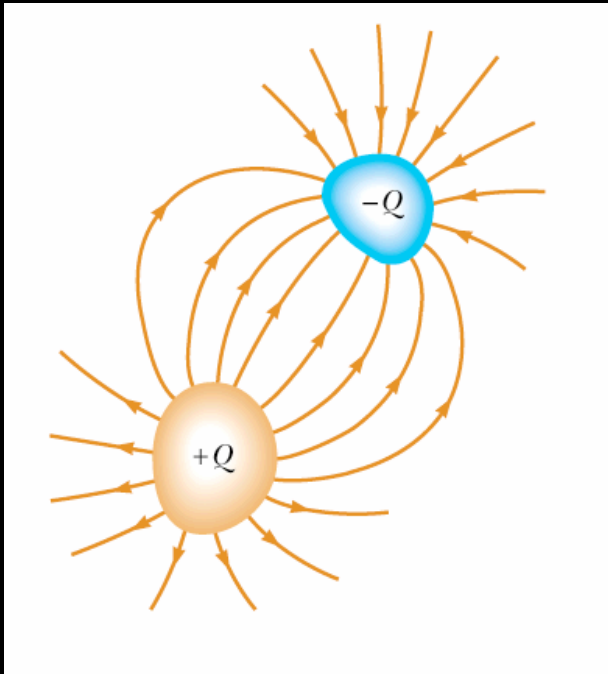


Capacitance and Dielectrics

- Electric devices function through storing and manipulating the flow of charges.
- This is achieved through several types discrete circuit components.
- Capacitors can store electric charge (and therefore electric potential energy) for a certain period of time.

Capacitance



Two
conductors
with equal
but opposite
charges



Electric
field



Potential
difference
between
the
charges

Now turn the problem around ...

Given a potential difference (a voltage)
between two conductors, what is the charge
on them?

$$C \equiv \frac{Q}{\Delta V} \quad (C/V=F)$$

Calculating Capacitance

Assume a charge q on the plates



Calculate the e-field using Gauss' Law



Calculate the potential



Find the capacitance

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

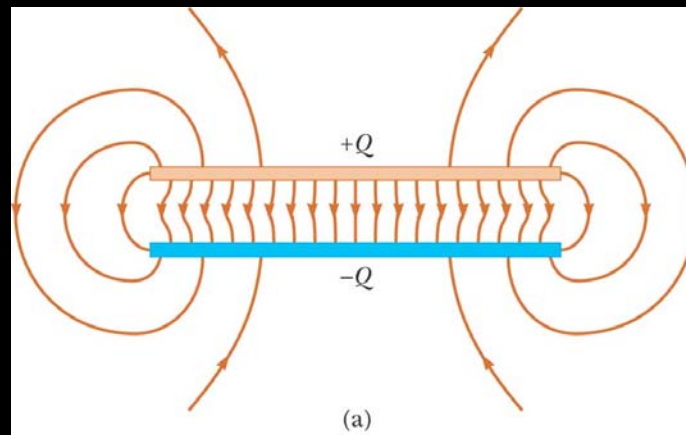
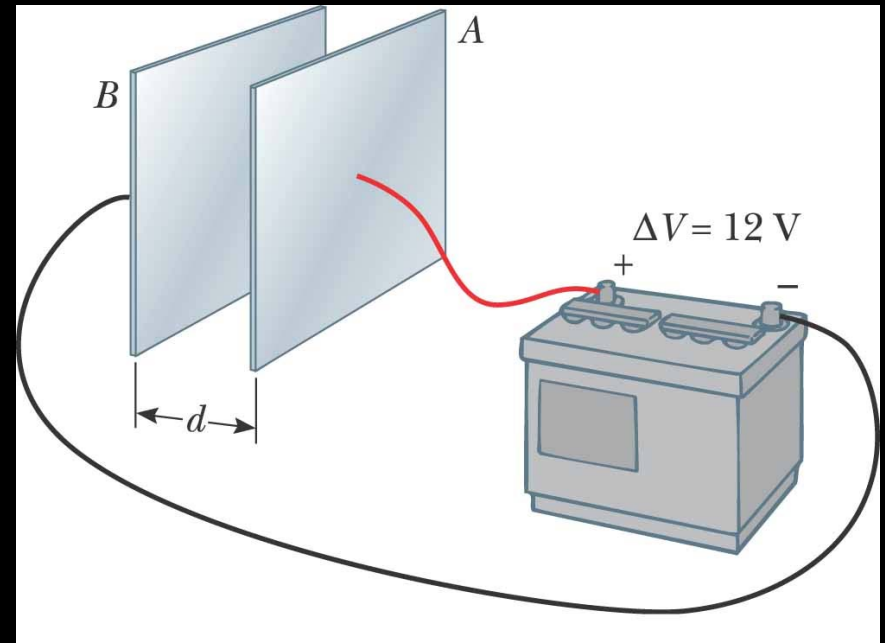
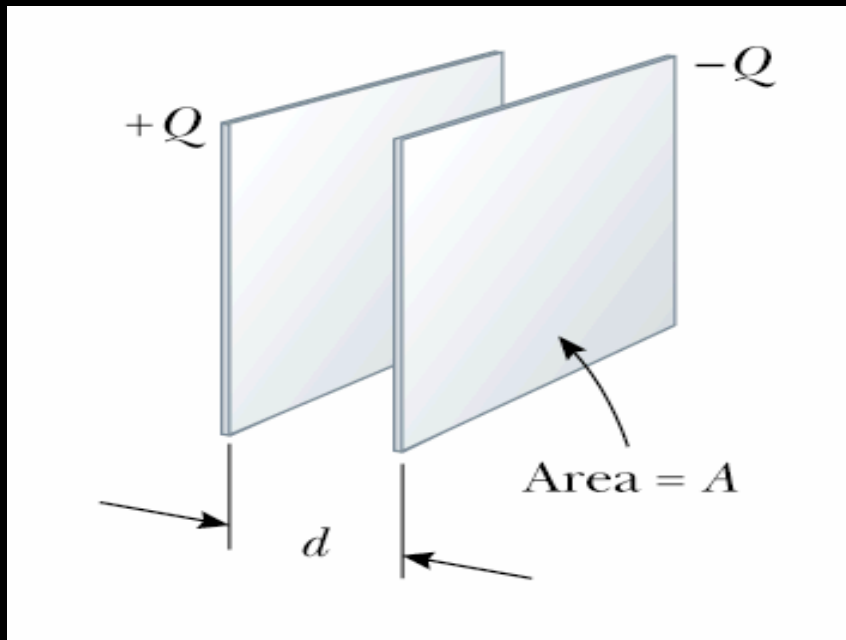
$$\epsilon_0 EA = q$$

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}$$

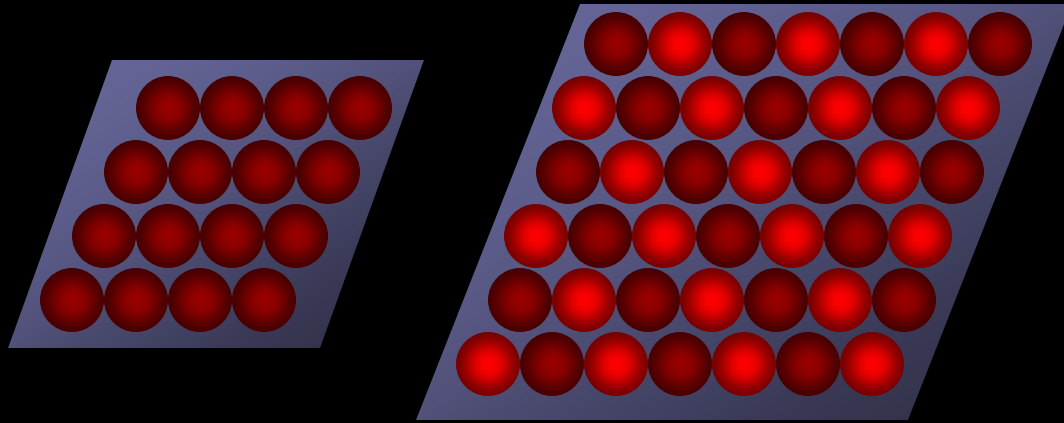
$$\Delta V = \int_{-}^{+} E ds$$

$$C = \frac{q}{\Delta V}$$

Parallel Plate Capacitor



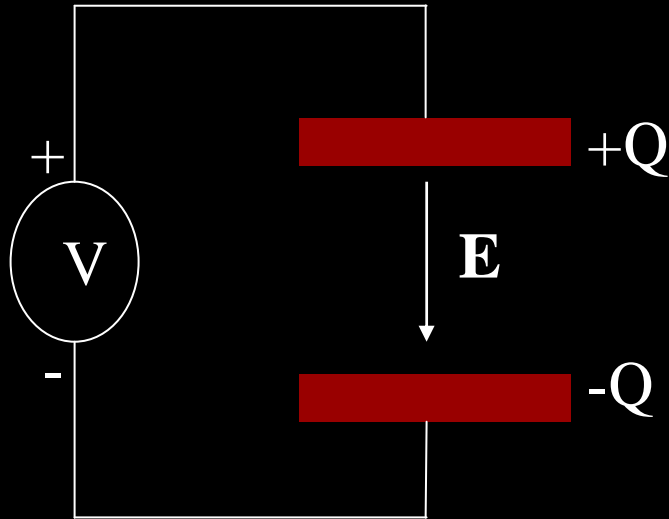
Parallel Plate Capacitor



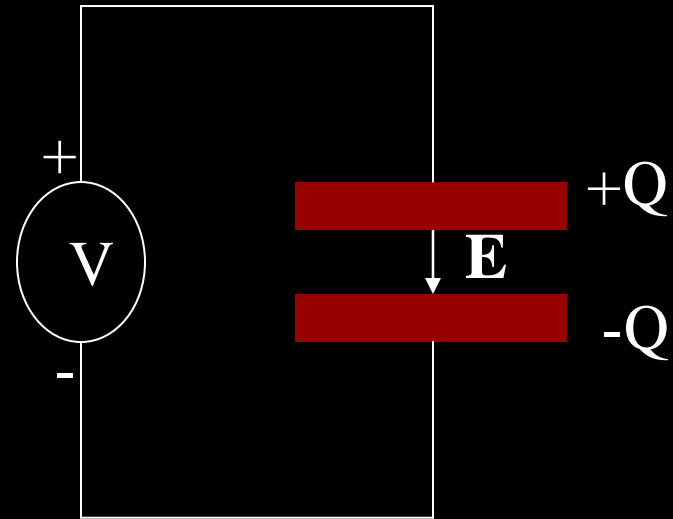
Increased area results in increased charge storage for a constant voltage.

Capacitance is proportional to area

Parallel Plate Capacitor



$$\Delta V = Ed = V$$



$$\Delta V' = Ed' < V$$

The potential difference between the battery terminals and the plates will create a field and charges will flow from the battery to the plates until the potential difference is zeroed.

Capacitance is inversely proportional to the distance between the plates.

Parallel Plate Capacitor

Surface charge density on the plates:

$$\sigma = \frac{Q}{A}$$

The field (assuming $d \ll A^{1/2}$):

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

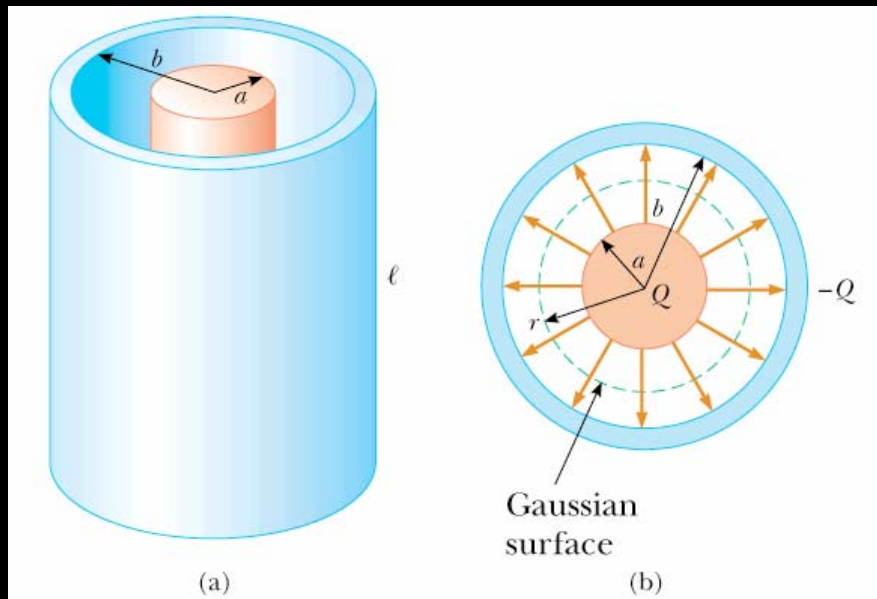
The potential difference:

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

The capacitance:

$$C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$$

Cylindrical Capacitor



$$E = \frac{2k_e \lambda}{r}$$

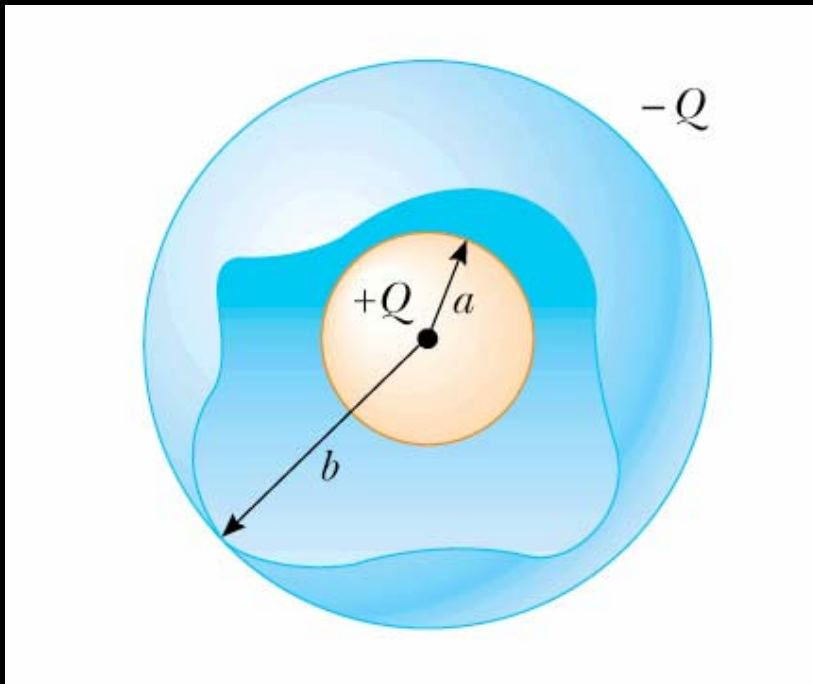
$$Q = \lambda l$$

$$V_b - V_a = -\int_a^b E dr = -2k_e \lambda \int_a^b \frac{dr}{r} = -2k_e \lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{\lambda l}{2k_e \lambda \ln\left(\frac{b}{a}\right)}$$

$$C = \frac{l}{2k_e \ln\left(\frac{b}{a}\right)}$$
$$\frac{C}{l} = \frac{1}{2k_e \ln\left(\frac{b}{a}\right)}$$

Spherical Capacitor



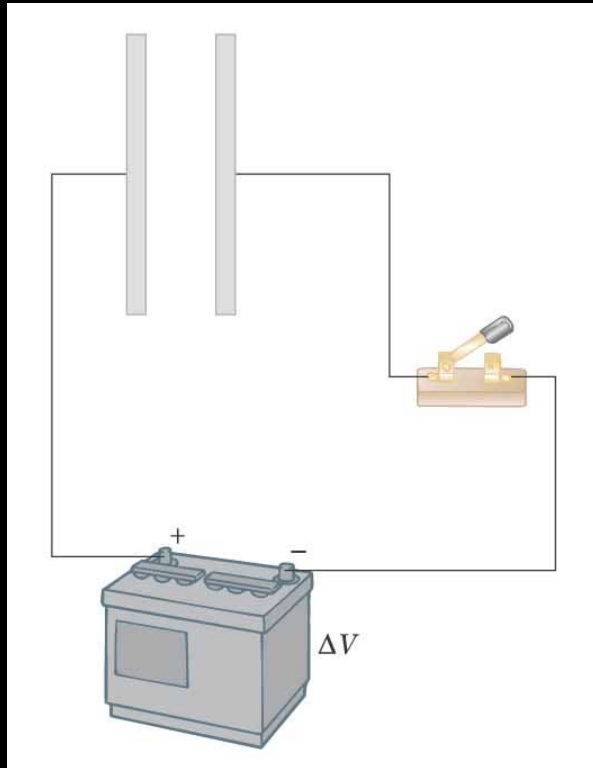
$$E = k_e \frac{Q}{r^2}$$

$$V_b - V_a = -\int_a^b E dr = \int_a^b \frac{dr}{r^2} = k_e Q \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q \left(\frac{1}{a} - \frac{1}{b} \right)}$$

$$C = \frac{ab}{k_e (b - a)}$$

Circuits with Capacitors

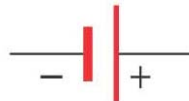


- A battery drives charges to accumulate on the capacitor plates.
- A switch controls the flow of charge.

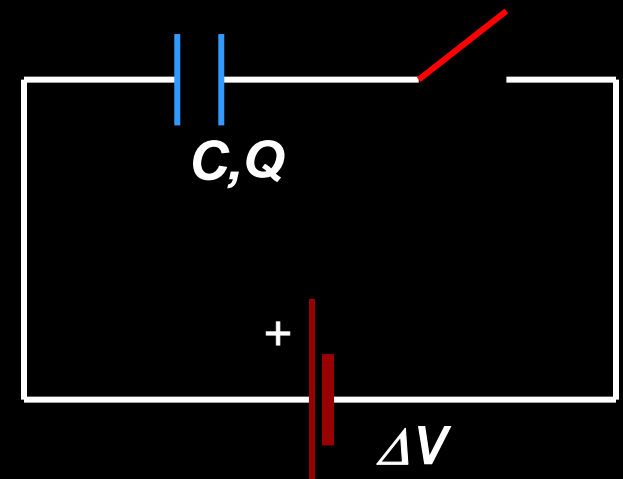
Capacitor symbol



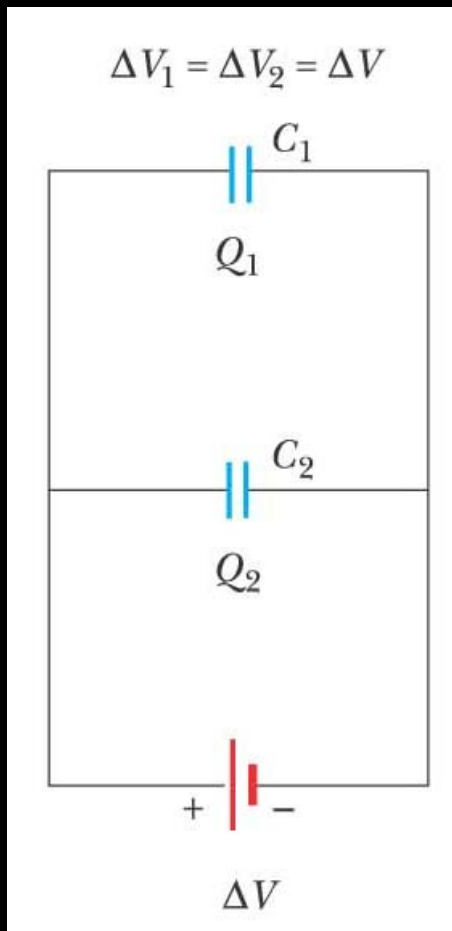
Battery symbol



Switch symbol



Parallel Combination



$$Q = Q_1 + Q_2$$

$$\Delta V = \Delta V_1 = \Delta V_2$$

$$Q_1 = C_1 \Delta V_1 = C_1 \Delta V$$

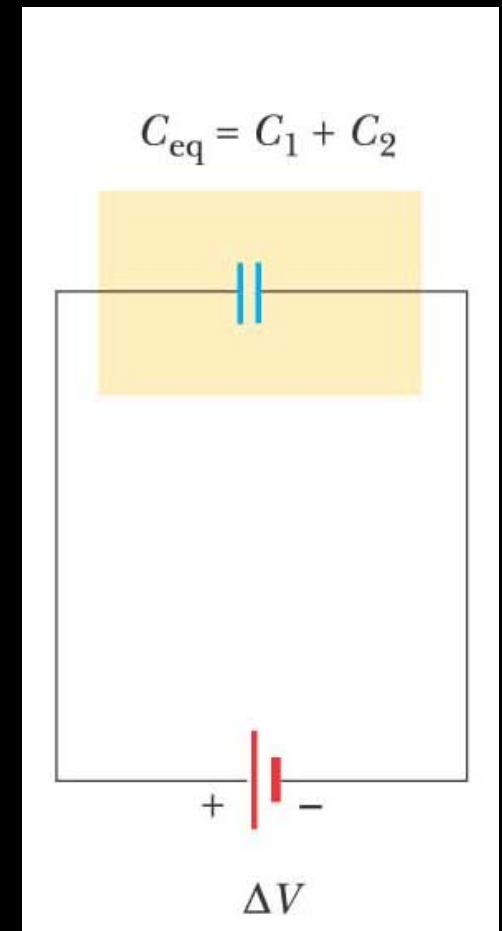
$$Q_2 = C_2 \Delta V_2 = C_2 \Delta V$$

$$Q = C_{eq} \Delta V$$

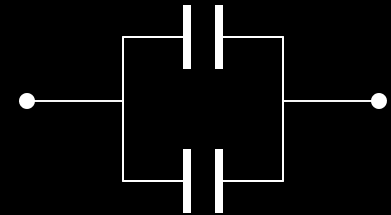
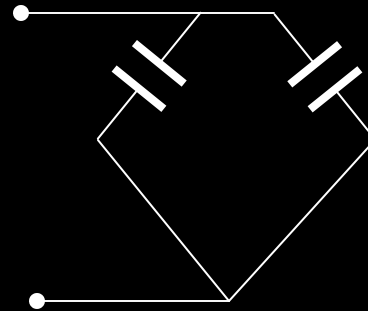
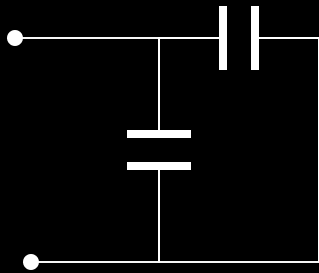
$$C_{eq} = C_1 + C_2$$

For parallel capacitors

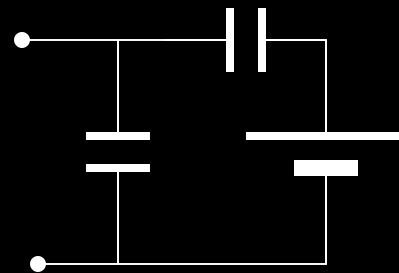
$$C_{eq} = \sum_{j=1}^n C_j$$



Parallel Connections

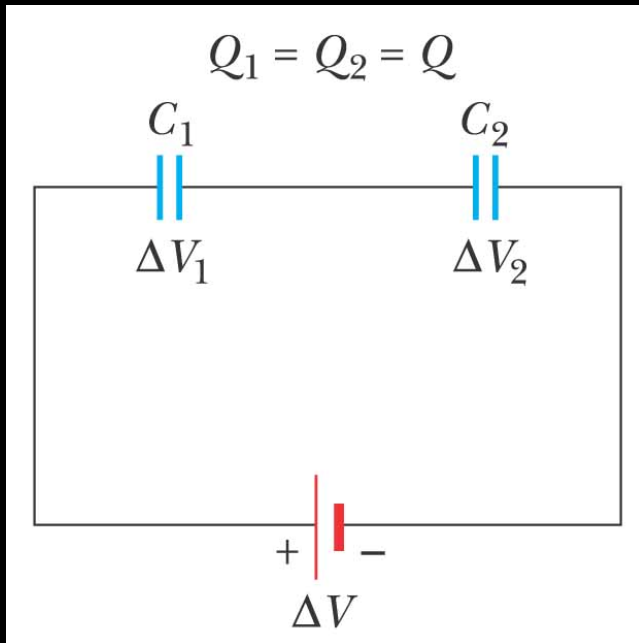


All are parallel connections



This is not

Series Combination



$$\Delta V = \Delta V_1 + \Delta V_2$$

$$Q = Q_1 = Q_2$$

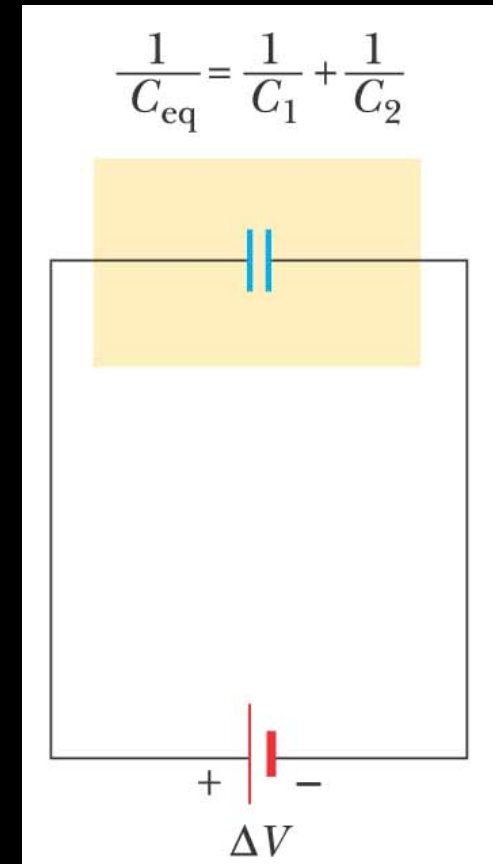
$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1}$$

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{Q}{C_2}$$

$$\Delta V = \frac{Q}{C_{eq}}$$

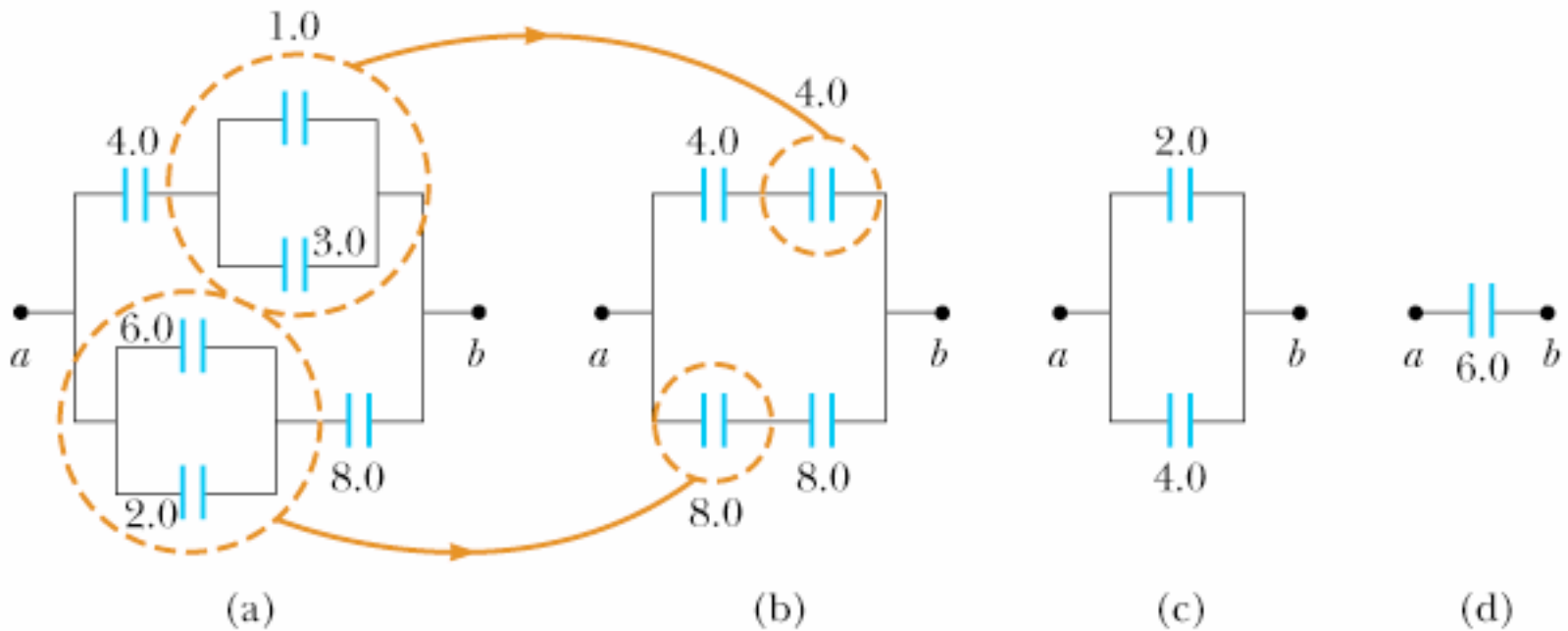
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^n \frac{1}{C_j}$$

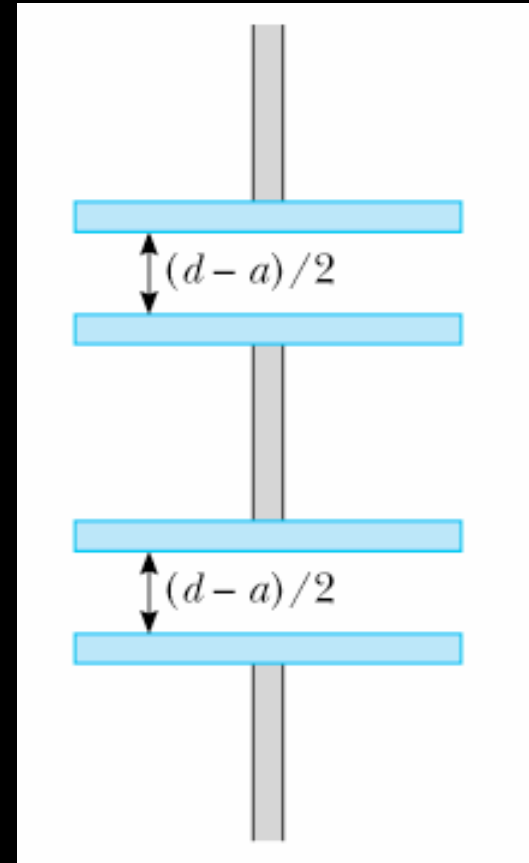
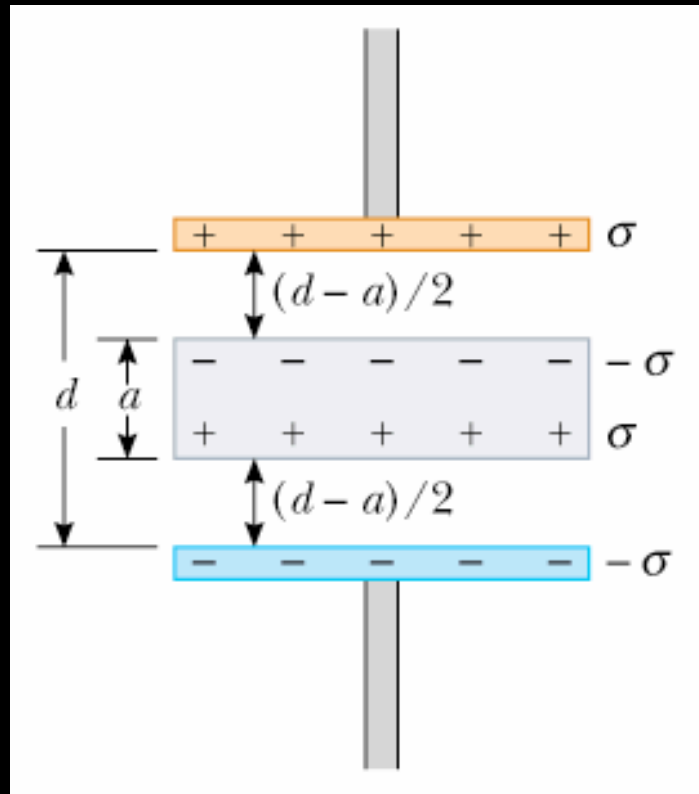


For series capacitors

Mixed Combination of Capacitors



Metallic Slab Inside a Capacitor



$$C = \frac{\epsilon_0 A}{d - a}$$

Energy Stored in a Capacitor

- Since capacitors can carry charge after a battery is disconnected, they can store energy.
- To calculate this energy, consider the work done to move a small amount of charge from one plate of the capacitor to the other.
- As more and more charges are moved, the work required increases.

$$dW = \Delta V dq = \frac{q}{C} dq$$

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

Concept Question

Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d . Suppose the plates are pulled apart until they are separated by a distance $D > d$. The electrostatic energy stored in the capacitor is

1. greater than
2. the same as
3. smaller than

before the plates were pulled apart.

Energy Density

- Defined as stored energy per unit volume.
- Consider the parallel plate capacitor.

$$U = \frac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2} (\epsilon_0 A d) E^2$$

$$\frac{\epsilon_0 A}{d}$$

$$(Ed)^2$$

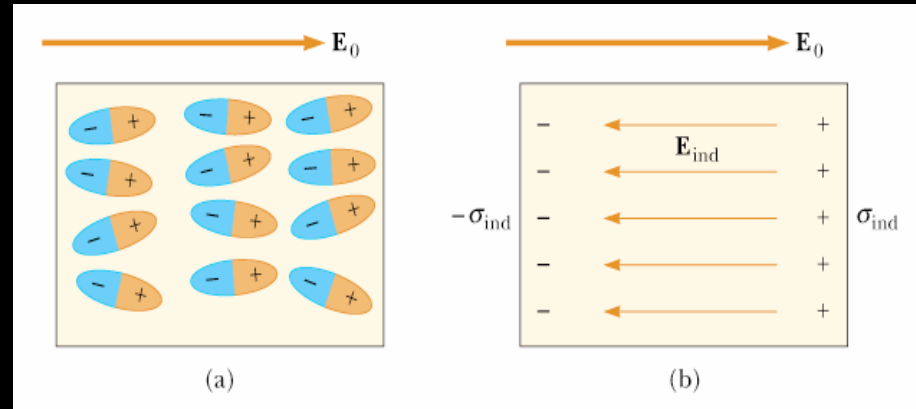
$$u_E = \frac{U}{Vol} = \frac{\frac{1}{2} (\epsilon_0 A d) E^2}{Ad}$$

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

Dielectrics

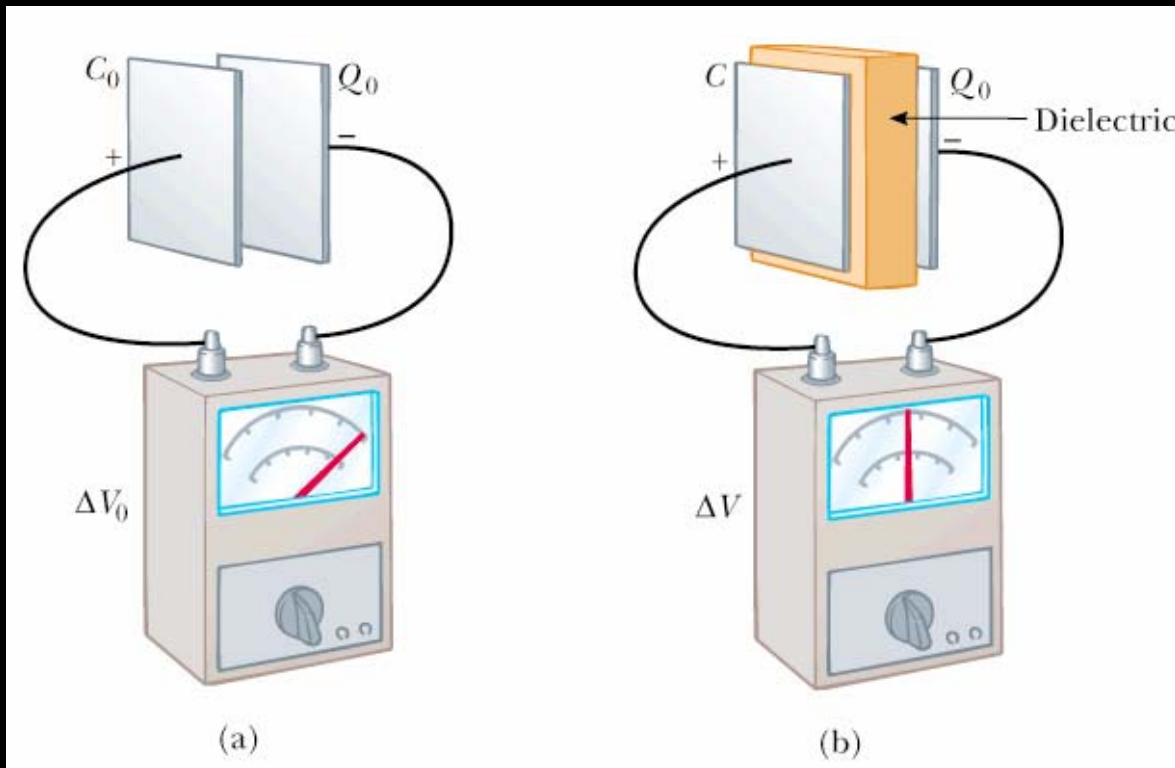
- Dielectrics are insulators like glass, rubber or wood.
- They are used to increase the charge storage capacity and/or maximum voltage in capacitors while providing mechanical support.
- They are characterized by a dielectric constant, κ , and are limited by their dielectric strength.

An Atomic View



- As we apply a voltage to the plates, the dielectric in between is polarized.
- The **dielectric constant** is a measure of the polarizability of the material.
- This creates a field in the opposite direction to the applied field and therefore reduces the effective voltage or under constant voltage allows a larger charge to be collected on the plates.
- The **dielectric strength** is the maximum electric field the material can withstand before breaking down (becoming a conductor).

Capacitors With Dielectrics



$$\Delta V = \frac{\Delta V_0}{\kappa}$$

$$C = \frac{Q_0}{\Delta V} = \kappa \frac{Q_0}{\Delta V_0}$$

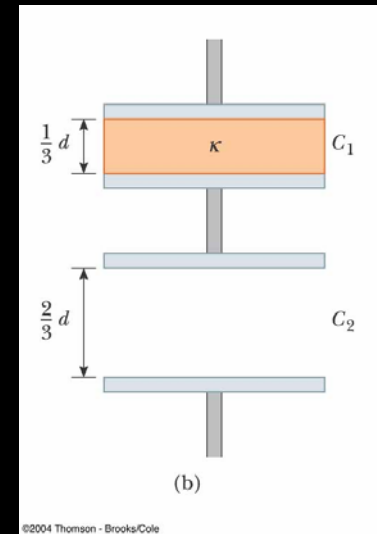
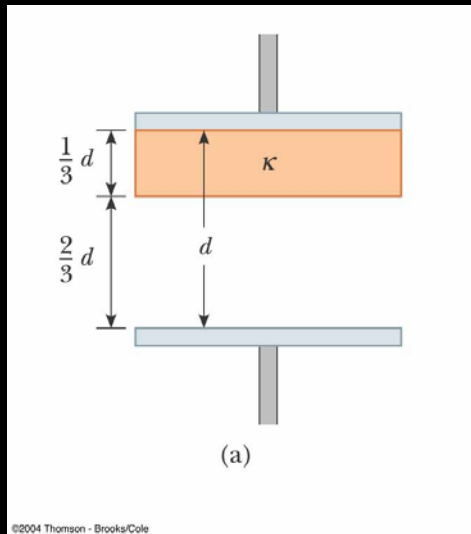
$$C = \kappa C_0$$

Concept Question

A parallel-plate capacitor is attached to a battery that maintains a constant potential difference V between the plates. While the battery is still connected, a glass slab is inserted so as to just fill the space between the plates. The stored energy

1. increases.
2. decreases.
3. remains the same.

A Partially Filled Capacitor



$$C = \frac{\kappa}{\frac{1}{3} + \kappa(1 - \frac{1}{3})} \frac{\epsilon_0 A}{d} = \frac{\kappa}{\frac{1}{3} + \frac{2}{3}\kappa} C_0$$

Summary

- Capacitors are geometries that can hold charges.
- Use Gauss' Law symmetries to calculate capacitance.
- Series and parallel connections.
- Dielectrics increase capacitance.

For Next Class

- Reading Assignment
 - Chapter 27 – Current and Resistance
- WebAssign: Assignment 4