Math 112
Dr. Sarah Raynor
September 18, 2009

TEST #1

NAME:

Pledge: I pledge on my honor that I have neither given nor received any assistance on this exam nor have I used any dishonest means to obtain my results.

Signature: _____

Note: This test is out of 60 points. To receive full credit you must SHOW ALL WORK!

Some Formulae You May find useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

Question	Score Possible	Score
1	30	
2	7	
3	5	
4	7	
5	11	

Total Score: _____/ 60

1. Compute the following: (5 points each)

(a)

$$\int_0^{\frac{\pi}{3}} \sec x \tan x dx$$

(b)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{2n} \sec^2(-\frac{\pi}{4} + \frac{\pi i}{2n})$$

(c)

$$\int_{-1}^{2} \frac{t+1}{(3+2t+t^2)^2} dt$$

(d) The derivative of $F(x) = \int_{x^3}^{0} \tan(t)dt$.

(e) The area of the region between $f(x) = x^2 - 5x$ and $g(x) = x - 8$.

(f) A domain on which the function $f(x) = \sin(x^2)$ is one-to-one, and its range on that domain.

2. (7 points) Suppose that you are given a three-dimensional shape and asked to find its volume. Describe in detail what your process would be to find the volume of the shape mathematically, from start to finish. If you give any formulae, justify them. Use full sentences. Note that you cannot assume your shape is a volume of rotation.

3. (5 points) Use a Riemann sum with $n = 4$ to estimate the area under the curve $f(x) = \sqrt{1 + x^3}$ from $x = -1$ to $x = 3$. Choose a method that makes your estimate an underestimate.

4. (7 points) Find the volume of the solid generated by rotating the region between $x = 4 - y^2$ and $x = -3y$ about the line $y = -1$.

5. A penny is dropped off of the Empire State Building, which is approximately 400m tall. Assume that its acceleration due to gravity is 10 m/s and that it is initially at rest. (It is dropped, not thrown.)

   (a) (4 points) Find the penny's position as a function of time. Make sure that your answer is clear and is in terms of the original units.

   (b) (4 points) Find the penny's average velocity during the time when it is in the air.

   (c) (3 points) At what time will the penny achieve its average velocity?