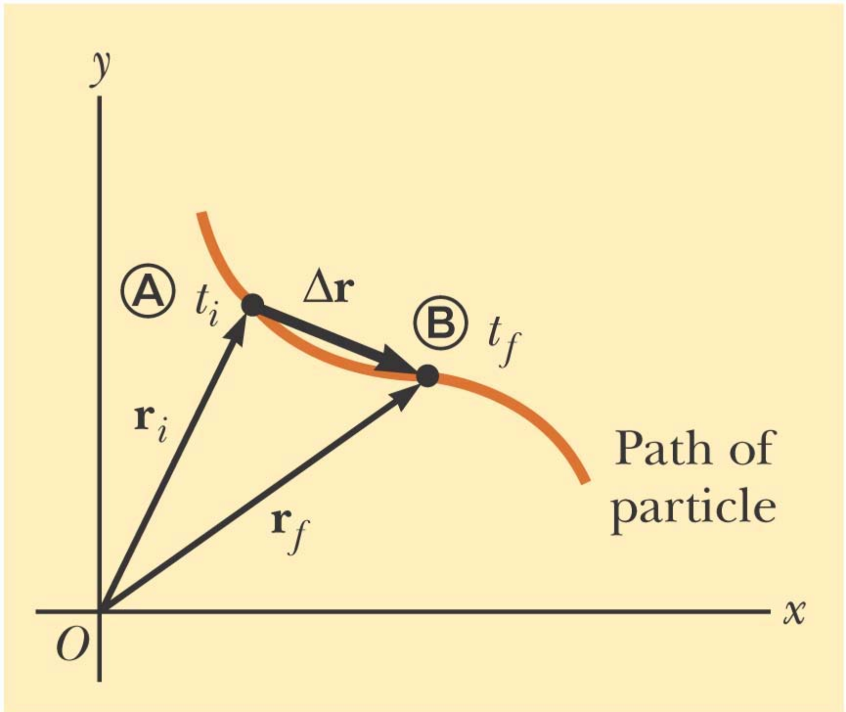
Motion in Two Dimensions

- Position, velocity and acceleration vectors
- 2D motion with constant acceleration
- Projectile motion

Position Vector

- **r** is drawn from origin to the position of the object at a given time.
- Displacement vector:

$$\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$$



Velocity

Average velocity

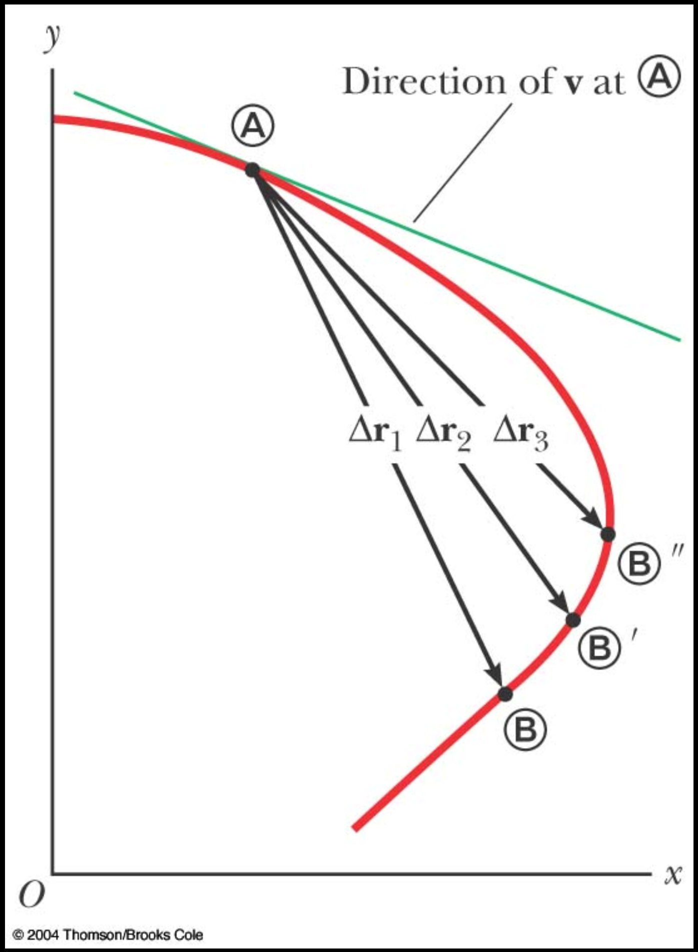
$$\bar{\mathbf{v}} \equiv \frac{\Delta \mathbf{r}}{\Delta t}$$

Speed

$$v = |\bar{\mathbf{v}}|$$

Instantaneous velocity

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$



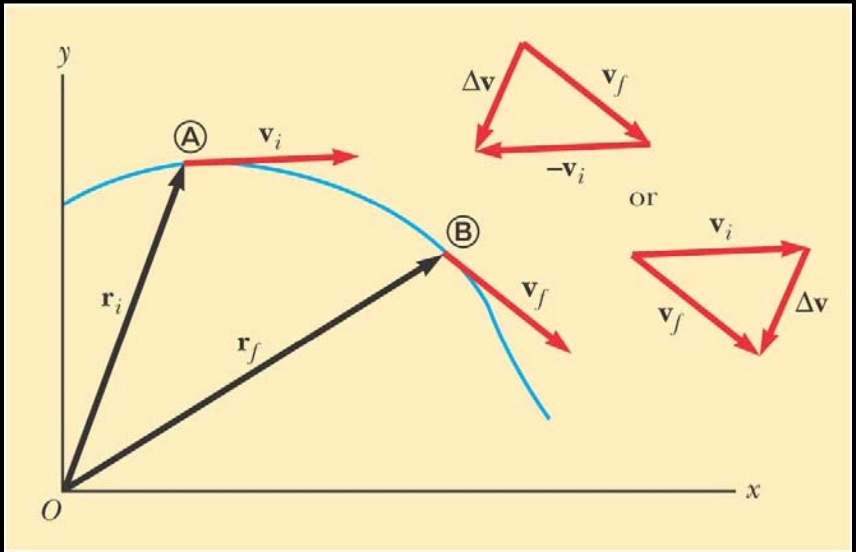
Acceleration

Average acceleration

$$\bar{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Instantaneous acceleration

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$



- Acceleration can cause a **change** in the **magnitude** or the **direction** of the velocity of an object, or both.

Two Dimensional Motion With Constant Acceleration

- Use 1D equations but replace scalars with vectors
- Use the superposition principle
  - Break down each vector to its components
  - Calculate each component independently

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$$

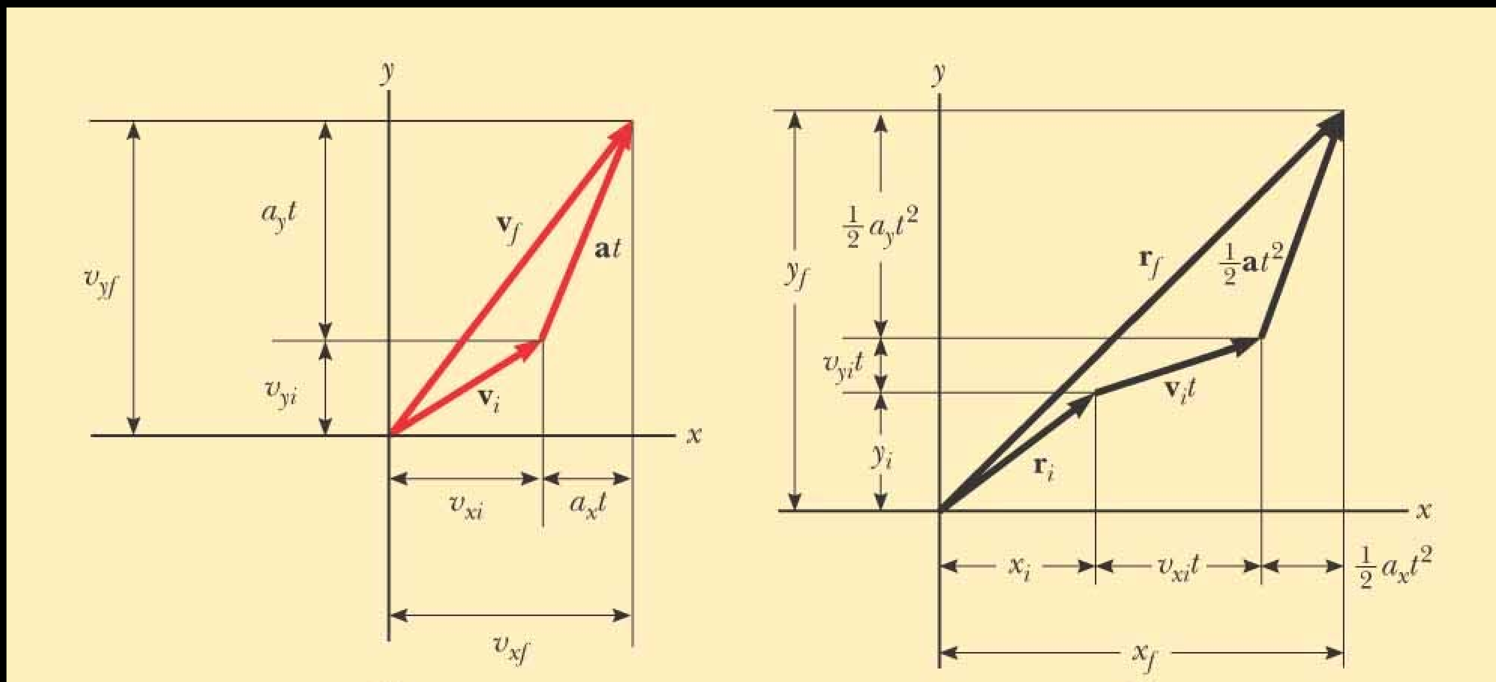
$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$$

Two Dimensional Motion With Constant Acceleration

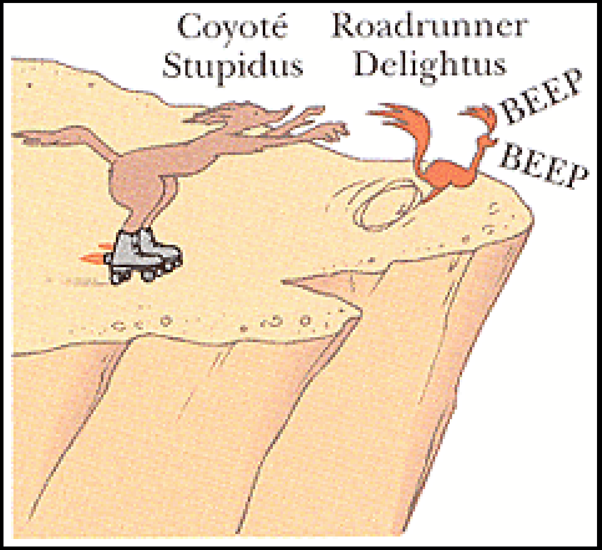


$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = x_f \hat{\mathbf{i}} + y_f \hat{\mathbf{j}}$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Example – P 4.65



The coyote has skates that give a constant acceleration of 15 m/s². He starts from rest and catches up the roadrunner at the edge of the cliff, 70 m later.

a) What is the speed of the coyote when he catches up with the roadrunner?

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$v_{xf} = v_{xi} + at$$

45.8m/s

Of course, the roadrunner makes a sharp turn right at the edge and the coyote flies off the cliff and maintains his horizontal acceleration. If the cliff is 100 m above the floor of the canyon,

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

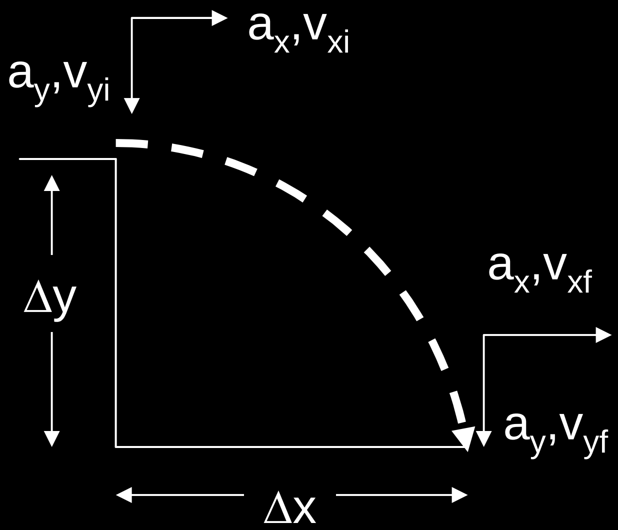
b) Where will the coyote land?

360m

c) What will its final velocity be?

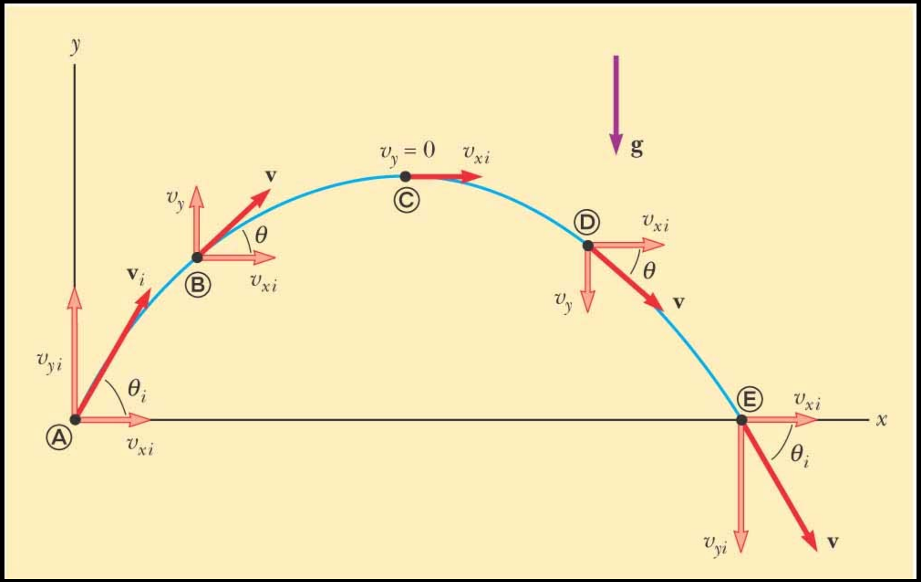
527.7m/s

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2}$$



Projectile Motion

- The object has an initial velocity, $\mathbf{v}_i$ and is launched at an angle, θ_i.
- Two assumptions:
  - Free-fall acceleration (g) is constant
  - Air resistance is negligible

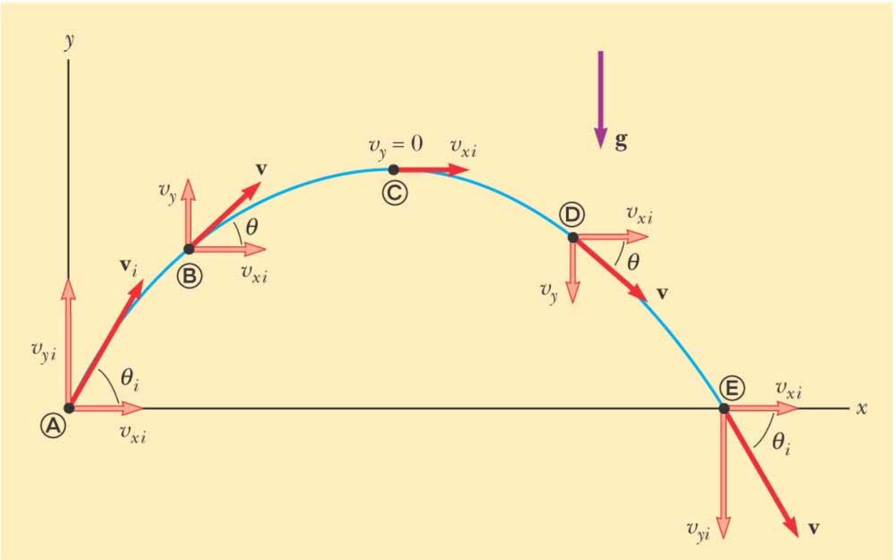


- Then the **trajectory** (path) of the projectile is a **parabola**.
- The velocity changes magnitude and direction
- The acceleration in the y-direction is constant (**-g**).
- The acceleration in the x-direction is **zero**.

Projectile Motion

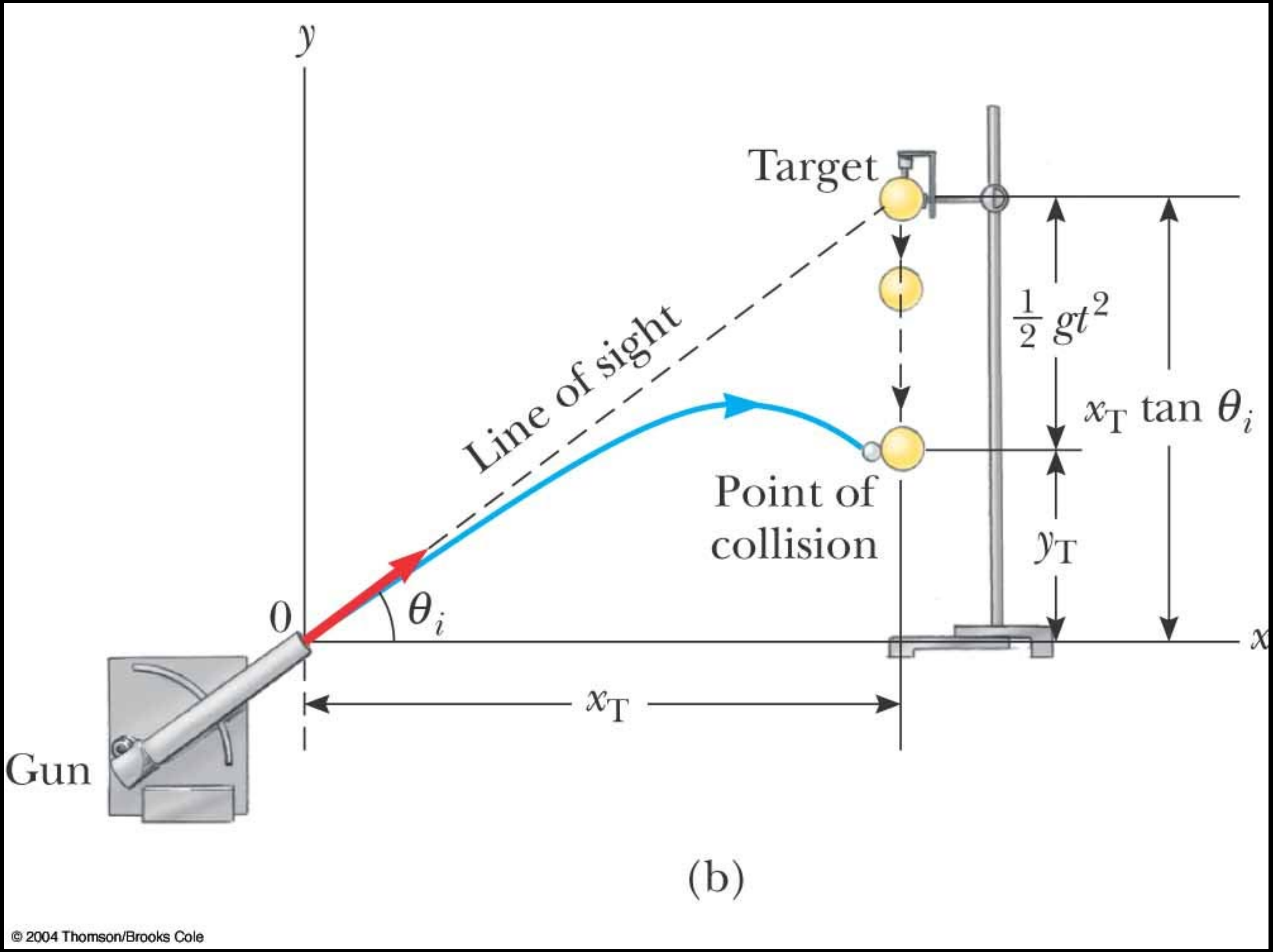
$$v_{xi} = v_i \cos\theta_i$$
$$v_{yi} = v_i \sin\theta_i$$

We'll analyze projectile motion as a superposition of two independent motions:



Constant velocity motion in the horizontal direction	Free-fall motion in the vertical direction
$v_{xf} = v_{xi} = \text{constant}$	$v_{yf} = v_{yi} + gt$
$x_f = x_i + v_x t$	$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

Demo (Bull's eye)



Example – 4.5

$v_i = 20$ m/s
$\theta_i = 30$
$h_{building} = 45$ m

a) $t = ?$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

t = 4.22 s

b) $v_f = ?$

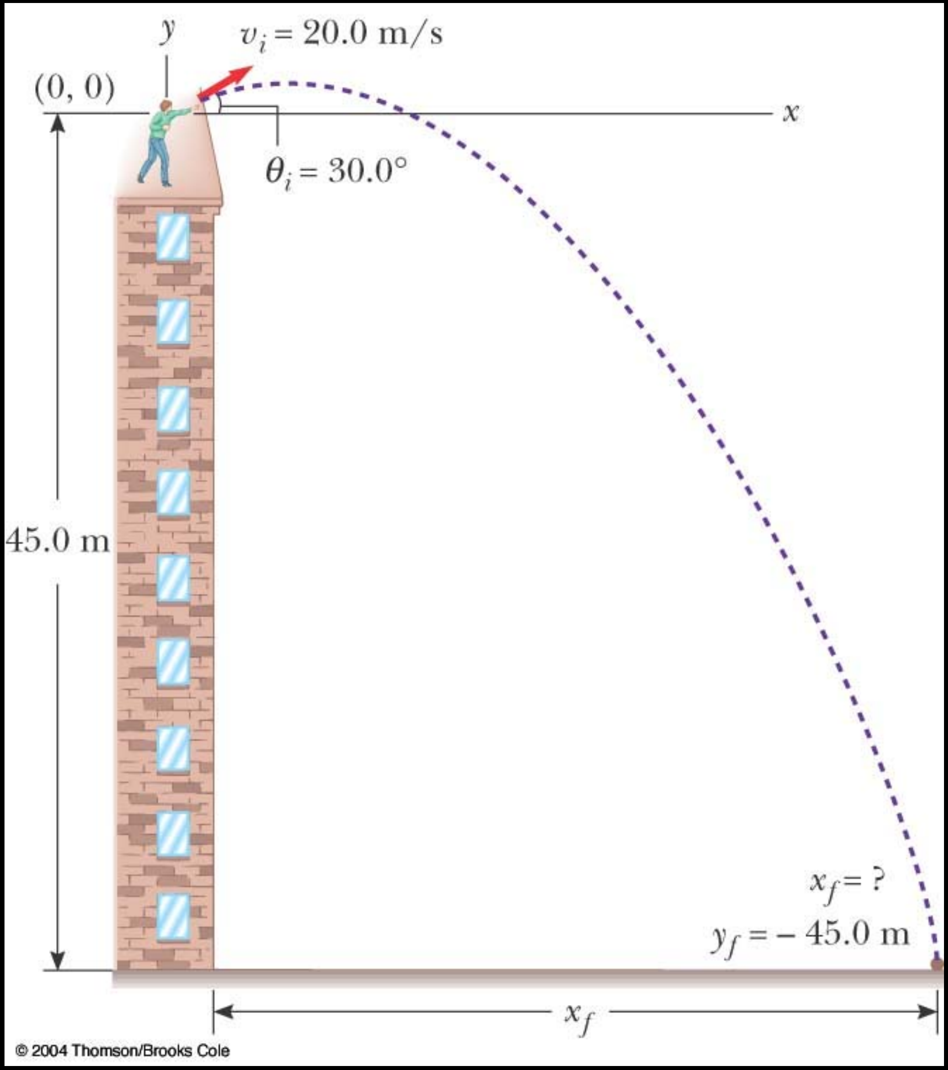
$$v_{yf} = v_{yi} + gt$$

$v_f = 35.9$ m/s

c) $x_f = ?$

$$x_f = x_i + v_x t$$

$x_f = 13.02$ m



Review

- Use the same basic equations as 1D but with vectors.
- Solve each direction separately.
- Projectile motion is superposition of 1D motion with constant velocity and free fall motion.