Math 112
Fall 2009
Dr. Raynor

Homework #3 Solutions

#1) First we need to find the equation of the tangent line:

$y = x^2$

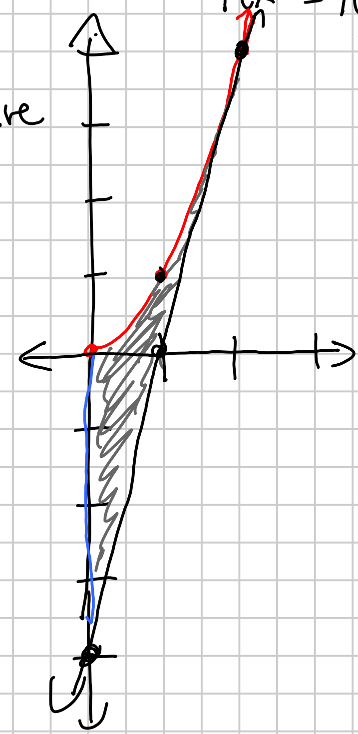
$y' = 2x$ at $x=2$, $y'=4$ so the tangent line is

$T(x) = 4(x-2)+4 = 4x-8+4 = \boxed{4x-4}$

y' value at 2

y value at 2

So, we have



The x-limits are 0 (y-axis) and 2 (the point of tangency)

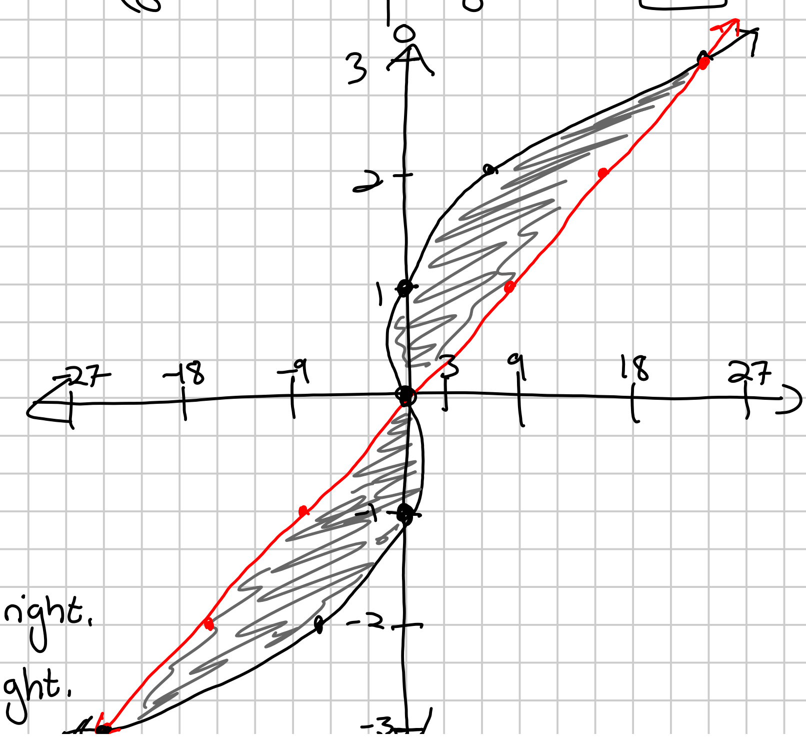
The parabola x^2 is the top function and the tangent line $4x-4$ is the bottom function, so the area is...

$$A = \int_0^2 (x^2 - (4x-4))\,dx = \int_0^2 (x^2 - 4x + 4)\,dx$$

$$= \left(\frac{x^3}{3} - 2x + 4x\right)\Big|_0^2 = \frac{8}{3} - 4 + 8 = \boxed{\frac{20}{3}}$$

#2) $x = y^3 - y$

$x = 8y$



Intersection Points:

$y^3 - y = 8y$

$y^3 - 9y = 0$

$y(y^2 - 9) = 0$

$y(y-3)(y+3) = 0$

$y = 0, 3, -3$

From -3 to 0, $y^3 - y$ is on the right.

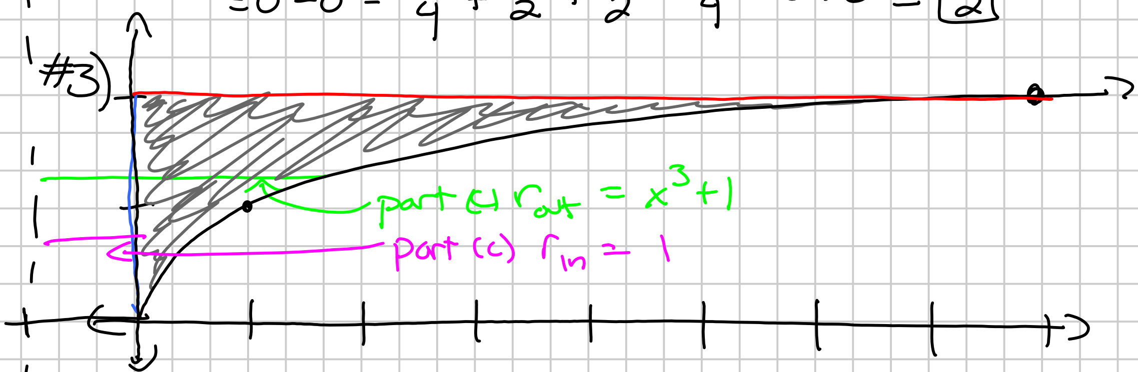
From 0 to 3, $8y$ is on the right.

So, $A = \displaystyle\int_{-3}^{0} (y^3 - y - 8y)\,dy + \int_{0}^{3} (8y - (y^3 - y))\,dy$

$$= \int_{-3}^{0} (y^3 - 9y)\,dy + \int_{0}^{3} (9y - y^3)\,dy = \left(\frac{y^4}{4} - \frac{9y^2}{2}\right)\Bigg|_{-3}^{0} + \left(\frac{9y^2}{2} - \frac{y^4}{4}\right)\Bigg|_{0}^{3}$$

$$= 0 - 0 - \frac{81}{4} + \frac{81}{2} + \frac{81}{2} - \frac{81}{4} - 0 + 0 = \boxed{\frac{81}{2}}$$

#3)



a) <u>Disk method</u>: $y = \sqrt[3]{x}$ so $x = y^3 \leftarrow$ radius
limits are $y=0$ (where $\sqrt[3]{x}$ crosses the y-axis)
and $y=2$ (where $\sqrt[3]{x}$ crosses $y=2$).

$$V = \pi\int_{0}^{2} (y^3)^2\,dy = \pi\int_{0}^{2} y^6\,dy = \frac{\pi y^7}{7}\Bigg|_{0}^{2} = \boxed{\frac{128\pi}{7}}$$

b) <u>Shell method</u>: $r = y$ because we are rotating around the x-axis and thus integrating dy.

h = right function − left function $= y^3 - 0 = y^3$

same limits as (a)

$$V = 2\pi\int_{0}^{2} y\cdot(y^3 - 0)\,dy = 2\pi\int_{0}^{2} y^4\,dy = 2\pi\,\frac{y^5}{5}\Bigg|_{0}^{2} = \boxed{\frac{64\pi}{5}}$$

c) Similar to (a), but <u>washer</u> method with a shifted radius:

outside radius $= x^3 + 1$ (see picture) Limits same as before
inside radius $= 1$

$$V = \pi\int_{0}^{2} \left((x^3+1)^2 - 1^2\right)dx = \pi\int_{0}^{2} (x^6 + 2x^3 + \cancel{1 - 1})\,dx = \pi\int_{0}^{2} (x^6 + 2x^3)\,dx$$

$$= \pi\left(\frac{x^7}{7}+\frac{x^4}{2}\right)\Big|_0^2 = \pi\left(\frac{128}{7}+8\right) = \boxed{\frac{184\pi}{7}}$$

4) $V = \pi\int_{-1}^{3}\left(4\cos^2\left(\frac{\pi x}{12}\right) - \frac{1}{1+x^2}\right)dx$

a) This integral is a washer method integral.

$V = \pi\int_a^b (r_{out}^2 - r_{in}^2)\,dx$, so the volume goes from $x=-1$ to $x=3$ and it is a volume of revolution whose outer radius is $\sqrt{4\cos^2\left(\frac{\pi x}{12}\right)} = 2\cos\left(\frac{\pi x}{12}\right)$ and whose inner radius is $\sqrt{\frac{1}{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$. The volume is generated by rotating these functions around the x-axis.

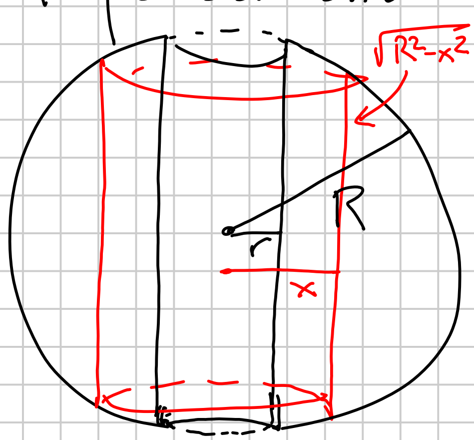
b) $b-a = 3-(-1) = 4$ and $n=4$, so $\Delta x = \frac{4}{4} = 1$.

So, the x-values are $-.5, +.5, 1.5, 2.5$

x_i	$f(x_i) = 4\cos^2\left(\frac{\pi x_i}{12}\right) - \frac{1}{1+x_i^2}$
-0.5	3.132
$+0.5$	3.132
$+1.5$	3.107
$+2.5$	2.380

$$V \approx \pi\cdot 1\cdot\left(3.132+3.132+3.107+2.380\right) = \boxed{36.913}$$

5)



Use shells:

The function for a circle is

$$x^2+y^2=R^2$$

So, $y = \pm\sqrt{R^2-x^2}$

The top is +, bottom is −.

So, the height is $+\sqrt{R^2-x^2} - -\sqrt{R^2-x^2} = 2\sqrt{R^2-x^2}$

The inside limit is the smallest radius shell that's included, which will be r. The outside limit is the largest included radius, R. So

$$V = 2\pi \int_r^R x \cdot 2\sqrt{R^2-x^2}\,dx = 4\pi \int_r^R x\sqrt{R^2-x^2}\,dx$$

$u = R^2 - x^2$

$du = -2x\,dx$

$x = r \rightarrow u = R^2 - r^2$

$x = R \rightarrow u = R^2 - R^2 = 0$

$$= -\frac{1}{2}\cdot 4\pi \int_{R^2-r^2}^{0} \sqrt{u}\,du$$

$$= 2\pi \int_0^{R^2-r^2} \sqrt{u}\,du$$

$$= 2\pi \cdot \frac{2}{3} u^{3/2} \Big|_0^{R^2-r^2} = \boxed{\frac{4\pi}{3}\left(\sqrt{R-r^2}\right)^3}$$