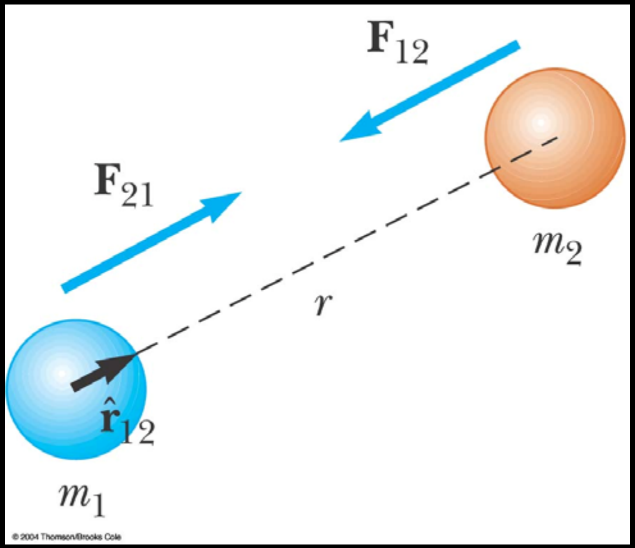
Universal Gravitation

- Newton's law of universal gravitation
- Free fall acceleration and gravitational force
- Kepler's laws
- The gravitational field

Newton's Law of Universal Gravitation



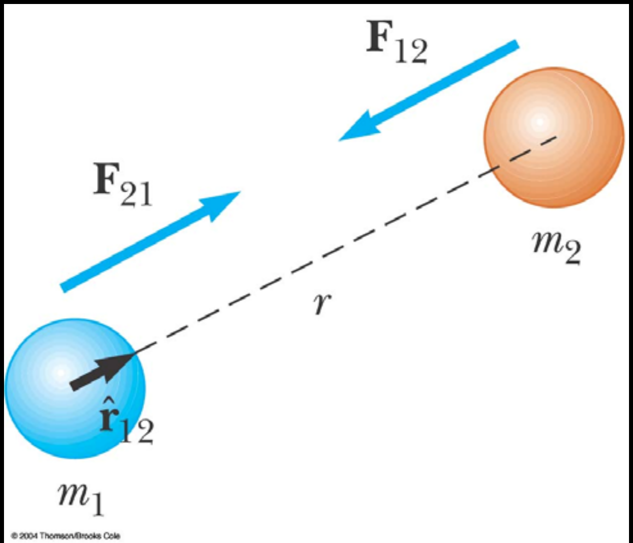
$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12}$$

$$G = 6.673\times10^{-11}\, Nm^2/kg^2$$

- The gravitational force is always attractive.
- It follows an inverse-square law.
- F_{12} and F_{21} are an action-reaction pair.
- m_1 and m_2 are attracted to each other with the same force.
- We can treat spherical mass distributions as if the mass was concentrated in the center of the sphere.

Example – P13.4

Two objects attract each other with a gravitational force of magnitude 0.93x10^{-8} N when separated by 19.3 cm. If the total mass of the two objects is 5.09 kg, what is the mass of each?



$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}$$

$$0.93\times10^{-8} = G\frac{(5.09 - m_2)m_2}{0.193^2}$$

$$m_2^{\ 2} - 5.09m_2 + \frac{(0.93\times10^{-8})(0.193^2)}{G} = 0$$

$$m_2 = 1.411kg \text{ or } 3.251kg$$

Free Fall Acceleration and Gravitational Force

For an object on the surface of Earth:

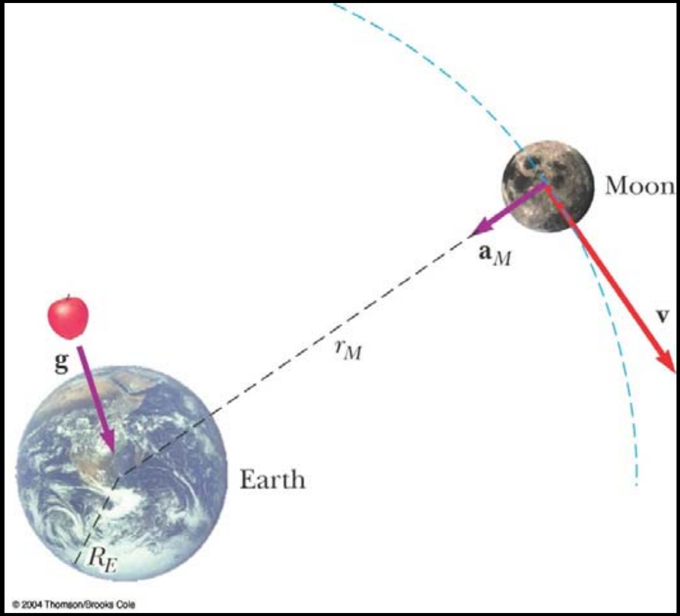
$$F_g = G\frac{M_E m}{R_E{}^2} = mg$$

$$g = G\frac{M_E}{R_E{}^2}$$

For an object at a height h above Earth:

$$F_g = G\frac{M_E m}{(R_E + h)^2} = mg_h$$

$$g_h = G\frac{M_E}{(R_E + h)^2}$$



Example – 13.3

$$g = 9.80 m/s^2$$
$$R_E = 6.37 \times 10^6 m$$
$$\rho_E = ?$$

$$g = G\frac{M_E}{R_E^{\ 2}}$$

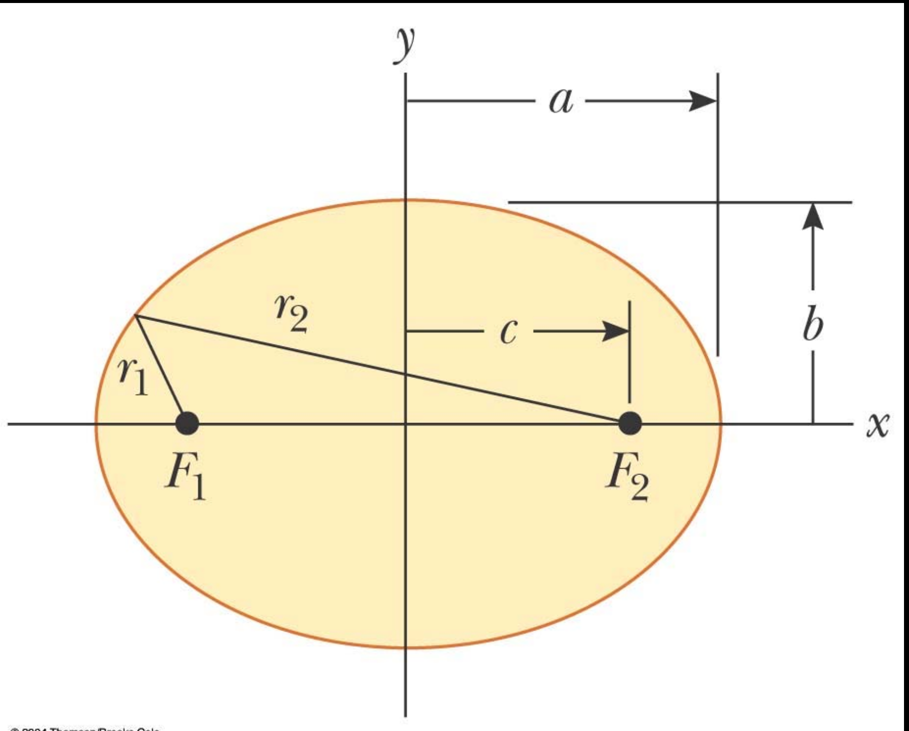
$$M_E = \frac{g}{G} R_E^{\ 2}$$

$$V_E = \frac{4}{3}\pi R_E^{\ 3}$$

$$\rho_E = \frac{M_E}{V_E} = \frac{\frac{g}{G} R_E^{\ 2}}{\frac{4}{3}\pi R_E^{\ 3}} = \frac{3g}{4\pi G R_E} = 5.51 \times 10^3 kg/m^3$$

Kepler's First Law

- Planets move in elliptical paths around the sun. The sun is in one of the focal points (foci) of the ellipse.
- F_1 and F_2 are the foci of the ellipse.
- $r_1 + r_2$ is constant.
- The major axis has a length 2*a* and the minor axis has a length 2*b*.



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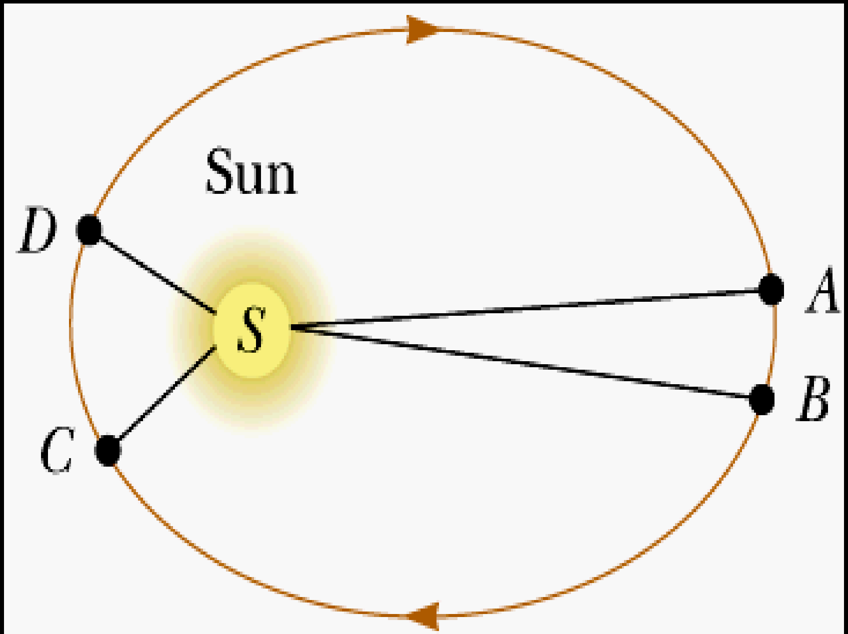
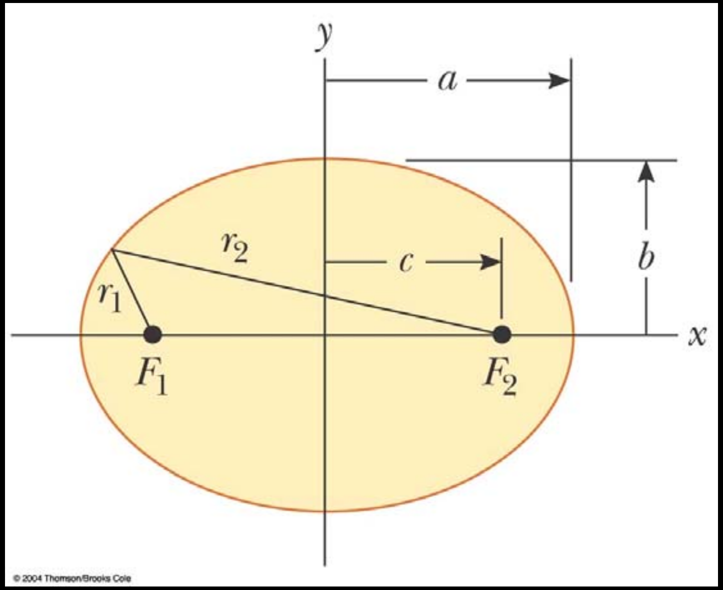
Kepler’s Second Law

- The radius vector drawn from the sun to a planet sweeps out equal areas in equal time intervals (Law of equal areas).



Kepler's Third Law

- The square of the orbital period, T, of any planet is proportional to the cube of the semi-major axis of the elliptical orbit, a.



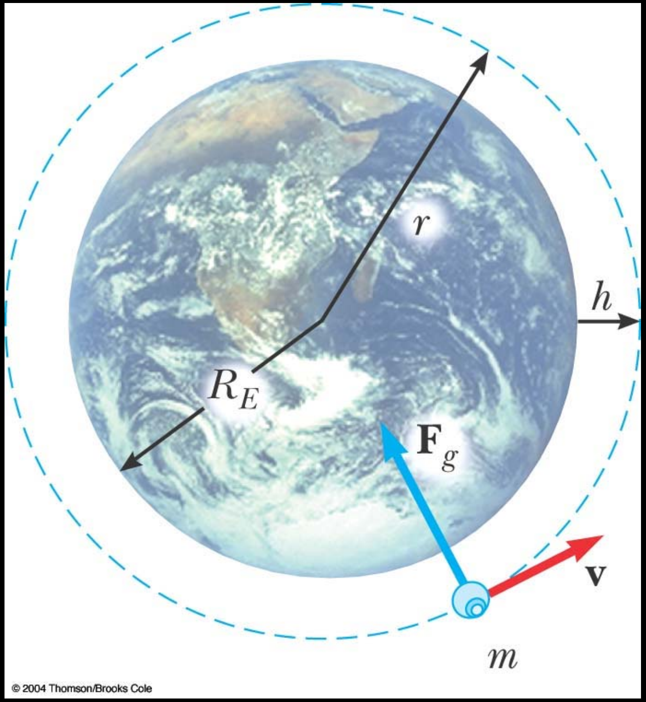
$$T^2 \propto a^3$$

$$\frac{T^2}{a^3} = const.$$

Thus, for any two planets:

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

Example – 13.5



For a satellite orbiting Earth at constant speed, find v as a function of G, h, R_E and M_E.

$$F_r = F_g = G\frac{M_E m}{r^2} \longrightarrow m\frac{v^2}{r} = G\frac{M_E m}{r^2}$$

$$v = \sqrt{G\frac{M_E}{r}} = \sqrt{G\frac{M_E}{R_E + h}}$$

If the orbit is geosynchronous, what is its speed?

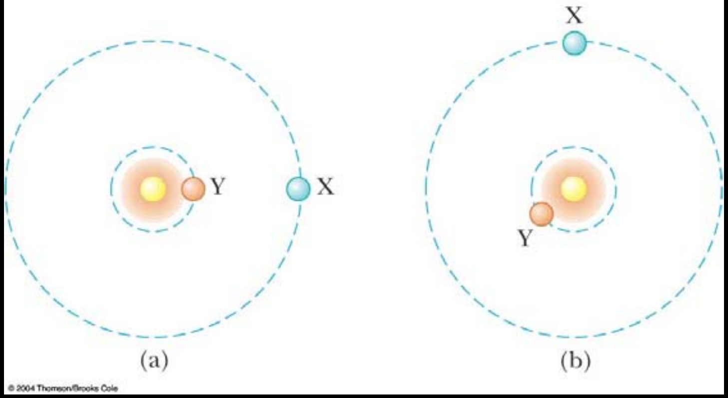
$$T = \frac{2\pi r}{v} \longrightarrow T = \frac{2\pi r}{\sqrt{G\frac{M_E}{r}}}$$

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 \longrightarrow r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} = 4.23\times10^7\,m$$

$$v = \sqrt{G\frac{M_E}{r}} = 3.07\times10^3\,m/s$$

Example – P13.18

Two planets X and Y travel counterclockwise in circular orbits about a star as in the figure. The radii of their orbits are in the ratio 3:1. At some time, they are aligned as in figure a, making a straight line with the star. During the next five years, the angular displacement of planet X is 90.0°, as in figure b. Where is planet Y at this time?



$$m\frac{v^2}{r} = G\frac{M_{Star}m}{r^2}$$

$$v^2 = G\frac{M_{Star}}{r} = \omega^2 r^2$$

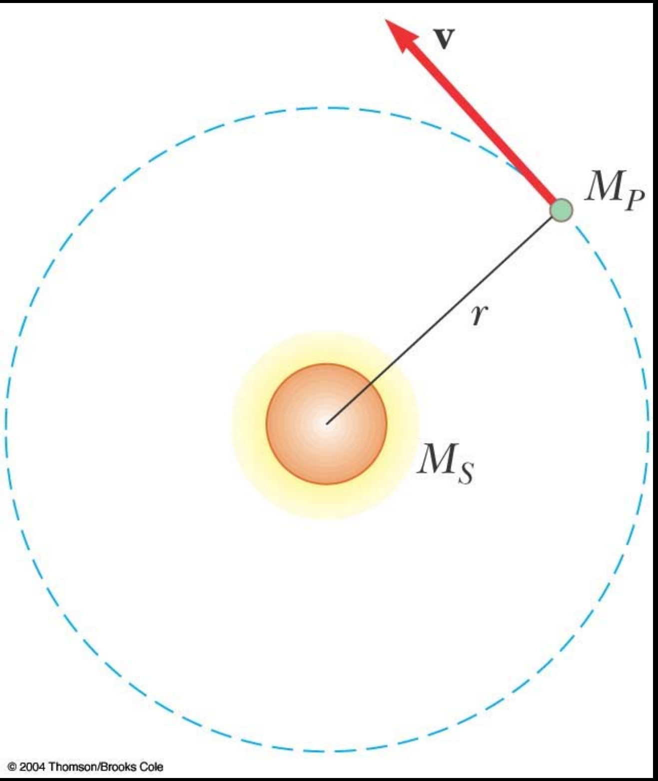
$$GM_E = \omega_X{}^2 r_X{}^3 = \omega_Y{}^2 r_Y{}^3$$

$$\omega_Y = \omega_X\left(\frac{r_X}{r_Y}\right)^{3/2} = \frac{\pi/2}{5}(3)^{3/2}$$

$$\theta_Y = 5\omega_Y = \frac{\pi}{2}(3)^{3/2} = 8.16 rad = 1.3 rev$$

In the Solar System

- Most planets (except Mercury) are on an almost circular orbit.



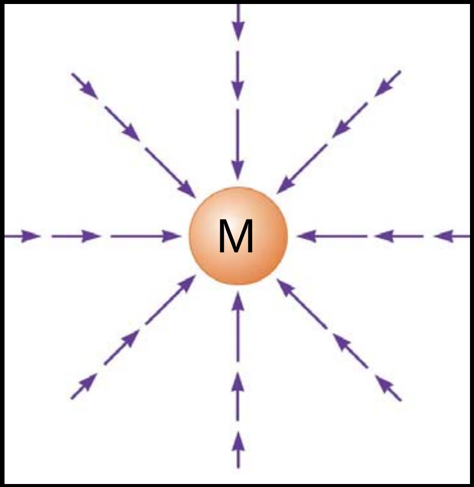
For Earth: $$\frac{b}{a} = 0.99986$$

For the solar system planets: $$\frac{T^2}{a^3} = 2.97 \times 10^{-19} \frac{s^2}{m^3}$$

The Gravitational Field

- An object with mass, M, will attract another object with mass, m, in its vicinity with the gravitational force, F_g.

$$\mathbf{F}_g = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$



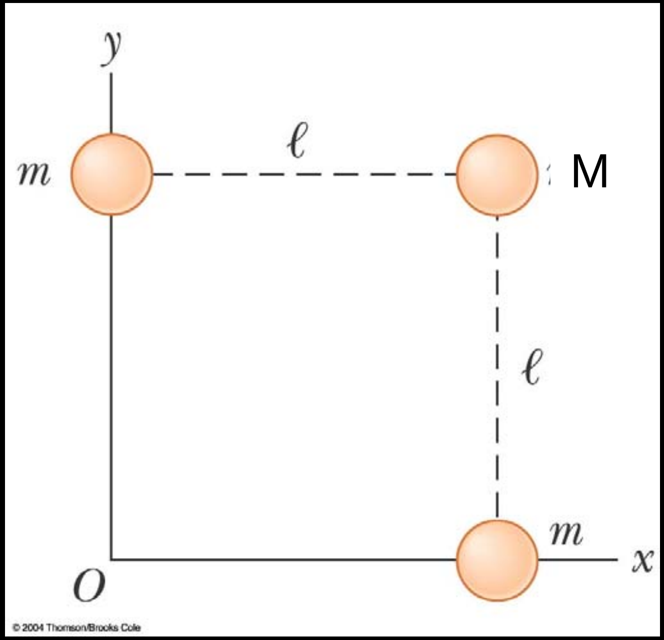
- While the force is a quantity between M and m, M will exhibit the gravitational effect regardless of the existence of m.

- This inherent effect is quantified by the gravitational field, g which is the gravitational force exerted by M per unit mass on a test mass, m.

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = G\frac{M}{r^2}\hat{\mathbf{r}}$$

Example – P13.23

Three objects -- two of mass *m* and one of mass *M* -- are located at three corners of a square of edge length *l* as in the figure. Find the gravitational field **g** at the fourth corner due to these objects. (Express your answers in terms of the edge length *l*, the masses *m* and *M*, and the gravitational constant *G*).



$$g_x = \frac{Gm}{l^2} + \frac{GM}{2l^2} = G\frac{2m+M}{2l^2}$$

$$g_y = G\frac{m}{l^2} + G\frac{M}{2l^2} = G\frac{2m+M}{2l^2}$$

$$\mathbf{g} = g_x\hat{\mathbf{i}} + g_y\hat{\mathbf{j}} = G\frac{2m+M}{2l^2}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

$$g = \sqrt{g_x^{\ 2} + g_y^{\ 2}} = G\frac{2m+M}{2l^2}\sqrt{2}$$

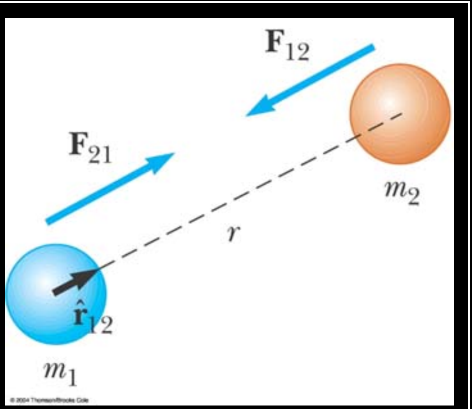
Review

Gravitational Force

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}$$

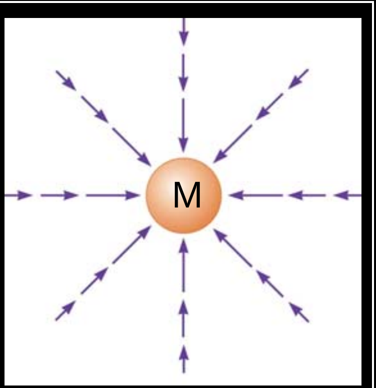
$$G = 6.673\times10^{-11}\ Nm^2/kg^2$$

$$g_h = G\frac{M_E}{(R_E+h)^2}$$

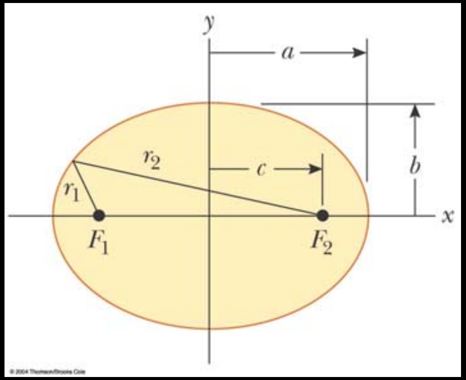


Gravitational Field

$$\mathbf{g} = \frac{\mathbf{F}_g}{m} = -G\frac{M}{r^2}\hat{\mathbf{r}}$$



Planetary Motion



$$\frac{T^2}{a^3} = const.$$