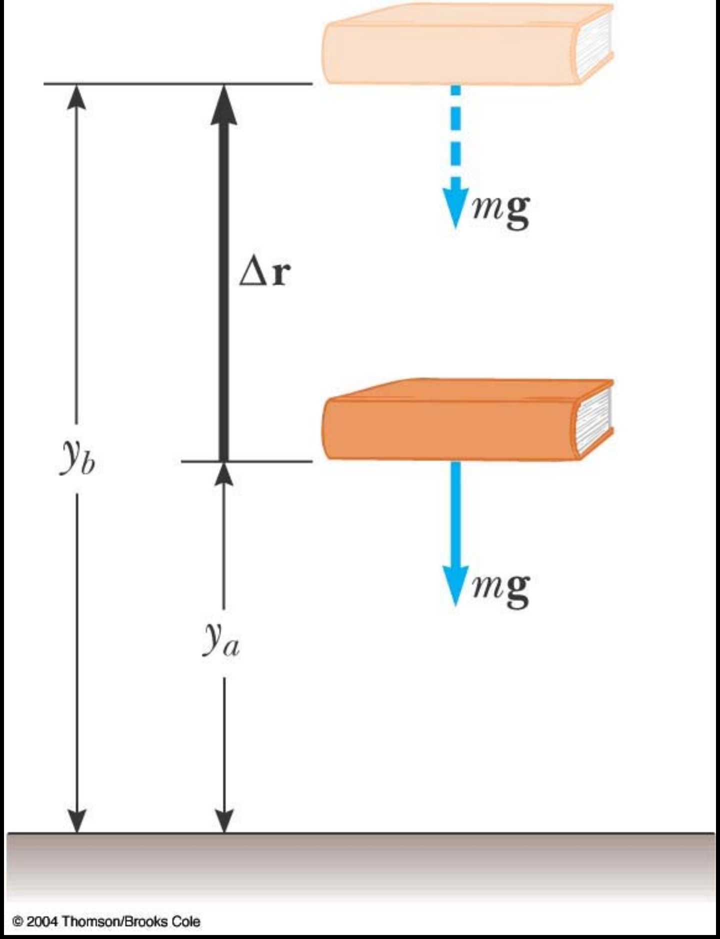
Potential Energy

- Potential Energy
- Conservation of Mechanical Energy
- Conservative and Non-conservative Forces
- Energy Diagrams and Equilibrium

Potential Energy

- Think of it as stored energy in a system that can do work or change the system's kinetic energy.
  - Gravitational
  - Elastic
  - Chemical, electrical, etc.
- When work gets done on a system, its potential and/or kinetic energy increases.

Gravitational Potential Energy



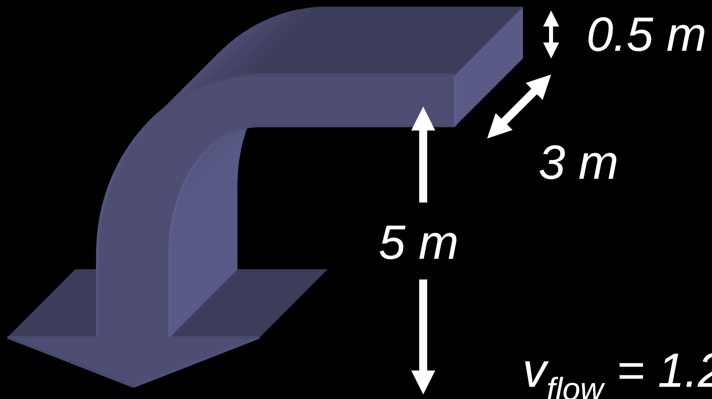
$$W = \left(\mathbf{F}_{app}\right) \cdot \Delta \mathbf{r} = \left(mg\hat{\mathbf{j}}\right) \cdot \left[\left(y_b - y_a\right)\hat{\mathbf{j}}\right] = mg\left(y_b - y_a\right)$$

$$U_g \equiv mgy$$

$$W = \Delta U_g$$

Gravitational potential energy only depends on height (y) and not on lateral distance (x).

Example – P8.3



v_{flow} = 1.20 m/s
Eff = 25%

P = ?

The mass of water flow per second:

$$m = \rho V = (1 gr/cm^3)(300cm \times 50cm \times 120cm/s) = 1.8 \times 10^6\, gr/s = 1800 kg/s$$

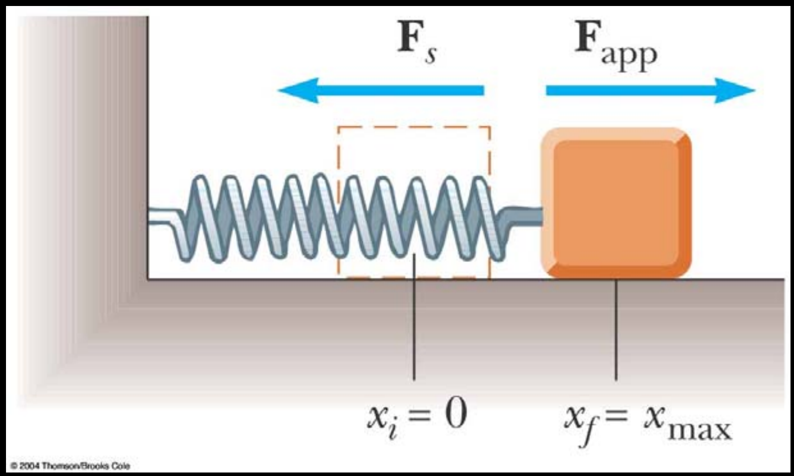
The change in potential energy per second of this mass as it falls down 5 m:

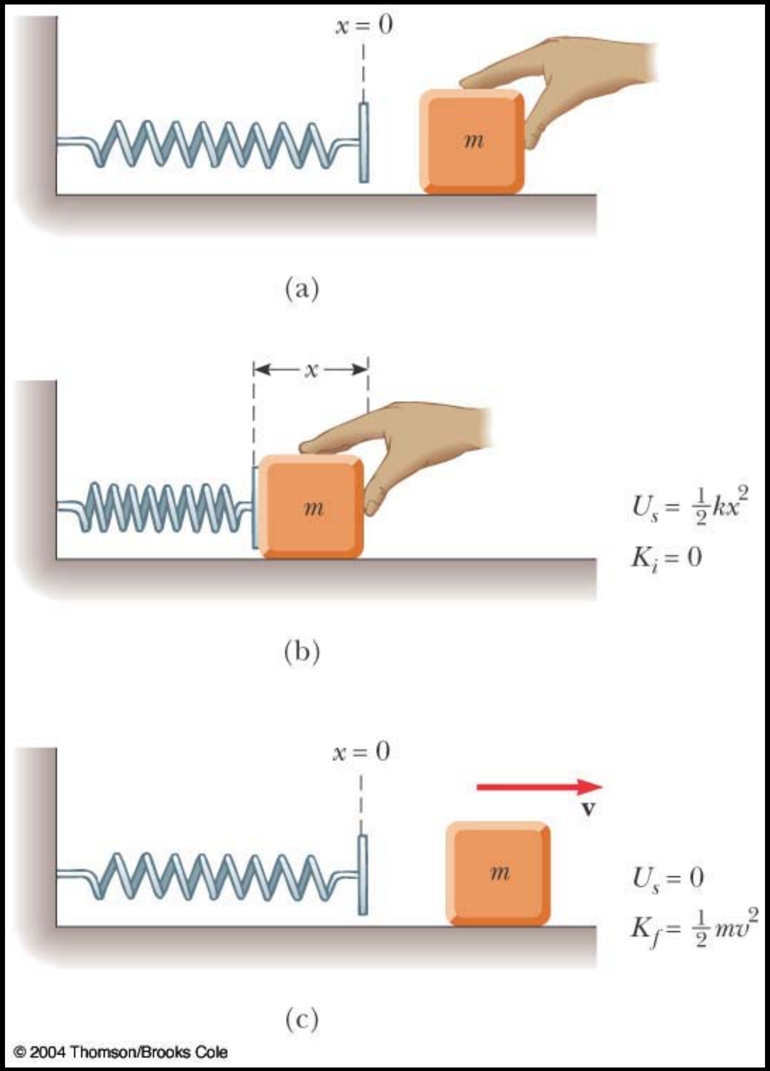
$$\Delta U/\Delta t = mgh = 1800 \times 9.8 \times 5 = 8.82 \times 10^4\, J/s$$

Power generated by this change in energy:

$$P = 0.25 \Delta U/\Delta t = 22kW$$

Elastic Potential Energy





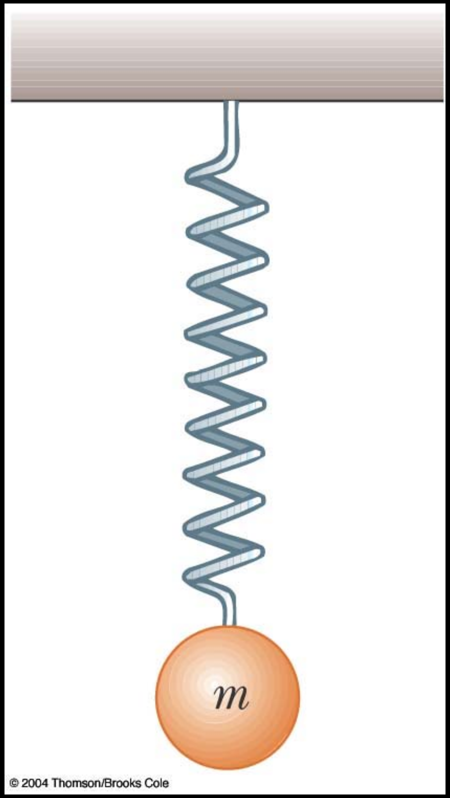
The work done by the spring

$$W_{F_{app}} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

The potential energy stored in the spring

$$U_s = \frac{1}{2}kx^2$$

Example - QQ8.7/8



A mass m is bobbing up and down on a spring.

Describe the various forms of energy of this system.

(a) At the highest point

Gravitational and elastic potential energy

(b) At the point where the kinetic energy is highest

Kinetic energy, gravitational potential energy

(c) At the lowest point

Gravitational and elastic potential energy

Conservative Forces

- Conservative Forces:
  - The work done by conservative forces is independent of path.
  - The work done over a closed path is zero.
  - We can always associate a potential energy with them.
  - We can only associate a potential energy with them.
  - Gravity, springs are examples.

Work done by a conservative force:

$$W_C = U_i - U_f = -\Delta U$$

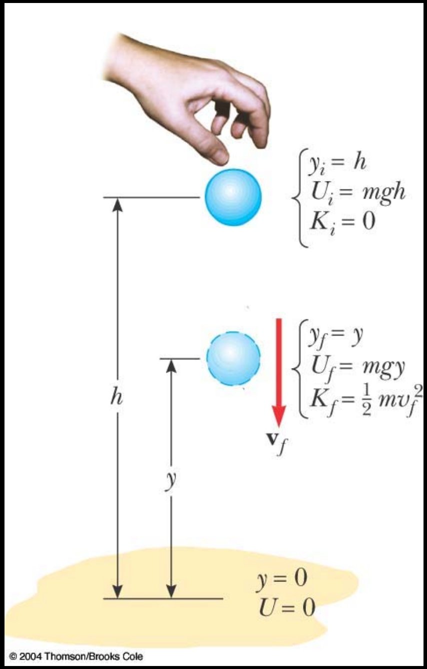
Relationship between a conservative force and potential energy

$$W_C = \int_{x_i}^{x_f} F_x dx = -\Delta U \longrightarrow F_x = -\frac{dU}{dx}$$

Mechanical Energy and Energy Conservation

Mechanical Energy = Kinetic Energy + Potential Energy

In an isolated system, mechanical energy is conserved when only conservative forces act upon the system.



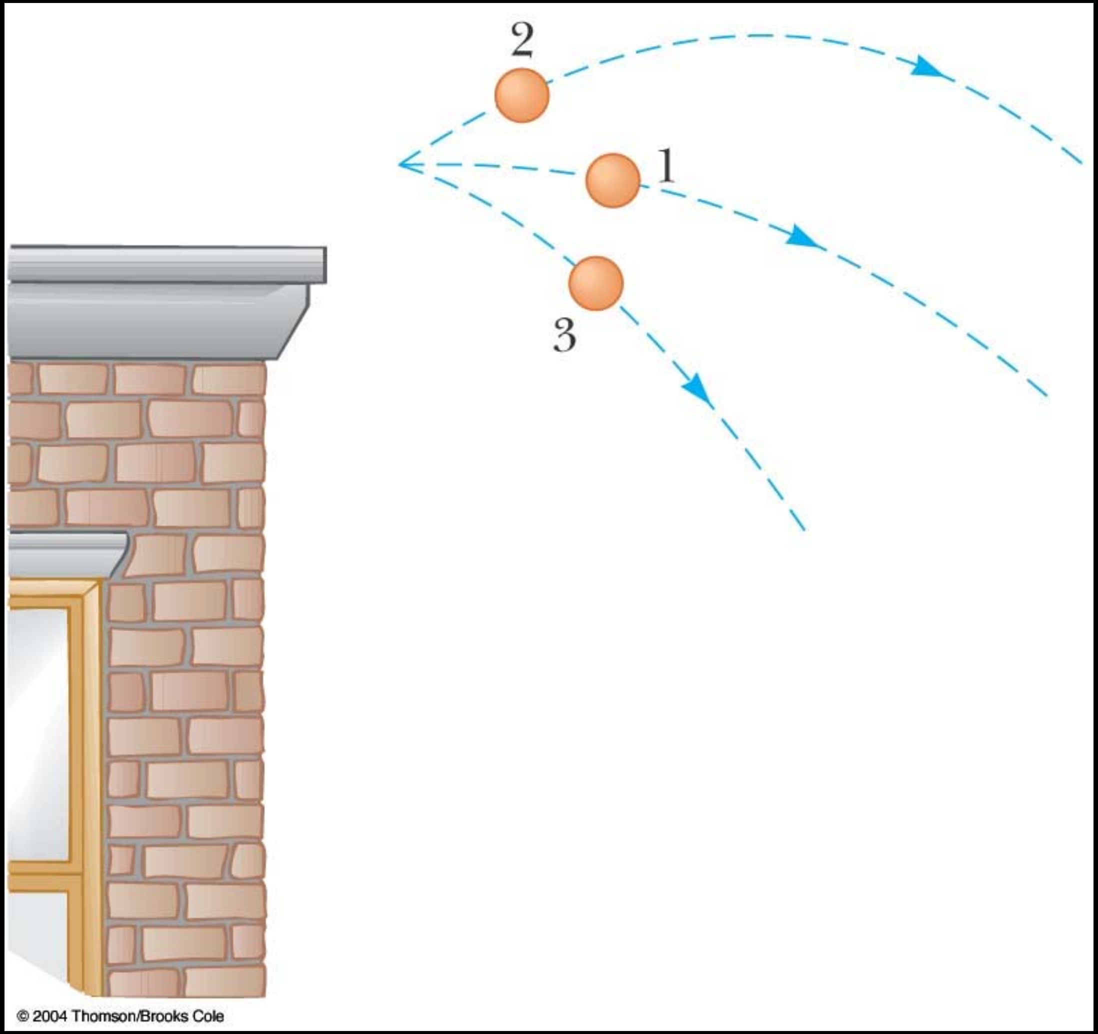
$$E_i = U_i = mgh$$

$$E_f = U_f + K_f = mgy + \frac{1}{2} mv_f^2$$

$$E_f = E_i \longrightarrow g(h - y) = \frac{1}{2} v_f^2$$

Roller Coaster

Example – QQ8.6



Three balls are thrown at different angles from the roof of a building with the same speed.

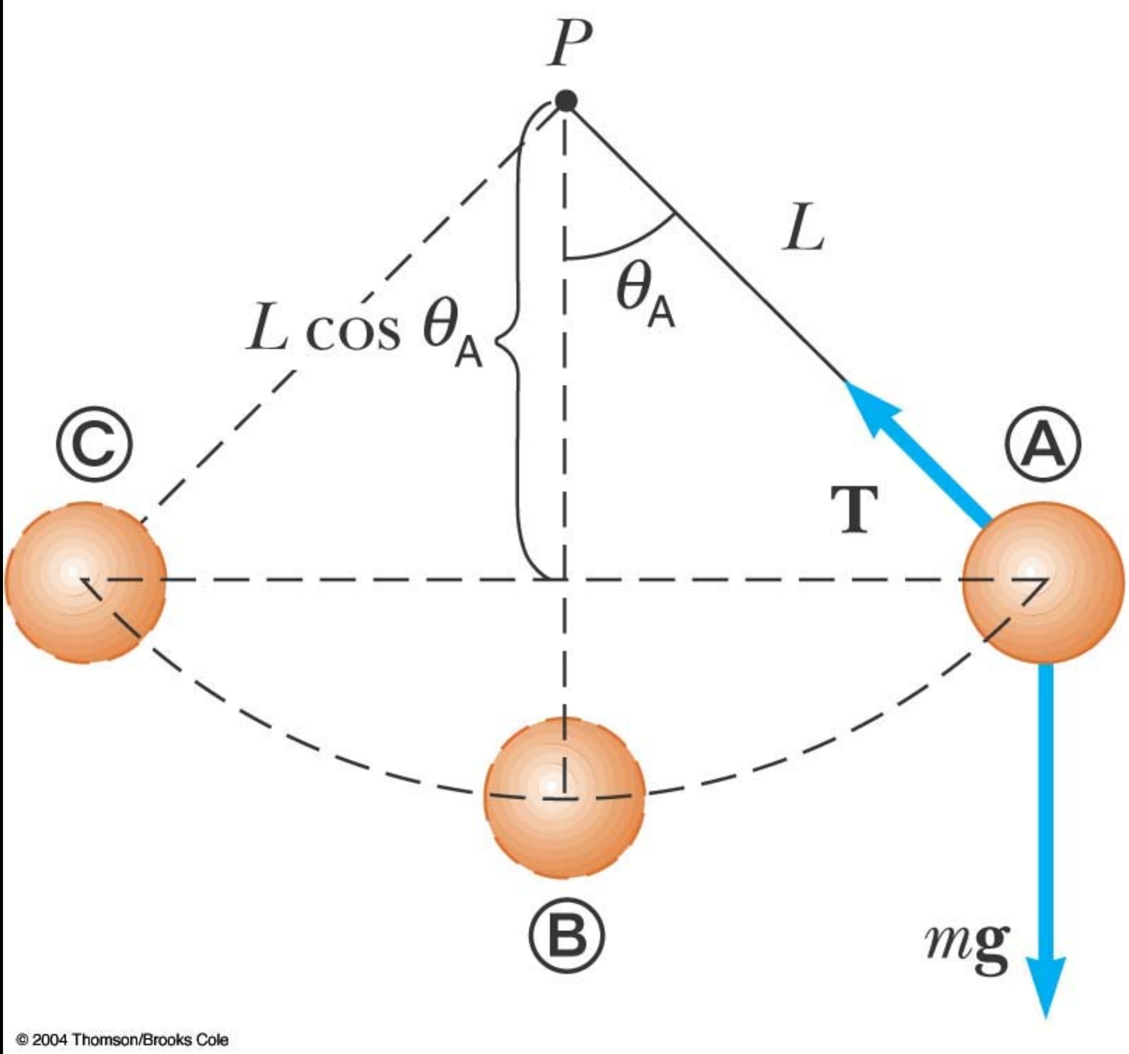
a) Describe the motion of the balls.

Projectile motion (parabola)

b) Rank their speed when they hit the ground.

All the same.

Example 8.3



$v_B = ?$
$T_B = ?$

$$K_B + U_B = K_A + U_A$$

$$mgL(1-\cos\theta_A) = \frac{1}{2} m v_B^{\ 2}$$

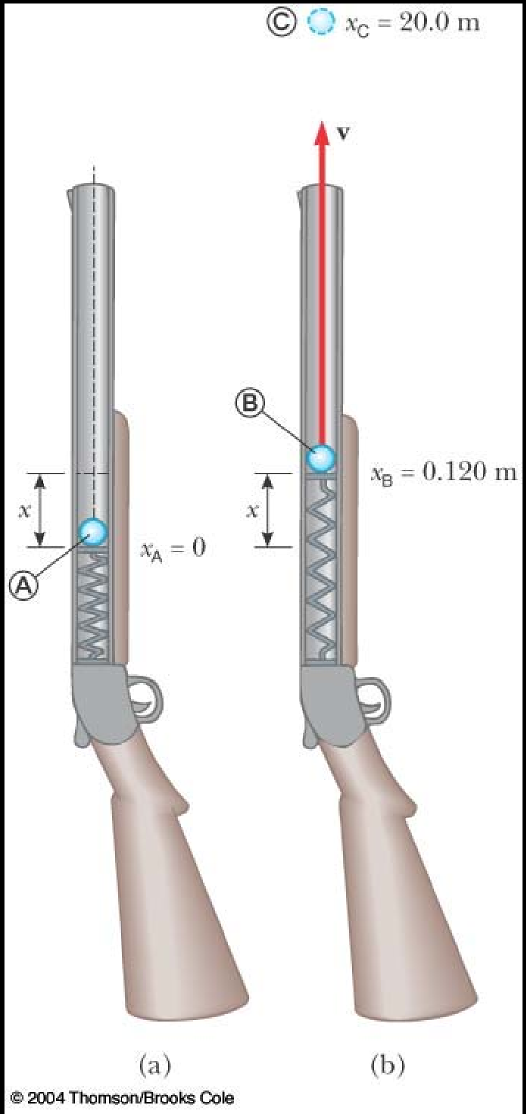
$$v_B = \sqrt{2gL(1-\cos\theta_A)}$$

$$\sum F_r = mg - T_B = -m\frac{v_B^{\ 2}}{L}$$

$$T_B = mg + m2g(1-\cos\theta_A)$$

$$T_B = mg(3-2\cos\theta_A)$$

Example 8.5



m = 35.0 g
Δx = 0.120 m
h = 20.0 m

k = ?

$$E_C = E_A$$

$$\frac{1}{2}k\Delta x^2 = mgh$$

$$k = \frac{2mgh}{\Delta x^2} = 953 N/m$$

v_B = ?

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv_B{}^2 + mg\Delta x$$

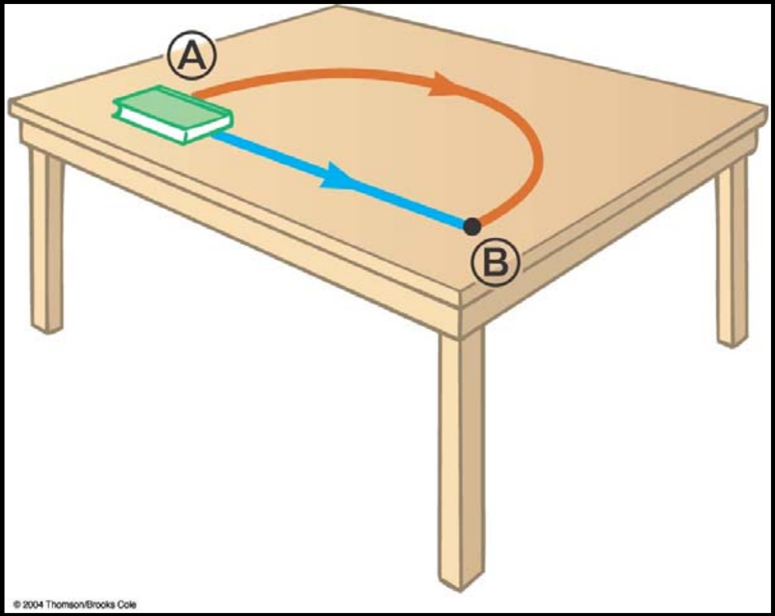
$$v_B = \sqrt{\Delta x^2 \frac{k}{m} - 2g\Delta x} = 19.7 m/s$$

Nose Basher!

Non-conservative Forces

- A non-conservative force will change the mechanical energy of the system.
- The total work done by the force depends on the path taken by the object.
- Example: Friction

The amount of work is done against friction and depends on the path. The change in the mechanical energy of the book is irreversibly converted other forms of energy (heat).



Work Done by Non-conservative Forces

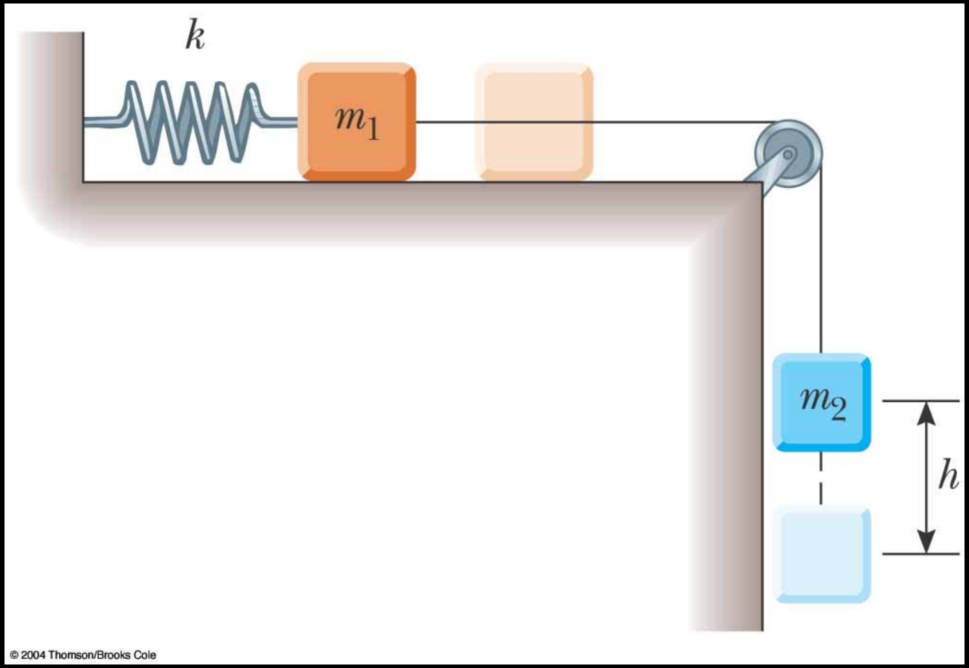
- Work can be done by an applied force.
  - Lifting an object increases the potential energy of the object.

$$\Delta U = W = \mathbf{F}_{app} \cdot \mathbf{d} = F_{app} d \cos\theta$$

- Work can be done by kinetic friction.
  - As an object moves on a surface with friction, it will lose kinetic energy.

$$\Delta K = -f_k \cdot d$$

Example 8.10



$$\Delta E_{mech} = \Delta U_s + \Delta U_g$$

$$\Delta E_{mech} = \frac{1}{2}kh^2 - m_2 gh$$

$$\Delta E_{mech} = -f_k h = -\mu_k m_1 gh$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

Review

Gravitational
Potential Energy

$$U_g \equiv mgy$$

Elastic
Potential Energy

$$U_s = \frac{1}{2}kx^2$$

Work done by a conservative force: $W_C = U_i - U_f = -\Delta U$

Mechanical Energy = Kinetic Energy + Potential Energy

M.E. is conserved when only conservative forces act on a system (spring, gravity)

Example of a non-conservative force: friction