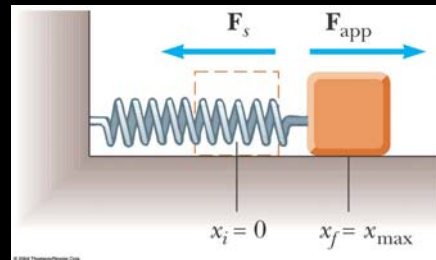


Electric Potential

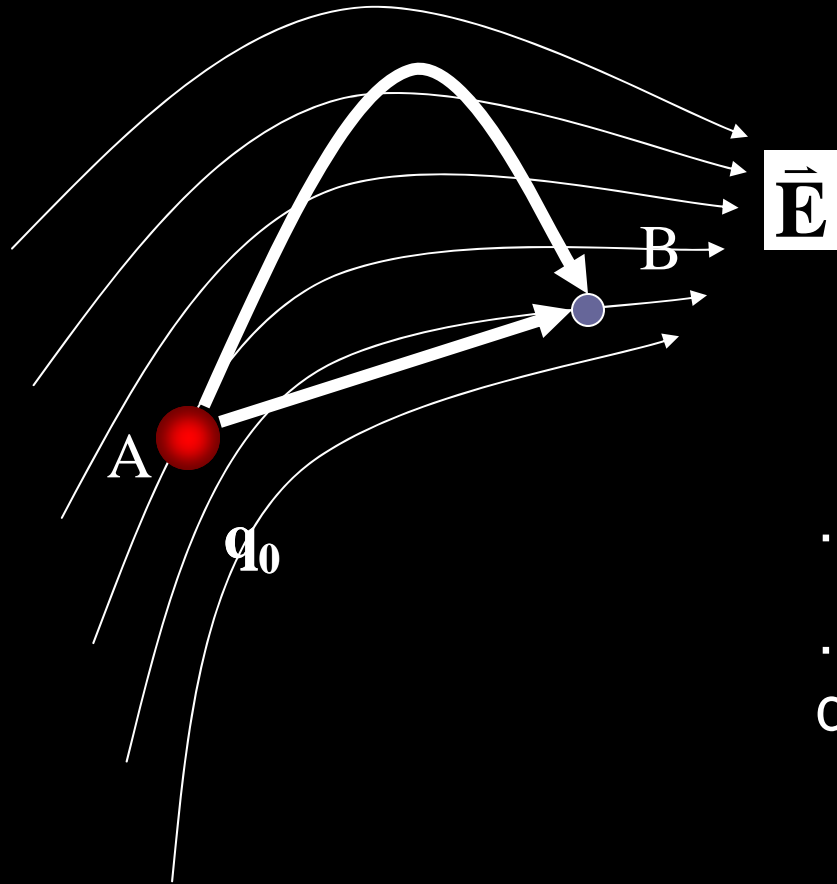
- Flashback to PHY113...
 - Work done by a conservative force is independent of the path of the object.
 - Gravity and elastic forces are examples.
 - This leads to the concept of potential energy and helps us avoid tackling problems using only forces.



$$W = \int_{x_i}^{x_f} F_s dx = -\Delta U$$

- Electrostatic forces are also conservative.

Electric Potential Energy



$$\vec{F} = q_0 \vec{E}$$

$$\Delta U = - \int_{path} \vec{F} \cdot d\vec{s} = -q_0 \int_{path} \vec{E} \cdot d\vec{s}$$

...but \mathbf{F} is a conservative force

...so the path we take
does not matter

$$\Delta U = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

Electric Potential

- Work, ΔU , is dependent on the magnitude of the test charge, q_0 .
- We'd like to have a quantity independent of the test charge and only an attribute of the electric field.

$$V = \frac{U}{q_0}$$

Electric Potential
(J/C=V)

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Potential Difference
between A and B

$$V_P = -\int_{\infty}^P \mathbf{E} \cdot d\mathbf{s}$$

Electric Potential at P

Unit Pit Stop

Potential

$$V = J/C$$

Energy

$$J = N \cdot m$$

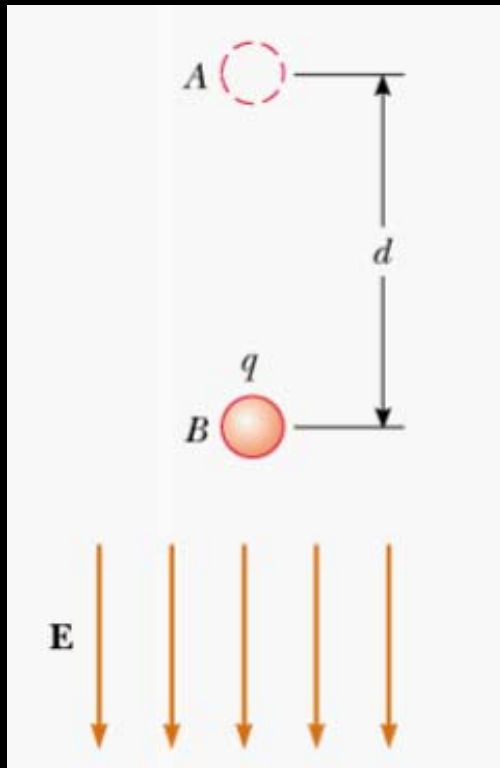
Electric Field

$$N/C = \frac{N}{C} \left(\frac{V \cdot C}{J} \right) \left(\frac{J}{N \cdot m} \right) = V/m$$

Electron-Volt

$$1eV = e(1V) = (1.6 \times 10^{-19} C)(1J/C) = 1.6 \times 10^{-19} J$$

Electric Potential in a Uniform Field



$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E \cos 0^\circ ds = -E \int_A^B ds$$

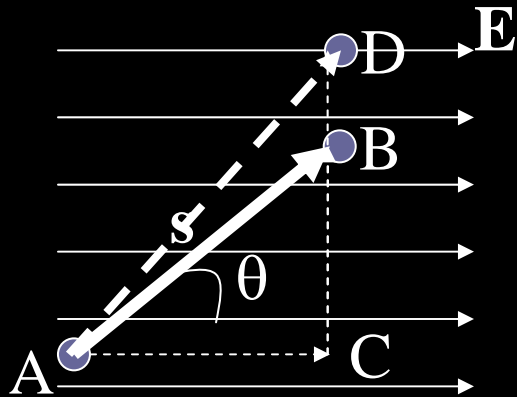
$$\Delta V = -Ed$$

$$\Delta U = -q_0 Ed$$

Electric field lines point to decreasing potential.

A positive charge will lose potential energy and gain kinetic energy when moving in the direction of the field.

Equipotential Surfaces



$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\mathbf{E} \cdot \int_A^B d\mathbf{s} = -\mathbf{E} \cdot \mathbf{s}$$

$$\Delta V = -(\mathbf{E} \cdot \mathbf{s})_{AC} - (\mathbf{E} \cdot \mathbf{s})_{CB}$$

$$\Delta V = -Es_{AC} \cos 0^\circ - Es_{CB} \cos 90^\circ$$

$$\Delta V = -Es \cos \theta$$

$$\Delta U = -q_0(\mathbf{E} \cdot \mathbf{s})_{AC} - q_0(\mathbf{E} \cdot \mathbf{s})_{CB}$$

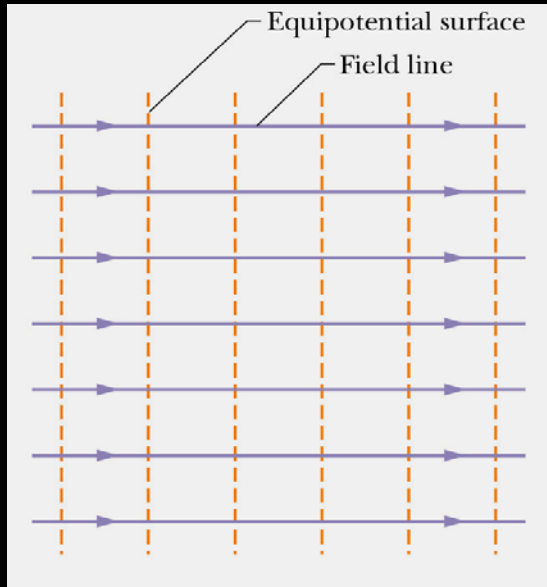
$$\Delta U = -q_0 Es_{AC} \cos 0^\circ$$

$$\Delta U = -q_0 Es \cos \theta$$

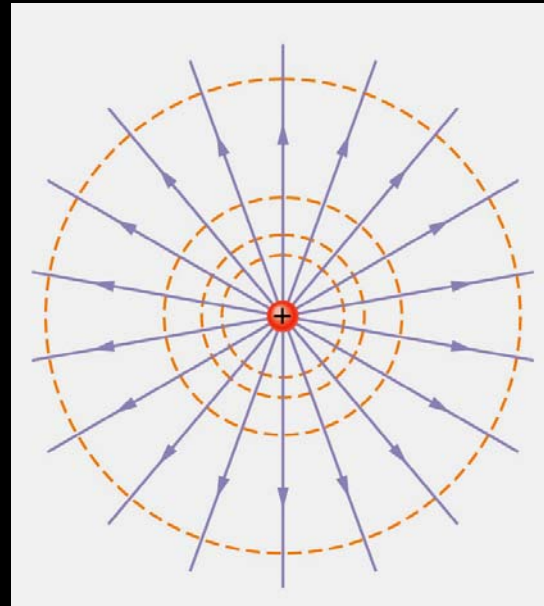
$$\Delta V_{AD} = -\int_A^D \mathbf{E} \cdot d\mathbf{s} = -\mathbf{E} \cdot \int_A^D d\mathbf{s} = -\mathbf{E} \cdot \mathbf{s} = -(\mathbf{E} \cdot \mathbf{s})_{AC} - (\mathbf{E} \cdot \mathbf{s})_{CD} = -Es \cos \theta$$

No work is done moving a charged particle perpendicular to a field (along equipotential surfaces)

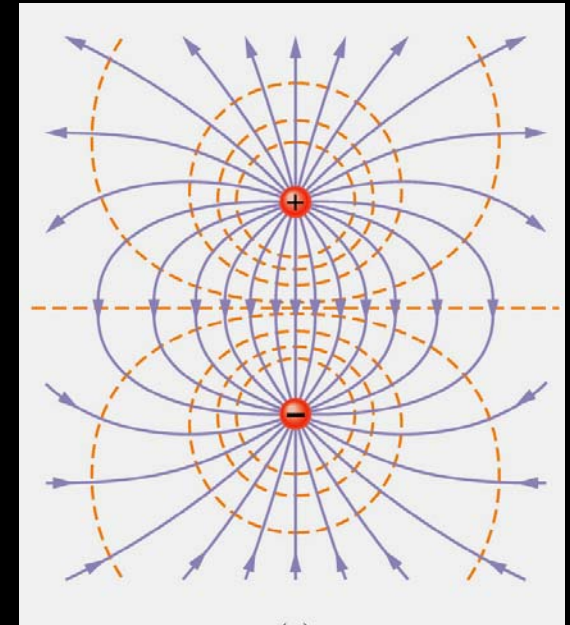
Equipotential Surfaces



Uniform Field

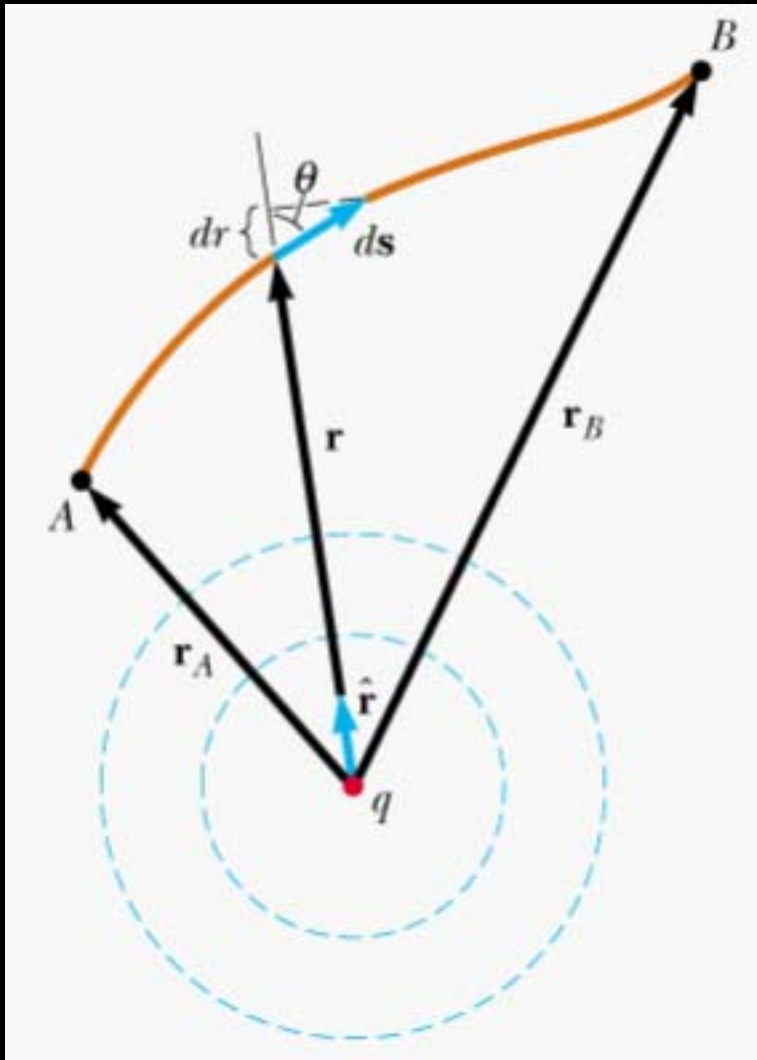


Point Charge



Electric Dipole

Electric Potential of a Point Charge



$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s} = k_e \frac{q}{r^2} ds \cos \theta = k_e \frac{q}{r^2} dr$$

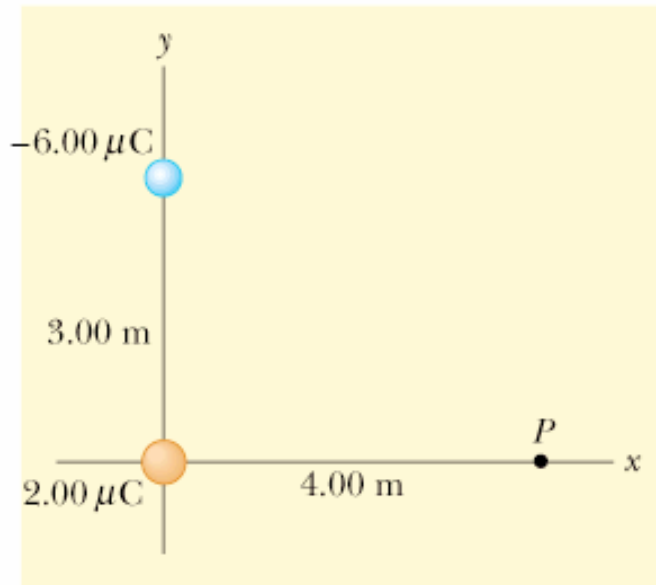
$$V_B - V_A = -\int E_r dr$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

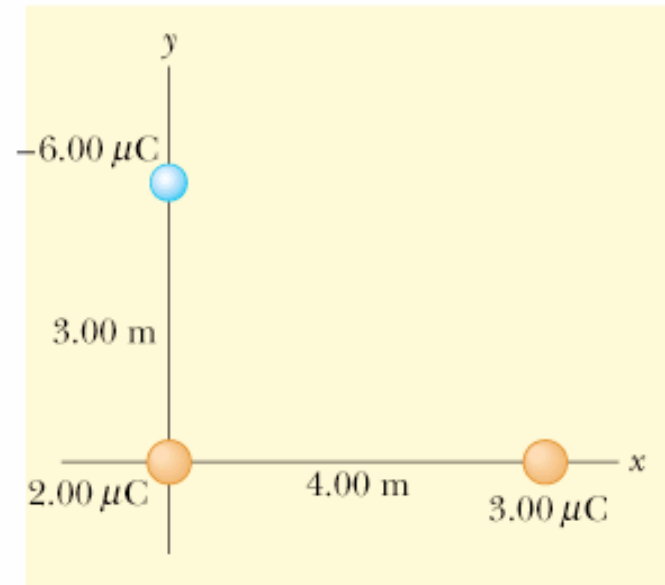
$$V_{A=\infty} = 0$$

$$V = k_e \frac{q}{r}$$

Two Point Charges



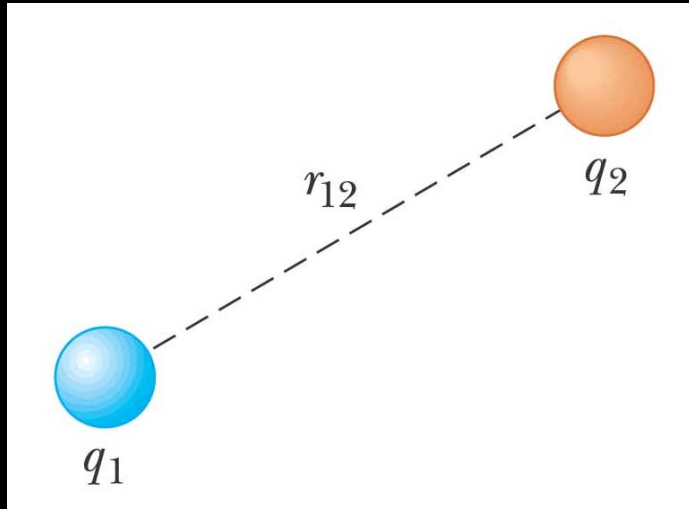
(a)



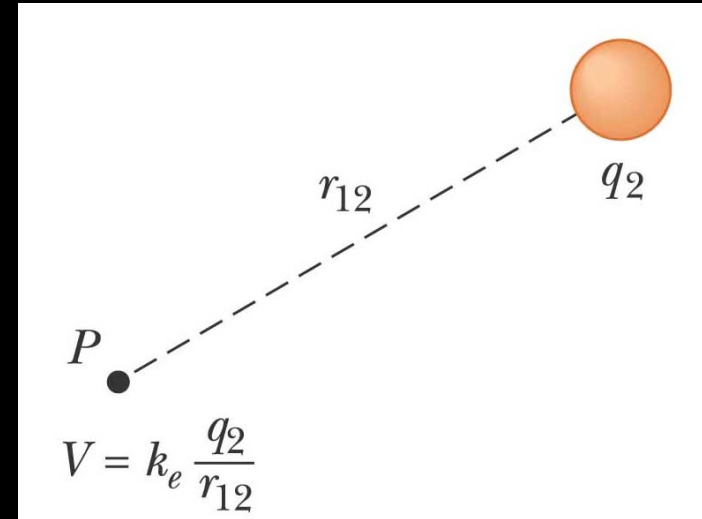
(b)

$$V_P = k_e \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right)$$

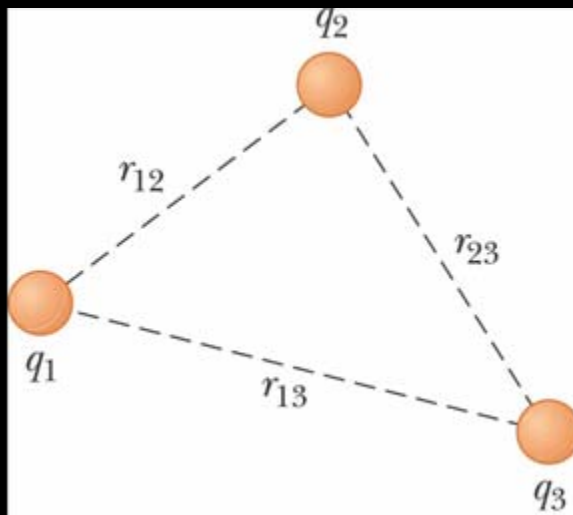
A System of Point Charges



$$U = k_e \frac{q_1 q_2}{r_{12}}$$



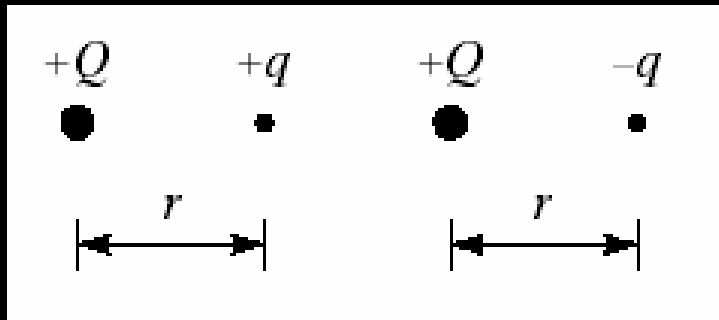
$$V = k_e \frac{q_2}{r_{12}}$$



$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}} \right)$$

Concept Question

Two test charges are brought separately into the vicinity of a charge $+Q$. First, test charge $+q$ is brought to a point a distance r from $+Q$. Then this charge is removed and test charge $-q$ is brought to the same point. The electrostatic potential energy of which test charge is greater:



1. $+q$
2. $-q$
3. It is the same for both.

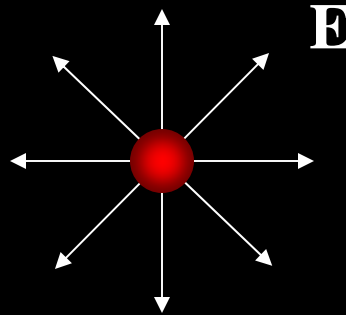
Getting From V to E

$$V = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$dV = -\mathbf{E} \cdot d\mathbf{s}$$



$$E_x = -\frac{dV}{dx}$$



$$E_r = -\frac{dV}{dr}$$

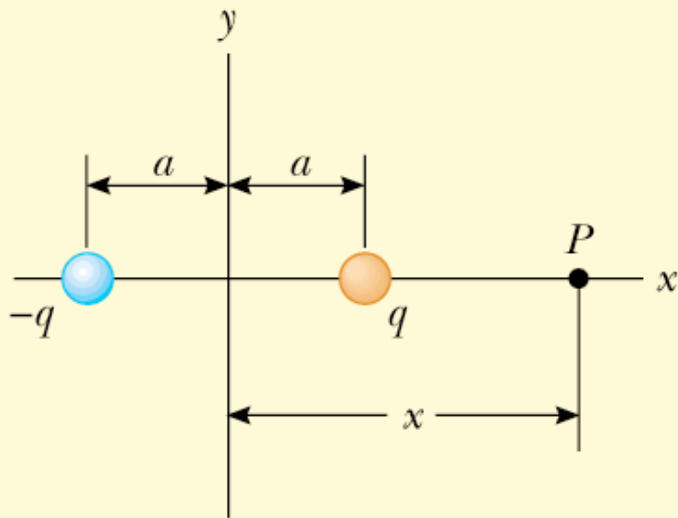
In general:

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

Electric Potential of a Dipole



$$V_P = \frac{2k_e qa}{x^2 - a^2}$$

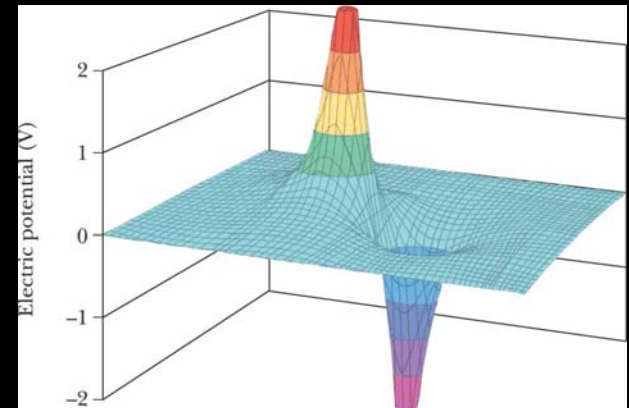
$$V \approx \frac{2k_e qa}{x^2} \quad (x \gg a)$$

$$E_x = -\frac{dV}{dx} = \frac{4k_e qa}{x^3}$$

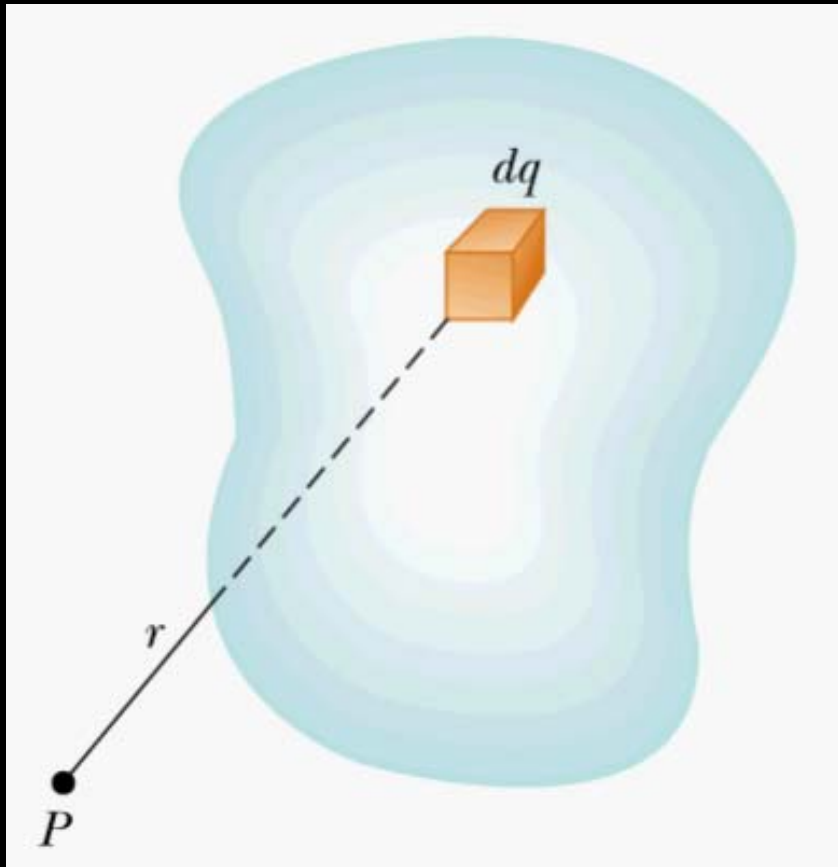
Between the Charges

$$V_P = \frac{2k_e qx}{a^2 - x^2}$$

$$E_x = -\frac{dV}{dx} = -2k_e q \left(\frac{a^2 + x^2}{(a^2 - x^2)^2} \right)$$



Electric Potential Due to Continuous Charge Distributions



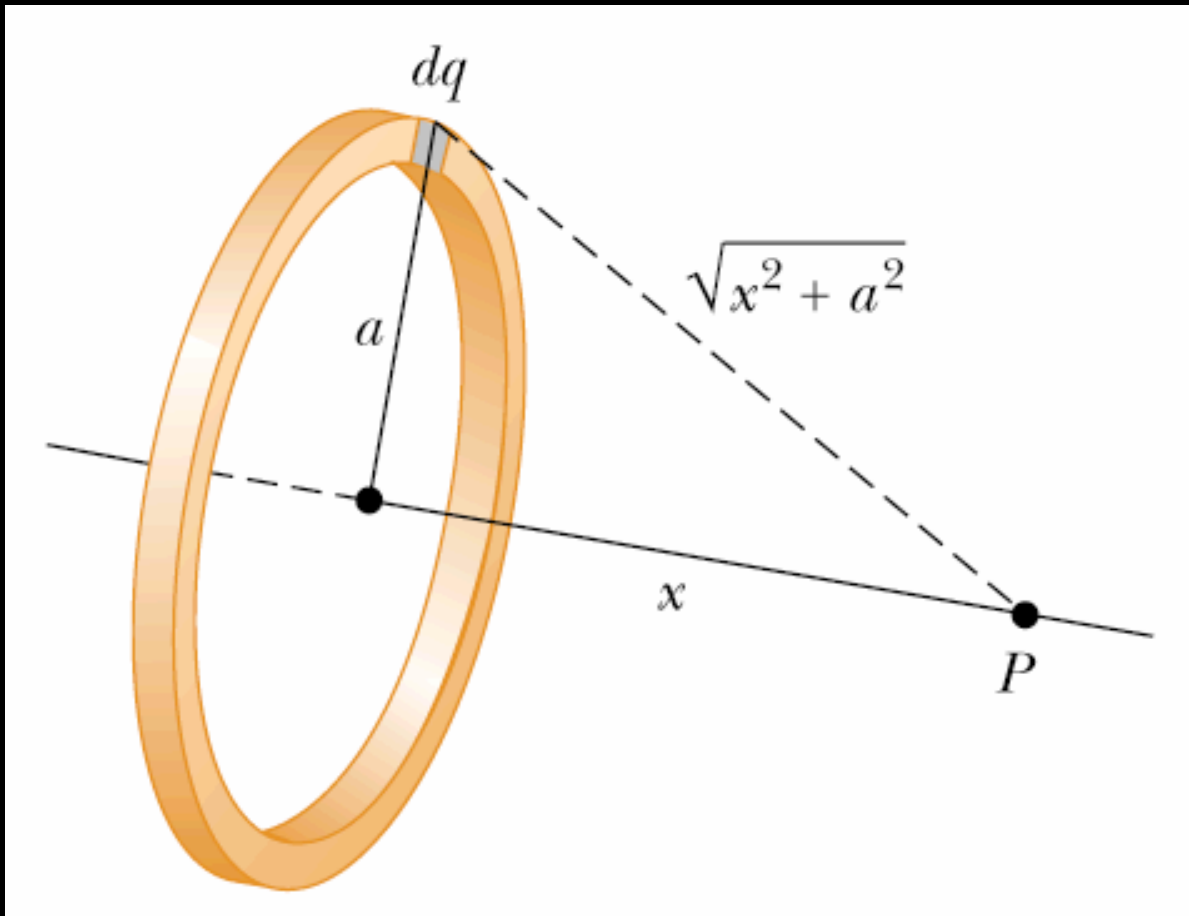
Start with an infinitesimal charge, dq .

$$dV = k_e \frac{dq}{r}$$

Then integrate over the whole distribution

$$V = k_e \int \frac{dq}{r}$$

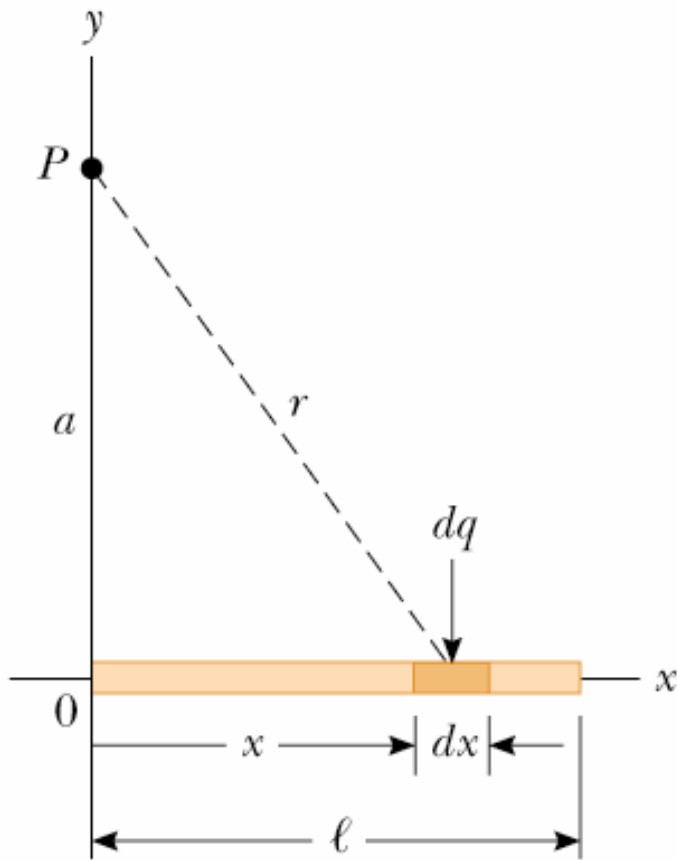
Electric Potential Due to a Uniformly Charged Ring



$$V = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

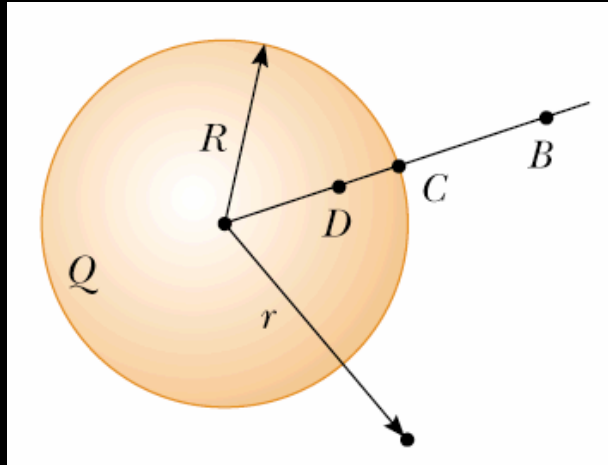
$$E = \frac{k_e Q x}{(x^2 + a^2)^{3/2}}$$

Electric Potential Due to a Finite Line of Charge



$$V = \frac{k_e Q}{l} \ln \left(\frac{l + \sqrt{l^2 + a^2}}{a} \right)$$

Electric Potential Due to a Uniformly Charged Sphere



$$E_r = \frac{k_e Q}{R^3} r$$

$r < R$

$$V_D - V_C = -\int_R^r E_r dr = \frac{k_e Q}{2R^3} (R^2 - r^2)$$

$$V_D = \frac{k_e Q}{2R} \left(3 - \frac{r^2}{R^2} \right)$$

$$E_r = k_e \frac{Q}{r^2}$$

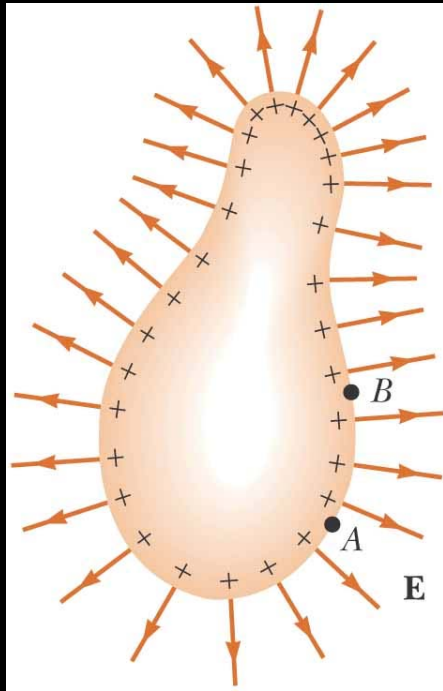
$r > R$

$$V_B = -\int_{\infty}^r E_r dr = -k_e Q \int_{\infty}^r \frac{dr}{r^2}$$

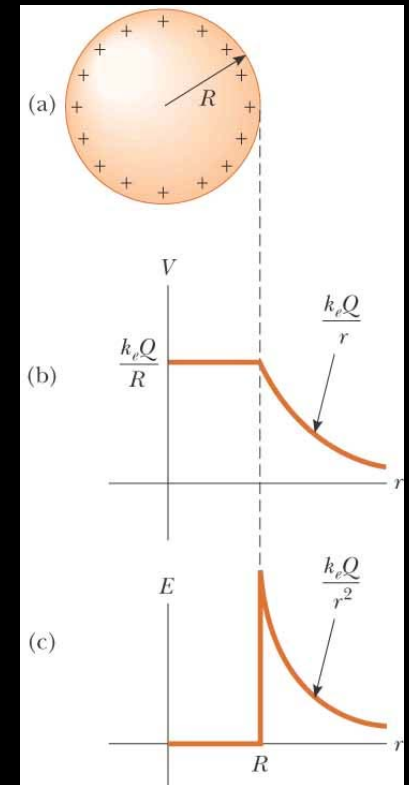
$$V_B = k_e \frac{Q}{r}$$

$$V_C = k_e \frac{Q}{R}$$

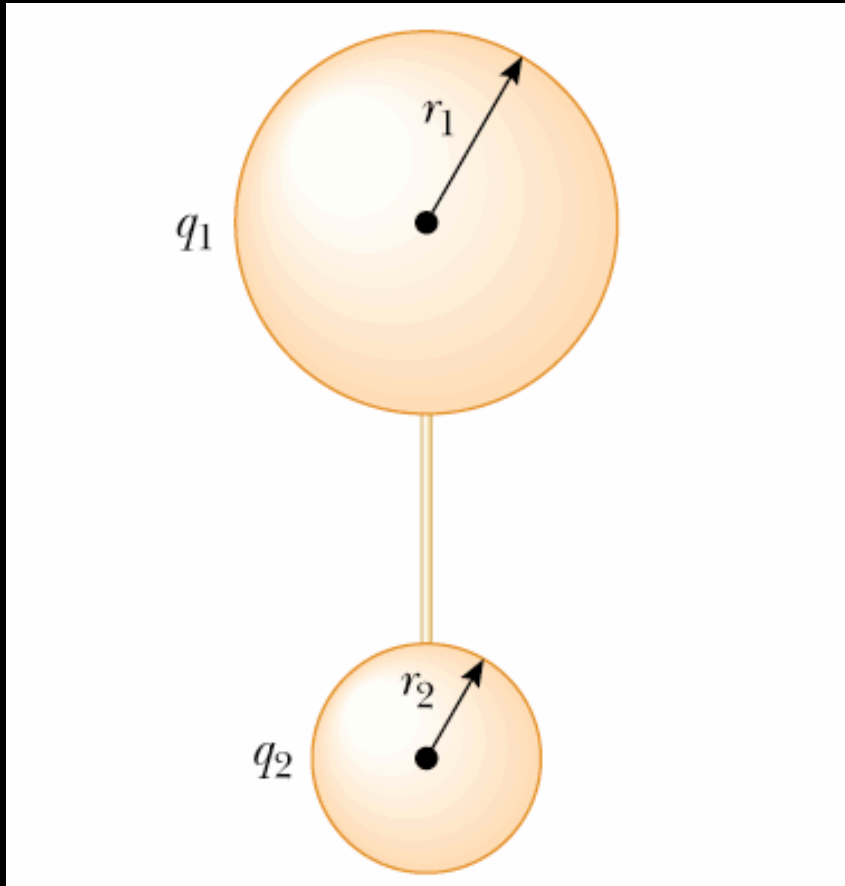
Potential Due to a Charged Conductor



- Charges always reside at the outer surface of the conductor.
- The field lines are always perpendicular to surface.
- Then $\mathbf{E} \cdot d\mathbf{s} = 0$ on the surface at any point.
- Which means, $V_B - V_A = 0$ along the surface.
- The surface is an equipotential surface.
- Finally, since $\mathbf{E} = 0$ inside the conductor, the potential V is constant and equal to the surface value.



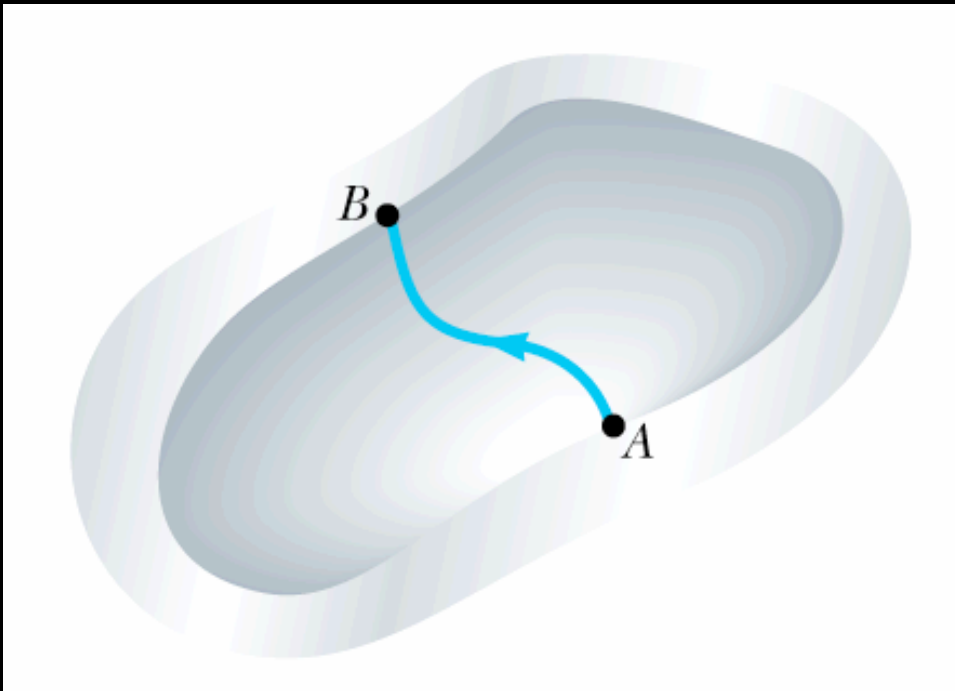
Connected Charged Conducting Spheres



$$\frac{q_1}{q_2} = \frac{r_1}{r_2}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

Cavity Within a Conductor



$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

We can always find a path where $\mathbf{E} \cdot d\mathbf{s}$ is non-zero.

But, since $\Delta V = 0$ for all paths, \mathbf{E} must be zero everywhere in the cavity.

A cavity without any charges enclosed by a conducting wall is field free.

Summary

- Charges can have different electric potential energy at different points in an electric field.
- Electric potential is the electric potential energy per unit charge.
- All points inside a conductor are at the same potential.

For Next Class

- Reading Assignment
 - Chapter 26 – Capacitance and Dielectrics
- WebAssign: Assignment 3