Math 112
Dr. Sarah Raynor
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# TEST #2

## NAME:

**Pledge:** I pledge on my honor that I have neither given nor received any assistance on this exam nor have I used any dishonest means to obtain my results.

**Signature:** _____

**Note: This test is out of 72 points. To receive full credit you must SHOW ALL WORK!**

Some Formulae You May find useful:

$$\sin(\frac{\pi}{6}) = \frac{1}{2}$$

$$\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$$

$$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$$

| Question | Score Possible | Score |
|---|---|---|
| 1 | 30 | |
| 2 | 18 | |
| 3 | 7 | |
| 4 | 17 | |

Total Score: _____/ 72

1. Compute the following (5 points each)

   (a) $x$ if $\ln(x) = 2\ln(4) - \ln(x-6)$.

   (b) $\frac{d}{dx}(2^{x^2})$.

   (c) $\int \tan^{-1}(x)dx$.

(d) $\lim_{x\to 0} \frac{\sin(x)}{\sin^{-1}(x)}$.

(e) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2(x)}{\tan(x)} dx$

(f) $\lim_{x\to\infty} x^{\frac{1}{x}}$

2. Consider the function $f(x) = \cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$.

(a) (3 points) Find a domain including the first quadrant on which $f(x)$ is one-to-one.

(b) (5 points) Using your work from (a), find the domain and range of $f^{-1}(x)$.

(c) (2 points) Find $f^{-1}(\frac{-1}{\sqrt{3}})$.

(d) (8 points) Find a formula for the derivative of $f^{-1}(x)$ **THAT DOES NOT HAVE ANY TRIG FUNCTIONS IN IT**.

3. (7 points) State the formula for integration by parts. Explain in **FULL SENTENCES** what everything in the formula means, where it comes from, and how to use it.

4. You have taken out a loan for \$100,000 at a 9% annual interest rate. You intend to pay \$1000 per month to pay off this loan. This situation can be modelled by a function M(t) which solves the equation

$$\frac{dM(t)}{dt} = 0.0075M(t) - 1000,$$

where $t$ is in units of months.

(a) (7 points) Find an expression for $M(t)$, the amount of money you owe as a function of time. (Hint: Let $y = M(t) - 1000/0.0075 = M(t) - 400000/3$. Then $y(t)$ should be a familiar function.)

(b) (5 points) When will your loan be paid off? That is, how long will it be until you don't owe any more money? You should give an exact answer, not a decimal.

(c) (5 points) Sketch a graph of the function $M(t)$ on the attached graph paper, including any asymptotes and intercepts. You don't need to try to find critical points or inflection points. (There aren't any.)