PHY 114 – Summer 2009 – Final Exam Solutions

Conceptual Question 1:

A spherical rubber balloon has a charge uniformly distributed over its surface. As the balloon is inflated, how does the electric field E vary

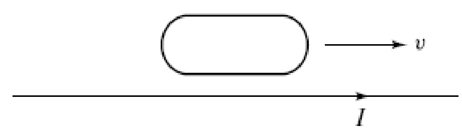
(a) outside the balloon, at some point well away from the surface?
(b) at the outer surface of the balloon?
(c) inside the balloon?

Assume the balloon remains spherical during inflation.

(a) Since the field outside for a spherically symmetric charge distribution is essentially identical to a point charge at the center of the distribution, the field far away stays the same.
(b) As the radius of the balloon increases, the field on the surface decreases as the square of the radius.
(c) Since there are no charges inside the balloon, the flux is zero. And since there is spherical symmetry, the field is zero as well.

Conceptual Question 2:

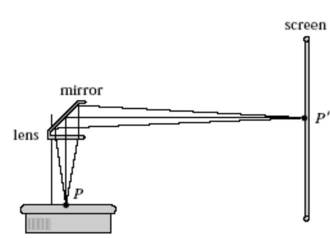
In the following figure, a closed loop moves at a constant speed parallel to a long, straight, current-carrying wire. Is there a current in the loop? If so, is this current circulating clockwise or counterclockwise?



The magnetic field created by the current in the straight wire is a function of the radial distance from the wire. However, as the loop moves parallel to the wire, and does not get closer or farther away, the flux through it does not change over time. Therefore, there is no induced current in the loop.

Conceptual Question 3:

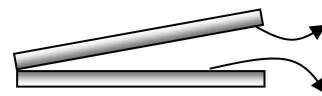
The lens in an overhead projector forms an image P` of a point P on an overhead transparency. If the screen is moved closer to the projector, how must we adjust the lens to keep the image on the screen in focus?



Since the screen is moved closer, the image distance is being decreased. To compensate, the object distance must increase and therefore, the lens must be moved up.

Conceptual Question 4:

Two glass slides form a narrow wedge as shown in the figure. Taking into account only the reflections from surfaces A and B, describe how the interference pattern will look like when the slides are illuminated with white light and the pattern is viewed from above.



Interference occurs between waves that reflect from A and B so that the spacing between the two slides acts as the thin film. The light reflected from A does not have a 180° phase change upon reflection while the light reflected from B does. Therefore the condition for constructive interference is: $2t = (m + 1/2)\lambda$ and the condition for destructive interference is: $2t = m\lambda$. At the point where the two slides meet the destructive interference condition is satisfied for every wavelength (t = 0) so we will see a dark fringe. As the distance between the slides increases, the condition for constructive interference will be satisfied for light of larger wavelengths. So looking from above we will see a succession of fringes with a rainbow of colors interrupted by white or dark bands where the interference conditions are satisfied by multiple wavelengths.

Conceptual Question 5:

Light of frequency *f* illuminating a long narrow slit produces a diffraction pattern.

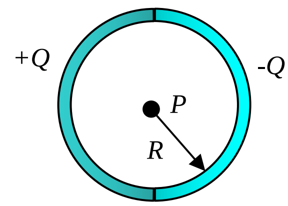
(a) If we switch to light of frequency 1.3*f*, does the pattern expand away from the center or contract toward it?

(b) Does the pattern expand or contract if, instead, we submerge the equipment in clear corn syrup?

(a) If the frequency of light is increased, its wavelength will be less. Since the diffraction pattern is proportional to the ratio of the wavelength to the slit width, the pattern will contract (less diffraction).

(b) Since the wavelength of light is going to be shorter in another medium with higher index of refraction, again the pattern will contract.

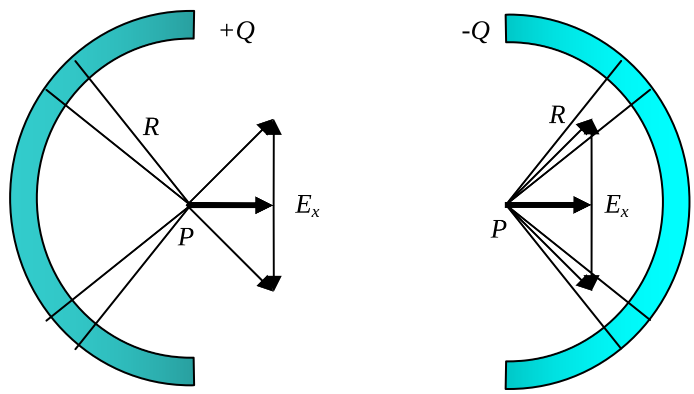
Problem 1:



In the figure two semicircular curved plastic rods, one uniformly charged with +*Q*, the other uniformly charged with -*Q*, form a circle of radius *R* centered at *P*.

(a) What is the magnitude and direction of the electric field at *P*?

(b) What is the electric potential at *P*?



This is very similar to the practice test question. We can treat each semi-circle separately. For the +*Q* half, dividing the arc into infinitesimal pieces with length *dl*, charge *dq* and subtending an angle dθ, we can see that every piece above the horizontal has a symmetric counterpart below the horizontal for which the vertical components of the electric field cancel leaving only the horizontal (x) component. Also, since the charge is positive, the field at *P* points away from the arc. In the same way the –*Q* half will only produce a field that has an x component pointing toward the negative half. Therefore the field contributions from the two sides will be equal in magnitude and in the same direction, so they will add. Looking at the left half (+*Q*):

(a) The rod length is: $l = \pi R$. The charge density is, $\lambda = Q/l$. Then, $dq = \lambda dl = \lambda R d\theta$

$$dE = k_e \frac{dq}{R^2} \text{ and } dE_x = k_e \frac{dq}{R^2}\cos\theta = k_e \frac{\lambda r d\theta}{R^2}\cos\theta$$

$$E_x = \int_{-90^\circ}^{90^\circ} k_e \frac{\lambda r d\theta}{R^2}\cos\theta = k_e \frac{\lambda}{R}\int_{-90^\circ}^{90^\circ}\cos\theta d\theta = 2k_e \frac{\lambda}{R} = 2k_e \frac{Q}{\pi R^2}$$

The total field is: $\boxed{E_T = 2E_x = 4k_e \frac{Q}{\pi R^2}}$, pointing towards the +x-direction.

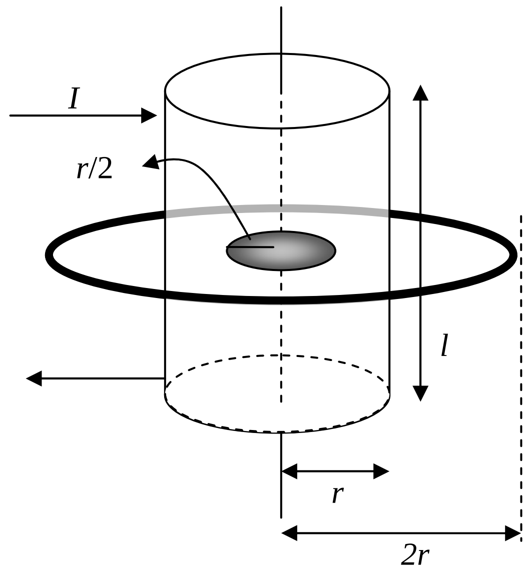
(b) For potential, again, each semicircle can be treated separately. For the left half,

$$dV = k_e \frac{dq}{R} = k_e \frac{\lambda R d\theta}{R} \text{ and } V_L = \int_{-\pi/2}^{\pi/2} k_e \lambda d\theta = k_e \lambda \int_{-\pi/2}^{\pi/2} d\theta = k_e \pi\lambda = k_e \frac{Q}{R}$$

For the right half, $V_R = \int_{-\pi/2}^{\pi/2} k_e \lambda d\theta = k_e \lambda \int_{-\pi/2}^{\pi/2} d\theta = k_e \pi\lambda = -k_e \frac{Q}{R}$

Therefore the total potential at *P* is: $\boxed{V_T = V_L + V_R = 0}$

Problem 2:



A solenoid of radius r and length l has N turns and carries a current I.

(a) Calculate the flux through the surface of a disk of radius $r/2$ that is positioned perpendicular to and centered on the axis of the solenoid.

(b) If the current in the solenoid is increasing at a rate of 0.5 A/s, what is the induced current in a metallic ring positioned perpendicular to and centered on the axis of the solenoid? The ring has a radius $2r$ and resistance per unit length of R_L.

(a) The field produced by the solenoid is ideally a uniform field, confined to the inside of the solenoid and directed along the axis of the solenoid.

$$B_{sol} = \mu_0 n I = \mu_0 \left(\frac{N}{l}\right) I \,.$$

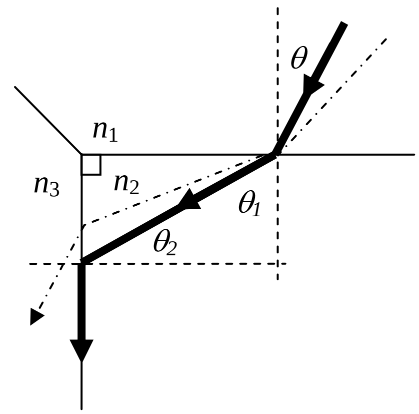
The flux through the disc is: $\boxed{\Phi_B = \int_{disk} \vec{B}_{sol} . d\vec{A} = B_{sol} A_{disk} = \mu_0 \left(\frac{N}{l}\right) I \pi \left(\frac{r}{2}\right)^2 = \frac{\mu_0 N \pi r^2}{4l} I}$

(b) The induced emf is equal to the rate of change of the magnetic flux through the ring. For the area we only take the area of the solenoid because the field is confined within the solenoid.

$$E = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -\pi r^2 \frac{d}{dt}\left[\mu_0 \left(\frac{N}{l}\right) I\right] = -\pi r^2 \mu_0 \left(\frac{N}{l}\right) \frac{dI}{dt} = -0.5\pi r^2 \mu_0 \left(\frac{N}{l}\right)$$

The current is, $\boxed{I = \frac{E}{R} = -0.5\pi r^2 \mu_0 \left(\frac{N}{l}\right) \frac{1}{R_L (2\pi 2r)} = -\frac{r \mu_0 N}{8 l R_L}}$

Problem 3:



In the arrangement at the right, light is initially in material 1 and is incident on material 2. After refraction, it travels through material 2 and is incident on material 3 at the critical angle. The indexes of refraction are, n_1 = 1.6, n_2 = 1.4, n_3 = 1.2.

(a) Find θ.

(b) If θ is increased, will the light ray escape into material 3?

(a) The critical angle between 2 and 3 is: $\theta_c = \arcsin\left(\frac{n_3}{n_2}\right) = \arcsin\left(\frac{1.2}{1.4}\right) = 59°$ and $\theta_2 = \theta_c$

Then $\theta_1 = 90° - \theta_2 = 31°$ and $\boxed{\theta = \arcsin\left(\frac{n_2}{n_1}\sin\theta_1\right) = \arcsin\left(\frac{1.4}{1.6}\sin 31°\right) = 26.8°}$

(b) If θ is increased, θ_1 increases and θ_2 will decrease. Being incident at less than the critical angle, the light will escape in to material 3. See the dash-dot line as an example.

Problem 4:

Two parallel slits are illuminated with monochromatic light of wavelength 500 nm. An interference pattern is formed on a screen some distance from the slits, and the fourth dark band is located 1.68 cm from the central bright band on the screen.

(a) What is the path length difference corresponding to the fourth dark band?

(b) What is the distance on the screen between the central bright band and the first bright band?

Assume that the angles involved are small enough that $\sin\theta \approx \tan\theta$.

(a) The fourth dark band occurs at m = 3. $\boxed{\delta = (m + 1/2)\lambda = \frac{7}{2}(500nm) = 1750nm}$

(b) For dark bands, $y_m = (m + 1/2)\frac{\lambda L}{d}$.

For the fourth dark band, $y_3 = \frac{7\lambda L}{2d} = \frac{7(500nm)(L)}{2d} = (1750nm)\frac{L}{d} = 1.68cm$.

Then, $\frac{L}{d} = 9600$.

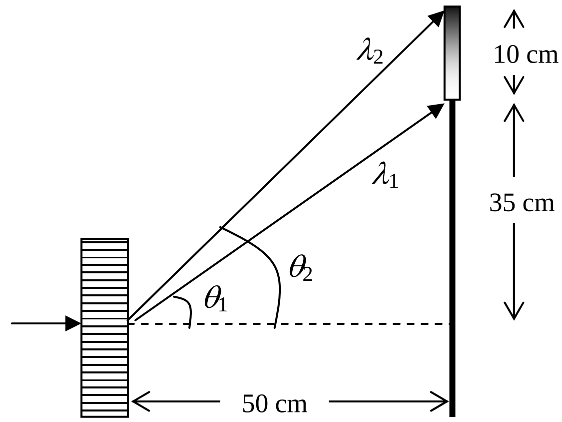
For the first bright band, $\boxed{y = \frac{\lambda L}{d} = (500nm)(9600) = 4.8mm}$

Problem 5:

In a spectrometer, a grating that has 1000 lines/mm is used to send an unknown spectrum of light to a 10 cm long detector array that is placed on a screen. The screen is 50 cm away and the detector can detect the full extent of the first order spectrum when its lower end is 35 cm away from the center of the slit.

(a) Find the minimum and maximum wavelengths of the spectrum.

(b) Does any part of the spectrum diffract into the second order?



The grating's slit spacing is, $d = 1/N = (1/1000)mm = 1\mu m$

(a) For the shortest wavelength, $\theta_1 = \arctan\dfrac{35cm}{50cm} = 35°$.

Then, $\boxed{d\sin\theta_1 = \lambda_1 = (1\mu m)\sin 35° = 573.5nm}$

For the longest wavelength, $\theta_2 = \arctan\dfrac{(35+10)cm}{50cm} = 42°$

Then, $\boxed{d\sin\theta_2 = \lambda_2 = (1\mu m)\sin 42° = 669nm}$

(b) In the second order, for the short wavelength side of the spectrum, $d\sin\theta = 2\lambda_2$.

$\sin\theta = \dfrac{2\lambda_1}{d} = \dfrac{2(573.5nm)}{1\mu m} = 1.147 > 1$ then no diffraction occurs in the second order.

Multiple Choice Questions:

1. A battery is used to charge a parallel-plate capacitor, after which it is disconnected. Then the plates are pulled apart to twice their original separation. This process will double the:
   a) capacitance
   b) surface charge density on each plate
   c) stored energy
   d) electric field between the two places
   e) charge on each plate

Since the battery is disconnected, the charge remains constant. Doubling the plate separation halves the capacitance and doubles the stored energy $\left(U = Q^2/2C\right)$

2. The rate at which electrical energy is used may be measured in:
   a) watt/second
   b) watt·second
   c) watt
   d) joule·second
   e) kilowatt·hour

The rate of energy change is power which is in units of watts.

3. Lines of the magnetic field produced by a long straight wire carrying a current:
   a) are in the direction of the current
   b) are opposite to the direction of the current
   c) leave the wire radially
   d) are circles concentric with the wire
   e) are lines similar to those produced by a bar magnet

The magnetic field will loop on itself and be perpendicular to the current that produces it.

4. In a purely resistive circuit the current:
   a) leads the voltage by 1/4 cycle
   b) leads the voltage by 1/2 cycle
   c) lags the voltage by 1/4 cycle
   d) lags the voltage by 1/2 cycle
   e) is in phase with the voltage

Since a resistor does not store energy, the current and voltage are in phase.

5. Maxwell's equations predict that the speed of light in free space is:
   a) an increasing function of frequency
   b) a decreasing function of frequency
   c) independent of frequency
   d) a function of the distance from the source
   e) a function of the size of the source

The speed of light in free space is constant.

6. Where must an object be placed in front of a converging lens in order to obtain a virtual image?
   a) At the focal point
   b) At twice the focal length
   c) Greater than the focal length
   d) Between the focal point and the lens
   e) Between the focal length and twice the focal length

When the object distance is less than the focal length, the image distance is negative and the image is virtual.

7. In a Young's double-slit experiment, the slit separation is doubled. This results in:
   a) an increase in fringe intensity
   b) a decrease in fringe intensity
   c) a halving of the wavelength
   d) a halving of the fringe spacing
   e) a doubling of the fringe spacing

If the slit separation is doubled, the angle should be halved for a given interference condition.

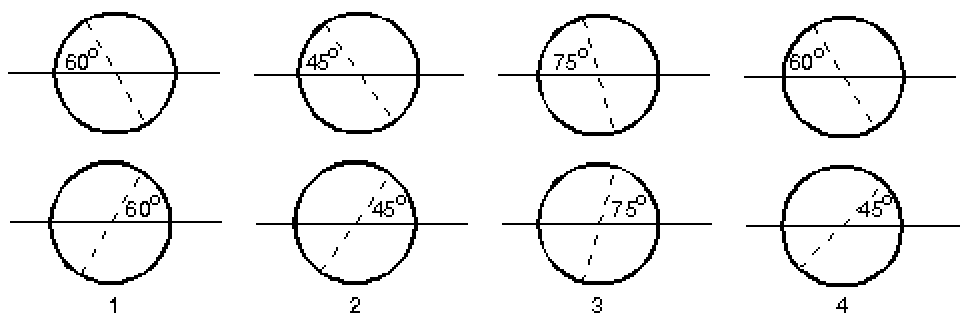
8. Monochromatic light, at normal incidence, strikes a thin film in air. If λ denotes the wavelength in the film, what is the thinnest film in which the reflected light will be a maximum?
   a) much less than λ
   b) λ/4
   c) λ/2
   d) 3λ/4
   ε) λ

In this case there is a 180° phase difference between the light reflected from the top and the bottom of the film so the constructive interference condition is $2nt = (m + 1/2)\lambda$. For minimum thickness, $m = 0$ and $t = \lambda/4n = \lambda_n/4$.

9. Two stars that are close together are photographed through a telescope. The black and white film is equally sensitive to all colors. Which situation would result in the most clearly separated images of the stars?
   a) Small lens, red stars
   b) Small lens, blue stars
   c) Large lens, red stars
   d) Large lens, blue stars
   e) Large lens, one star red and the other blue

The Rayleigh criterion for circular apertures is inversely proportional to lens diameter and directly proportional to wavelength.

10. The diagrams show four pairs of polarizing sheets, with the polarizing directions indicated by dashed lines. The two sheets of each pair are placed one behind the other and the front sheet is illuminated by unpolarized light. The incident intensity is the same for all pairs of sheets. Rank the pairs according to the intensity of transmitted light, least to greatest.



   a) 1, 2, 3, 4
   b) 4, 2, 1, 3
   c) 2, 4, 3, 1
   d) 2, 4, 1, 3
   e) 3, 1, 4, 2

Since intensity is proportional to the square of the cosine of the angle between the two polarizers, the order should be from the largest angle to the smallest angle. (1: 60°, 2: 90°, 3: 30°, 4: 75°)