Review of Mechanical Waves

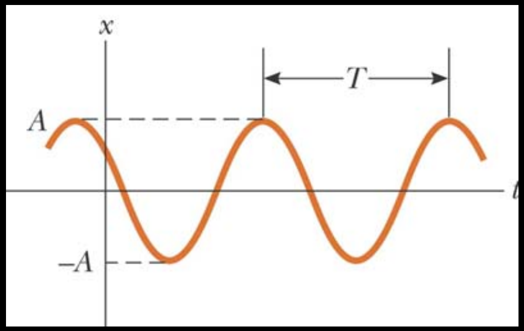
- Oscillations
- Traveling waves
- Sound Waves
- Wave interactions

Harmonic Oscillations

- When a restoring force acts on a mechanical system (spring on a block, gravity on a pendulum), the resultant motion is a simple harmonic motion.

$$x(t) = A\cos(\omega t + \phi)$$

- Harmonic oscillators move with an amplitude, frequency and period.



Frequency
(1/s=Hz)

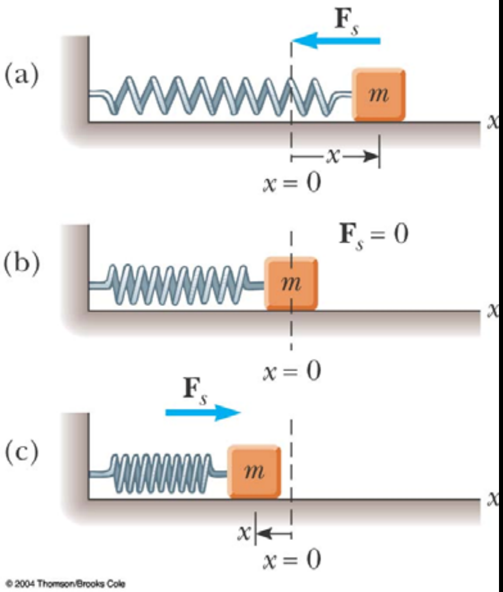
$$f = \frac{\omega}{2\pi}$$

Period
(s)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Harmonic Oscillation Systems

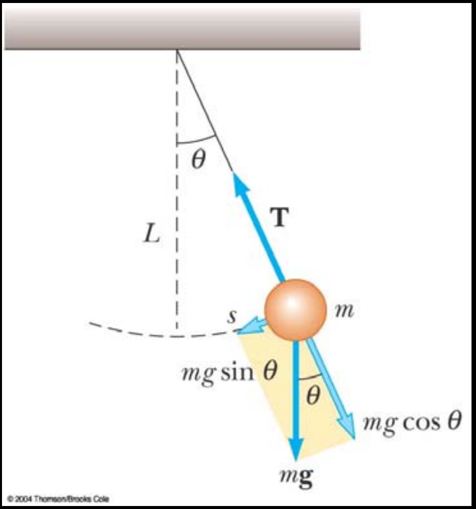
Spring-block system



$$x(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Simple pendulum



$$\theta = \theta_{\max}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{L}}$$

Energy of Harmonic Oscillators

- The total mechanical energy of a harmonic oscillator stays constant.

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}{}^2$$

for the spring-block system

- This energy is transferred back and forth between a potential energy (elastic / gravitational) and kinetic energy.
- The object undergoing harmonic motion will have maximum kinetic energy at the equilibrium position and maximum potential energy at the extremes of its motion.

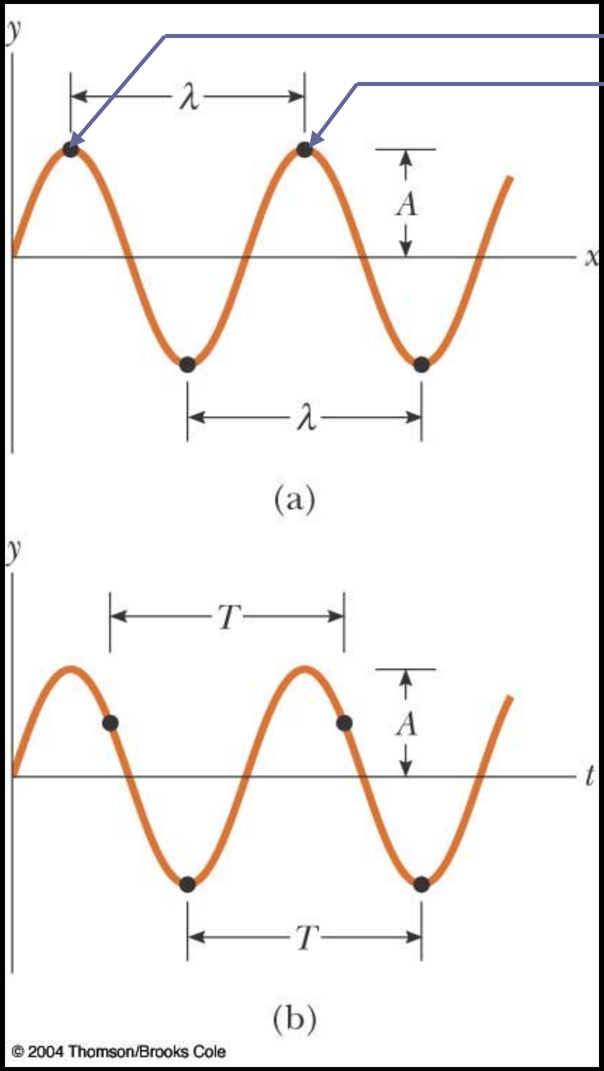
Traveling Waves

- Traveling waves (water waves, sound waves, waves on a string) are disturbances that move in a medium.

$$y = f(x - vt)$$

- Can be longitudinal (disturbance is parallel to wave motion) or transverse (disturbance is perpendicular to wave motion).
- Waves that are transmitted from one medium to another are not inverted.
- Waves that are reflected from an interface with a denser medium are inverted upon reflection.
- Waves that are reflected from an interface with a lighter medium are not inverted upon reflection.

Sinusoidal Waves



- λ : Wavelength
- T : Period
- A : Amplitude

$$f = \frac{1}{T}$$ Frequency (Hz)

$$k = \frac{2\pi}{\lambda}$$ Wave number (m^{-1})

$$\omega = \frac{2\pi}{T} = 2\pi f$$ Angular frequency (rad/s)

$$v = \lambda f = \frac{\omega}{k}$$ Speed (m/s)

$$v = \sqrt{\frac{T}{\mu}}$$ (String)

$$y(x,t) = A\sin(kx - \omega t + \phi)$$

Sound Waves

- Sound waves are longitudinal pressure waves, where air is compressed or decompressed periodically.

Air displacement: $s(x,t) = s_{\max}\cos(kx - \omega t)$

$$v = \sqrt{\frac{B}{\rho}}$$

Pressure variation: $\Delta P(x,t) = \Delta P_{\max}\sin(kx - \omega t)$

$$\Delta P_{\max} = \rho v \omega s_{\max}$$

- The intensity of a sound wave is the power per unit area of the wave.

$$I \equiv \frac{\mathcal{P}}{A} = \frac{1}{2}\rho v(\omega s_{\max})^2 = \frac{\Delta P_{\max}{}^2}{2\rho v}$$

Doppler Effect

- If either the source or the observer of a wave is in motion, the apparent frequency of the wave will depend on the relative velocities of the two.

Moving observer

$$f' = \left(\frac{v + v_O}{v}\right) f$$

Moving source

$$f' = \left(\frac{v}{v - v_S}\right) f$$

Both moving

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f$$

All signs are with source / observer moving toward the other

Wave Interactions

- When two or more waves pass through each other, they neither impede or destroy the other's motion.
- The total wave at a given point in time and space is the algebraic sum of the waves at that point.
- If the this sum is zero, the waves interfere destructively, if the sum is a maximum they interfere constructively.

Sinusoidal Wave Interactions

$y_1 = A\sin(kx - \omega t)$ $y_2 = A\sin(kx - \omega t + \phi)$ $\longrightarrow$ $y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$

- The amplitude of the resultant wave depends on the phase difference between the two.

$\phi = 0, 2\pi, 4\pi, \ldots 2n\pi$ Maximum amplitude

$\phi = \pi, 3\pi, \ldots (2n+1)\pi$ Minimum amplitude

- If the waves come from the same source but travel different paths:

$|r_2 - r_1| = \Delta r$ $\phi = 2\pi\frac{\Delta r}{\lambda}$

$\Delta r = 2n\frac{\lambda}{2}$ Constructive Interference

$\Delta r = (2n+1)\frac{\lambda}{2}$ Destructive Interference

Standing Waves

- If two waves travel in opposite directions, their interference will create a standing wave.

$$y_R = 2A\sin(kx)\cos(\omega t)$$

- This wave will have maxima (antinodes) and minima (nodes) at stationary points in space.

Nodes

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots, \frac{n\lambda}{2}$$

Antinodes

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots, \frac{(2n+1)\lambda}{4}$$

Standing Waves on Strings and Air Columns

- Standing waves created on a string with both ends fixed will only support a discrete number of frequencies.

Natural frequencies $$f_n = \frac{v}{\lambda_n} = \frac{vn}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

- An air column with both ends open supports all the natural frequencies of the system.
- If one end is closed, only odd multiples of the fundamental frequency is supported.