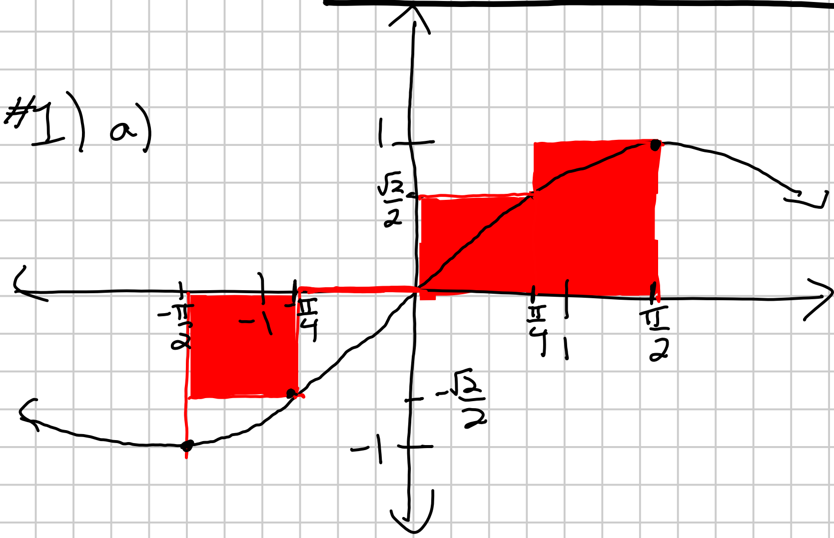
Math 112
Dr. Raynor
Fall 2009

Homework #1 Solutions

#1) a)



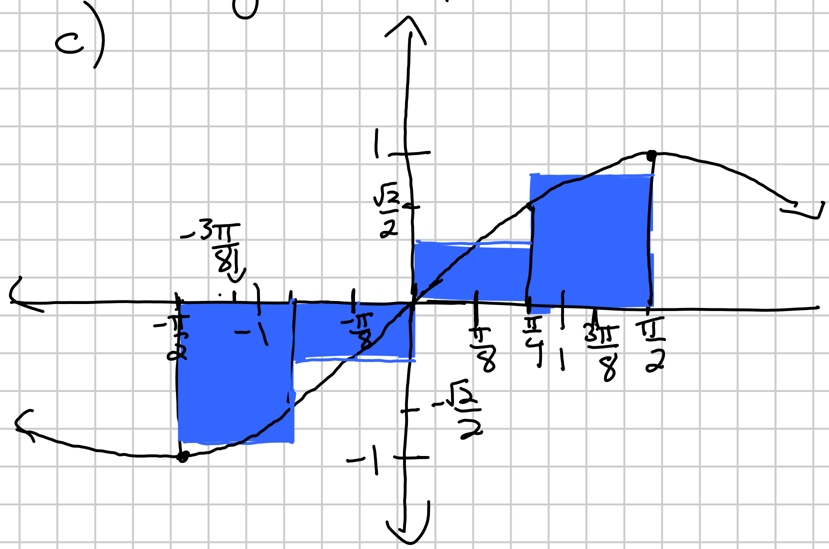
$\Delta x = \frac{\pi}{4} = \text{base}$

x_i	$f(x_i)$ = height
$-\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$
0	0
$\frac{\pi}{4}$	$+\frac{\sqrt{2}}{2}$
$\frac{\pi}{2}$	1

$$A \approx \frac{\pi}{4}\left(\frac{-\sqrt{2}}{2}\right) + \frac{\pi}{4}\cdot 0 + \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) + \frac{\pi}{4}(1) = \boxed{\frac{\pi}{4} \approx 0.7854}$$

b) My answer in part (a) is an overestimate because the function is increasing and thus all of the rectangles are above the curve, using the right endpoint method.

c)



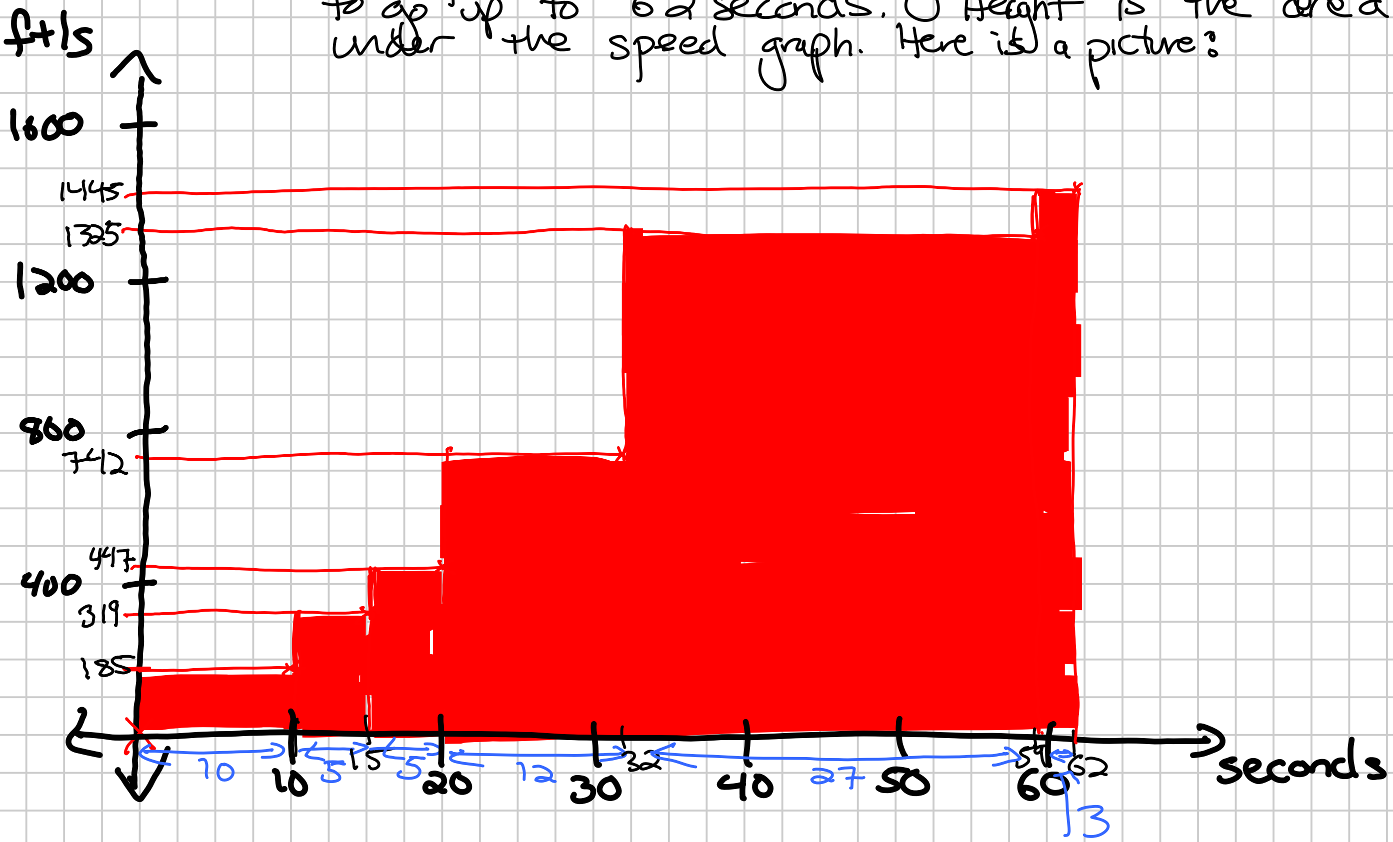
$\Delta x = \frac{\pi}{4} = \text{base}$

x_i	$f(x_i)$ = height
$-\frac{3\pi}{8}$	-0.9239
$-\frac{\pi}{8}$	-0.3827
$\frac{\pi}{8}$	$+0.3827$
$\frac{3\pi}{8}$	$+0.9239$

$$A \approx \frac{\pi}{4}\cdot{}^{-}.9239 + \frac{\pi}{4}\cdot{}^{-}.3827 + \frac{\pi}{4}\cdot .3827 + \frac{\pi}{4}\cdot .9239$$
$$= \boxed{0}$$

This answer is actually exact! The area in the two parts balances exactly between positive and negative. Or in other words, this is an odd function on an interval of the form $[-a, a]$ (with $a = \frac{\pi}{2}$) so the exact answer is 0, which is what we got using the midpoint rule.

2) To get an overestimate, we use a right-hand sum because the speed is an increasing function. We only want to go up to 62 seconds. Height is the area under the speed graph. Here is a picture:



So, the height 62 seconds after liftoff is approximately the red area

$$H \approx 10\cdot 185 + 5\cdot 319 + 5\cdot 447 + 12\cdot 742 + 27\cdot 1325 + 3\cdot 1445$$
$$= \boxed{54{,}694 \text{ feet above the earth's surface}}$$

#3) $\int_1^2 (x^2+3x)dx$ $a=1$ $b=2$ $b-a=1$

$f(x)=x^2+3x$

By definition, $\int_1^2 (x^2+3x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

Using the righthand method and n equally divided subintervals,

$$\Delta x = \frac{b-a}{n} = \frac{1}{n}$$

$$x_i = a + \frac{b-a}{n} i = 1 + \frac{i}{n}$$

$$f(x_i) = x^2+3x = \left(1+\frac{i}{n}\right)^2 + 3\left(1+\frac{i}{n}\right)$$

$$= 1 + \frac{2i}{n} + \frac{i^2}{n^2} + 3 + \frac{3i}{n}$$

$$= 4 + \frac{5i}{n} + \frac{i^2}{n^2}$$

$$f(x_i)\Delta x = \left(4+\frac{5i}{n}+\frac{i^2}{n^2}\right)\cdot\frac{1}{n} = \frac{4}{n} + \frac{5i}{n^2} + \frac{i^2}{n^3}$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(\frac{4}{n}+\frac{5i}{n^2}+\frac{i^2}{n^3}\right)$$

$$= \frac{4}{n}\sum_{i=1}^{n} 1 + \frac{5}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

Using the summation formulae: $= \frac{4}{\cancel{n}}\cdot\cancel{n} + \frac{5}{n^{\cancel{2}}}\cdot\frac{\cancel{n}(n+1)}{2} + \frac{1}{n^{\cancel{3}2}}\frac{\cancel{n}(n+1)(2n+1)}{6}$

So, the exact area is

$$A = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\left(4 + \frac{5}{2}\frac{n+1}{n} + \frac{(n+1)(2n+1)}{6n^2}\right)$$

$$= 4 + \frac{5}{2} + \frac{2}{6} = \frac{24+15+2}{6} = \boxed{\frac{41}{6}}$$

4) a) Using an equal partition & right endpoints, with $a=\pi$, $b=2\pi$, and $f(x)=\frac{\cos x}{x}$, we have: $\Delta x=\frac{b-a}{n}=\frac{2\pi-\pi}{n}=\frac{\pi}{n}$

and $x_i=a+\frac{b-a}{n}i=\pi+\frac{\pi i}{n}$

So, the Riemann sum becomes $\boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos\left(\pi+\frac{\pi i}{n}\right)}{\pi+\frac{\pi i}{n}}\cdot\frac{\pi}{n}}$

$\boxed{\text{or, } \lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos\left(\pi+\frac{\pi i}{n}\right)}{n+i}}$

b) Reading from the question, we see that
$\Delta x=\frac{2}{n}$ So $b-a=2$

$f(x_i)=\left(5+\frac{2i}{n}\right)^{10}$ so we could have $f(x)=x^{10}$

and $x_i=5+\frac{2i}{n}$, so $a=5$.

Since $b-a=2$, $b=7$.

Thus, $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(5+\frac{2i}{n}\right)^{10}=\boxed{\int_5^7 x^{10}\,dx}$