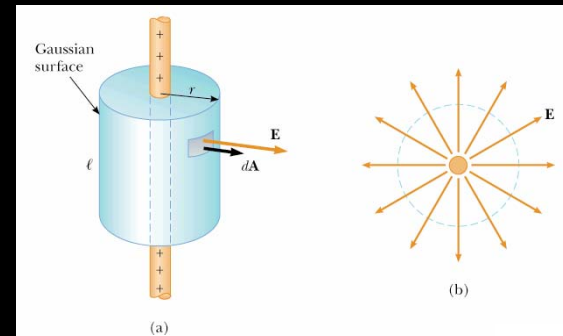


$$E = k_e \frac{Q}{r^2}$$

$$r > a$$

$$E = k_e \left(\frac{Q}{a^3} \right) r$$

$$r < R$$



$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Conductors and Gauss' Law

- The electric field is zero everywhere inside a conductor at electrostatic equilibrium.
- Any net charge on a conductor will reside on the surface.
- The electric field just outside a conductor is perpendicular to the surface and is proportional to the charge density.
- The charge density is highest near parts of the conductor with the smallest radius of curvature.

$$E = \frac{\sigma}{\epsilon_0}$$

Electric Potential

Electric Potential Energy

$$\Delta U = -q_0 \int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Potential Difference

$$\Delta V = \frac{\Delta U}{q_0} = - \int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

In a Uniform Field

$$\Delta V = -Ed$$

For a Point Charge

$$V = k_e \frac{q}{r}$$

For a Charge Distribution

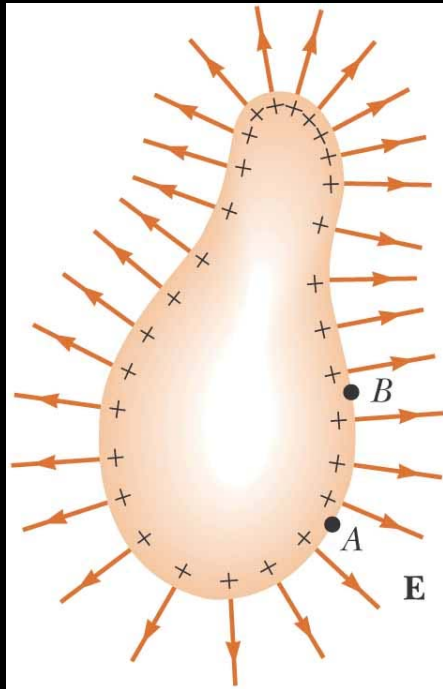
$$V = k_e \int \frac{dq}{r}$$

$$E_x = \frac{\partial V}{\partial x}$$

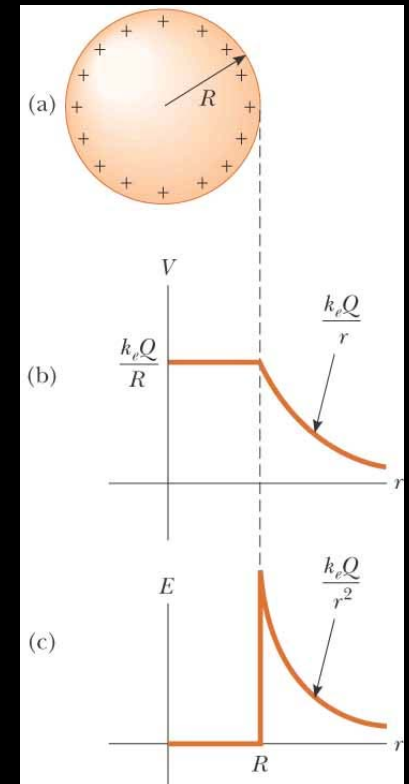
$$E_y = \frac{\partial V}{\partial y}$$

$$E_z = \frac{\partial V}{\partial z}$$

Potential Due to a Charged Conductor



- Charges always reside at the outer surface of the conductor.
- The field lines are always perpendicular to surface.
- Then $\mathbf{E} \cdot d\mathbf{s} = 0$ on the surface at any point.
- Which means, $V_B - V_A = 0$ along the surface.
- The surface is an equipotential surface.
- Finally, since $\mathbf{E} = 0$ inside the conductor, the potential V is constant and equal to the surface value.



Capacitors

$$C \equiv \frac{Q}{\Delta V} \quad (C/V=F)$$

Parallel Plate
Capacitor

$$C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$$

For series
capacitors

$$\frac{1}{C_{eq}} = \sum_{j=1}^n \frac{1}{C_j}$$

For parallel
capacitors

$$C_{eq} = \sum_{j=1}^n C_j$$

Stored Energy in a Capacitor

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

Capacitor with a dielectric

$$C = \kappa C_0$$

Current and Resistance

$$I(t) = \frac{dQ}{dt}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$R = \frac{\Delta V}{I}$$

$$\rho = \frac{1}{\sigma} \quad R = \rho \frac{l}{A}$$

$$R = R_0 [1 + \alpha(T - T_0)]$$

$$\mathcal{P} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

DC Circuits

Series

$$R_{eq} = \sum_i R_i$$

Parallel

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Kirchoff's
Rules

$$\sum_{\text{junction}} I_{in} = \sum_{\text{junction}} I_{out}$$

$$\sum_{\text{closed loop}} \Delta V = 0$$

RC Circuits

$$q(t) = C\mathcal{E}(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

$$I(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$q(t) = Qe^{-t/RC}$$
$$I(t) = \frac{dq}{dt} = -\frac{Q}{RC} e^{-t/RC}$$