Homework #5 Solutions

#2) $f(x) = e^{x^3 - x}$

$f(0) = 1$

$f(x) > 0$ for all x, so there's no x-intercept.

$\lim_{x \to \infty} e^{x^3-x} = e^{\infty} = \infty \qquad \lim_{x \to -\infty} e^{x^3-x} = e^{-\infty} = 0$

CPs: $f'(x) = (3x^2 - 1)e^{x^3-x} = 0 \qquad 3x^2 - 1 = 0$

$3x^2 = 1$

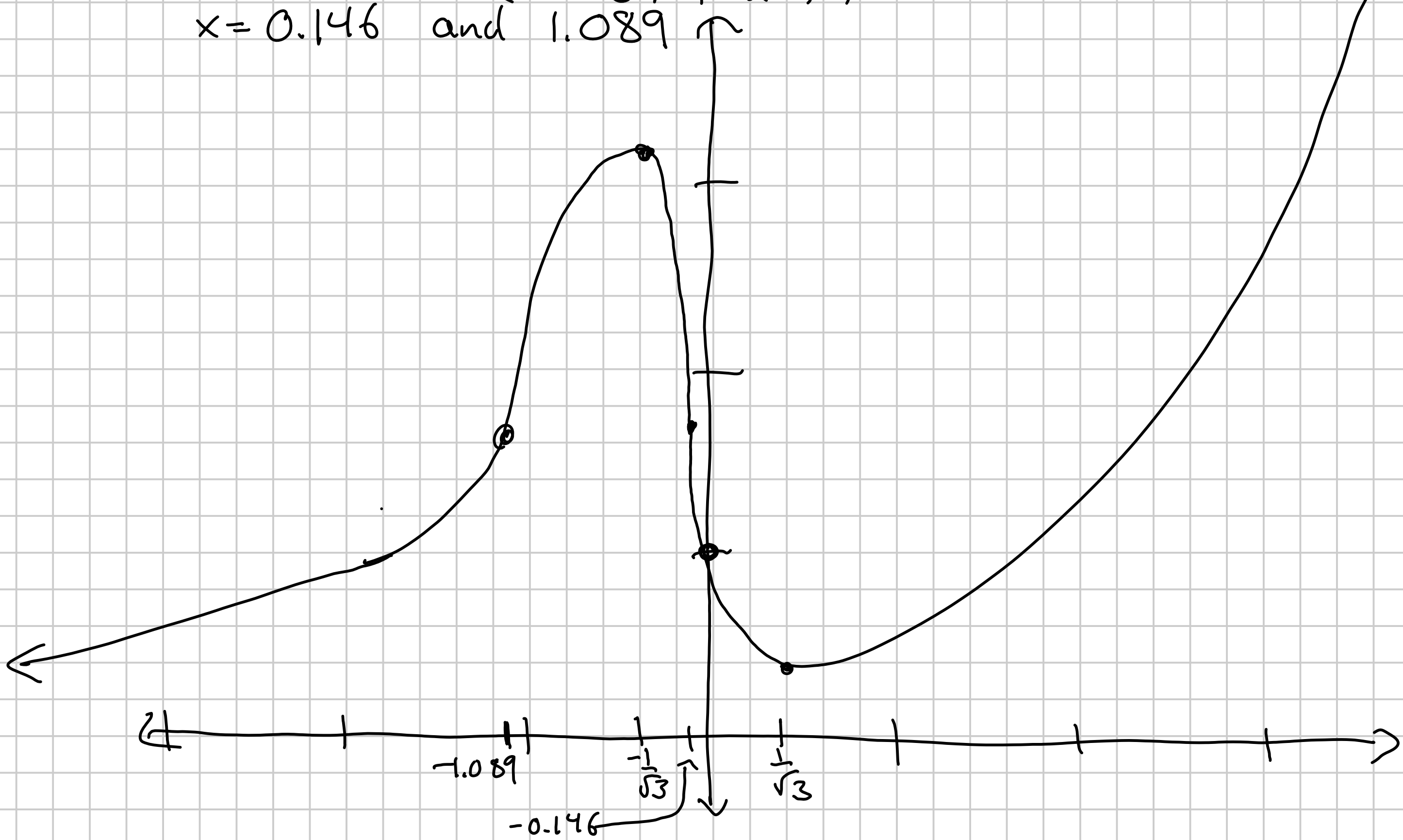
$x = \pm \frac{1}{\sqrt{3}}$

f'	+		−		+
f	incr.	$-\frac{1}{\sqrt{3}}$	decr.	$\frac{1}{\sqrt{3}}$	incr.

IPs: $f''(x) = (3x^2 - 1)^2 e^{x^3-x} + 6x e^{x^3-x}$

$= (9x^4 - 6x^2 + 6x + 1)e^{x^3-x}$

$x = 0.146$ and 1.089



3) $\frac{d}{dx}\left(x^{\cos x}\right) = \frac{d}{dx}\left(\left(e^{\ln x}\right)^{\cos x}\right) = \frac{d}{dx}\left(e^{(\cos x)(\ln x)}\right)$

$= ((\cos x)(\ln x))' e^{(\cos x)(\ln x)}$

$= \left((-\sin x)(\ln x) + (\cos x)\left(\frac{1}{x}\right)\right) x^{\cos x}$

$= \boxed{\left(-(\sin x)(\ln x) + \frac{\cos x}{x}\right) x^{\cos x}}$

4) $f(x) = C\cdot a^x \qquad f(1)=1,\ f(3)=4$

Plugin: $f(1) = C\cdot a^1 = C\cdot a = 1$

$f(3) = C\cdot a^3 = 4$ So, $\begin{array}{r} Ca^3 = 4 \\ \div\ Ca = 1 \\ \hline a^2 = 4 \\ a = 2 \end{array} \rightarrow Ca = 1$ so $C = \frac{1}{2}$

So, $\boxed{f(x) = \frac{1}{2}\cdot 2^x}$

5) $\lim_{t\to\infty}\left(1+\frac{2}{t}\right)^t = \lim_{t\to\infty}\left(1+\frac{1}{\left(\frac{t}{2}\right)}\right)^{\frac{t}{2}\cdot 2} = \left(\lim_{\frac{t}{2}\to\infty}\left(1+\frac{1}{\left(\frac{t}{2}\right)}\right)^{t/2}\right)^2$

$= \boxed{e^2}$

6) $f(x) = \frac{e^x}{x} \qquad x=1$

$f'(x) = \frac{xe^x - e^x\cdot 1}{x^2} = e^x\left(\frac{x-1}{x^2}\right)$

$f(2) = \frac{e^2}{2} \qquad f'(2) = e^2\left(\frac{2-1}{2^2}\right) = \frac{e^2}{4}$

Tangent line: $y - \frac{e^2}{2} = \frac{e^2}{4}(x-2)$

$y = \frac{e^2}{4}x - \cancel{\frac{e^2}{2}} + \cancel{\frac{e^2}{2}} \rightarrow \boxed{y = \frac{e^2}{4}x}$

7) $\ln(x+2) + \ln(x-3) = 1$

$\ln((x+2)(x-3)) = 1$

$\ln(x^2-x-6) = 1$

$x^2-x-6 = e$

$x^2-x-6-e = 0$

$x = \frac{1\pm\sqrt{1-4\cdot1(-6-e)}}{2}$

$x = \frac{1}{2} \pm \frac{\sqrt{25+4e}}{2}$

However, for $x = \frac{1}{2} - \frac{\sqrt{25+4e}}{2}$, $x-3<0$ (also $x+2<0$) so this root is not in the domain of the original question.

So, $\boxed{x = \frac{1}{2} + \frac{\sqrt{25+4e}}{2}}$ is the only solution.