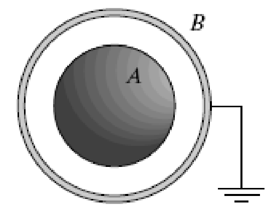
PHY 114 – Summer 2009

Midterm 1 Solutions

<u>Conceptual Question 1</u>

In the figure, *A* is a solid metallic sphere and *B* a concentric metallic shell. Suppose *A* is charged positively and *B* is grounded.

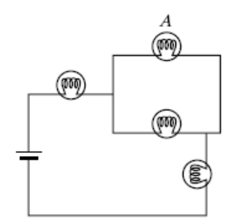


(a) Qualitatively compare the magnitude and distribution of charges on A and B.
(b) Is there an electric field (i) outside B (ii) between A and B; (iii) inside A? Explain.

(a) The positive charges on A will reside on the outer surface of the sphere and be distributed uniformly. This will attract an equal and opposite amount of charge (negative) to the inner surface of B with uniform distribution. The positive charges left behind on the outer surface of B will be neutralized by the negative charges that flow from the ground to the outer surface. Therefore the shell will be negatively charged overall.
(b) (i) Outside B, the electric field is going to be proportional to the enclosed charged in a Gaussian sphere (due to Gauss' Law and spherical symmetry). Since the total net charge on A and B is zero, there are no fields outside. (ii) Between A and B, again the field is proportional to the charge enclosed which is the charge on A, therefore there is a field. (iii) All the charge on A resides on the outer surface of the sphere; therefore there are no fields inside A.

<u>Conceptual Question 2</u>

In the circuit, all light bulbs are identical. Suppose bulb *A* is unscrewed from its socket.

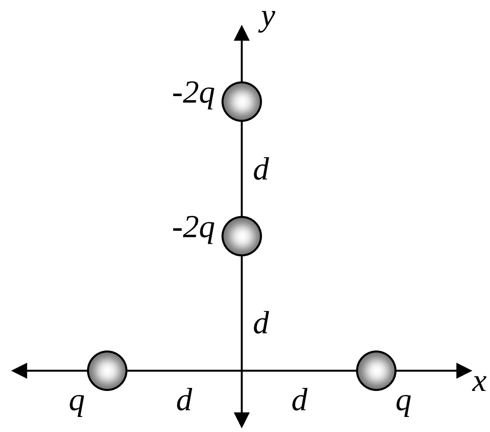


(a) How do the brightnesses of the three remaining bulbs change?
(b) How do these brightnesses compare with each other?

(a) When *A* is unscrewed, the current that goes through the bulb parallel to *A* increases and it gets brighter. However, since the equivalent resistance of the overall circuit increases, the overall current decreases and the other two bulbs get dimmer.
(b) Before being unscrewed, *A* and the bulb parallel to it have the same brightness, which is half the brightness of the other two. Afterwards, all the remaining three bulbs are in series and have the same current through them, so they have the same brightness.

Problem 1

Four charges are placed on a grid as shown in the figure.



(a) What is the net electric field at the origin?
(b) If an electron, initially at rest, is placed at the origin, which direction will it start to move?
(c) What is the electric potential at the origin? (assume $V_\infty = 0V$)

(a) The electric field from the two +q charges will cancel each other. The field due to the -2q charges will be in the +y direction. The magnitude of the field will be:

$$E = k_e \frac{2q}{d^2} + k_e \frac{2q}{(2d)^2} = k_e \frac{5q}{2d^2}$$

(b) Because an electron has a negative charge, it will move in the opposite direction of the field so it will initially move in the –y direction.

(c) $$V = k_e \frac{q}{d} + k_e \frac{q}{d} - k_e \frac{2q}{d} - k_e \frac{2q}{2d} = -k_e \frac{q}{d}$$

Problem 2

A parallel-plate capacitor in air has a plate separation of 1.5 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water ($\kappa = 80$). Assume that distilled water is an insulator.

(a) What is the charge on the plates before and after immersion?
(b) What is the capacitance and the potential difference after immersion?
(c) What is the change in energy of the capacitor?

Bonus (5 points): If, instead of being disconnected from the battery, the capacitor was immersed while still attached to the source, what would the change in the stored energy of the capacitor be?

(a) $C_i = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})(25\times10^{-4}m^2)}{0.015m} = 1.48pF$

Since the capacitor is disconnected from the source, the charge stays the same.

$$Q_i = C_i \Delta V = (1.48pF)(250V) = 369pC, \quad Q_f = Q_i = 369pC$$

(b) $\boxed{C_f = \kappa C_i = (80)(1.48\,pF) = 118\,pF}$, $\boxed{\Delta V_f = \frac{Q_f}{C_f} = \frac{369\,pC}{118\,pF} = 3.13V}$

(c) $U_i = \frac{Q_i^2}{2C_i} = \frac{(369\,pC)^2}{2(1.48\,pF)} = 46nJ$ and $U_f = \frac{Q_f^2}{2C_f} = \frac{U_i}{\kappa} = \frac{46nJ}{80} = 0.58nJ$.

Then, $\boxed{\Delta U = U_f - U_i = (0.58 - 46)nJ = -45.4nJ}$

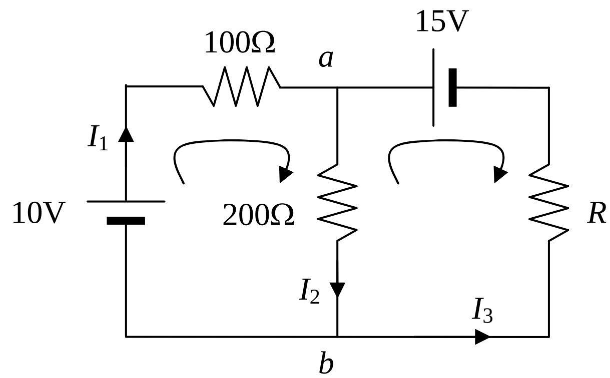
Bonus: $\Delta V_f = \Delta V_i$ and $U_f = \frac{1}{2}C_f \Delta V_f^2 = \frac{1}{2}\kappa C_i \Delta V_i^2 = \kappa U_i = 80(46nJ) = 3680nJ$. Then,

$\boxed{\Delta U = U_f - U_i = (3680 - 46)nJ = 3634nJ}$

Problem 3

Consider the circuit on the right.

(a) Identify all the currents and write a complete set of Kirchoff's equations.
(b) If the potential drop across the 200Ω resistor is 12V, find I_1.
(c) What is the power dissipated on R?



(a) Using the currents and clockwise loops as shown,

$$\boxed{\begin{array}{l} I_1 + I_3 = I_2 \\ 10 - 100I_1 - 200I_2 = 0 \\ -15 + RI_3 + 200I_2 = 0 \end{array}}$$

(b) $\Delta V_{200} = V_a - V_b = 12V = 200I_2$, then, $I_2 = \frac{12V}{200\Omega} = 60mA$

From the second equation, $10 - 100I_1 - 200(0.06) = 0$. Then, $\boxed{I_1 = -\frac{2V}{100\Omega} = -20mA}$

(c) From the first equation, $-20mA + I_3 = 60mA$. Then, $I_3 = 60mA + 20mA = 80mA$

From the third equation, $-15 + R(80mA) + 200(60mA) = 0$. Then, $R = 37.5\Omega$

Then, $\boxed{P_R = RI_3^2 = (37.5\Omega)(0.08A)^2 = 0.24W}$

Multiple Choice Questions

1. An electron traveling north enters a region where the electric field is uniform and points north. The electron:

   a) speeds up
   b) slows down
   c) veers east
   d) veers west
   e) continues with the same speed in the same direction

Because the electron is negatively charged, the force acting on it ($F = qE$) is in the opposite direction of the field. Therefore, the electron slows down.

2. Equipotentials are lines along which:

   a) the electric field is constant in magnitude and direction
   b) the electric charge is constant in magnitude and direction.
   c) a charge moving at constant speed requires that the maximum amount of work be done against electrical forces.
   d) a charge may be moved at constant speed without work against electrical forces.
   e) charges move by themselves.

Since an equipotential line is a line of constant potential, the electric potential energy does not change if the charge is not accelerating. Therefore, no work is done during the motion.

3. The capacitance of a cylindrical capacitor can be increased by:

   a) decreasing both the radius of the inner cylinder and the length
   b) increasing both the radius inner cylinder and the length
   c) increasing the radius outer cylindrical shell and decreasing the length
   d) decreasing the radius inner cylinder and increasing the radius of the outer cylindrical shell
   e) only by decreasing the length

The capacitance of a cylindrical capacitor ($C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$) is proportional to its length, L. It will also increase with increasing inner radius, a.

4. A segment of Nichrome wire is initially at 20.0°C. Using the data below, calculate the temperature to which the wire must be heated if its resistance is to be doubled (ρ = 1.5 x 10^{-6} Ω.m, α = 0.4 x 10^{-3} °C^{-1}).

a) 510 °C
b) 870 °C
c) 2520 °C
d) 3670 °C
e) 4220 °C

$R = 2R_0 = R_0\left[1+\alpha\left(T-T_0\right)\right]$. Then, $T = T_0 + \frac{1}{\alpha} = \left(20 + \frac{1}{0.4\times 10^{-3}}\right)°C = 2520°C$

5. You wish to double the rate of energy dissipation in a heating device. You could:

a) double the potential difference keeping the resistance the same
b) double the current keeping the resistance the same
c) double the resistance keeping the potential difference the same
d) double the resistance keeping the current the same
e) double both the potential difference and current

The rate of energy dissipation is the power. $P = IV = I^2R = \frac{V^2}{R}$