Math 331/631: Geometry
Syllabus, Fall 2007

I have discovered such wonderful things that I was amazed . . . out of nothing I have created a strange new universe. – János Bolyai, upon discovering non-Euclidean geometry

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1. Course Time & Location: MWF 11-12, Manchester 020.

2. Text: Marvin Greenberg, *Euclidean and Non-Euclidean Geometries*, 3rd ed.

3. Course description: The subject of geometry was first treated formally and *axiomatically* by Euclid around 300 B.C. He listed five 'postulates' or starting points. Assuming these five statements, Euclid showed that almost every fact we know and love about geometry in a plane was a direct consequence. Four of these postulates are very natural, but the fifth one, Euclid's parallel postulate, was disputed. It states that given a line and a point not on the line, there exists a unique line through the point that is parallel to the first line.

For over 2000 years, mathematicians tried unsuccessfully to show that this parallel postulate was a consequence of the first four. In the 1800's, two mathematicians, Bolyai and Gauss, staged a geometric coup; they showed the first 4 Euclidean postulates could hold while the parallel postulate was false and constructed *non-Euclidean geometry*, a rich and fascinating subject which forms the core of this course.

"The overthrow of Euclidean geometry is the most important event in the history of science for the epistemologist" – Hilary Putnam. Non-Euclidean geometry has great philosophical implications and will lead us to questioning everything we knew from high school geometry. In particular, most scientists agree the shape of the universe is non-Euclidean, though they cannot agree on its precise geometry. We examine hyperbolic geometry, which is non-Euclidean, in great detail in the last third of the course.

4. Homework. Homework will be assigned weekly and will be due on Wednesdays, at the start of class. The graduate version of the course will have more stringent homework assignments, in the form of additional problems and/or more difficult problems.

Academic integrity is something I take quite seriously. You are bound to uphold the University Honor Code. For this course, here are my expectations: the assignments that you submit should be your original work. Assignments should be completed individually, not as a group. The key ideas for the problems should be yours; if you want to use an idea that is not yours, you must reference how you came to understand it. Having said all of this, I encourage you to discuss the course material with your classmates (just not the key ideas underlying a problem).

5. Essay: In lieu of homework for chapter 8, 'Philosophical Implications', you will complete a 3-6 page typed essay on one of the considered topics. This should be a well-written, cohesive document, complete with an introduction and a thesis which is supported by well-developed paragraphs.

6. Midterm Exams: There will be two midterm exams but no final exam. The math 631 exam will be more challenging than the math 331 exam.

- 1st midterm: **F., Oct. 12**
- 2nd midterm: **M., Nov. 19**

7. Final Project: In lieu of a final exam, you will complete a final project for the course. A list of topics will be distributed in October, and you may suggest your own topic (with my approval). The end of each chapter of the text lists some ideas for projects. The final project will consist of two portions: (1) a 6-10 page mathematical paper on your topic, and (2) a 10-15 minute presentation on your topic – either a poster or a slideshow is recommended as a backdrop.

The default time for the presentations is our final exam period, Saturday, Dec. 15, 2-5pm. This timing is subject to debate, but changes require unanimous consent of the class. The paper is also at the time of the presentations.

8. Grade Calculation:

Homework	30%
Essay	5%
Midterm Exam 1	20%
Midterm Exam 2	20%
Final Project	25%

9. Math 331 vs. Math 631. The graduate version of this course will involve more challenging homework and midterm exams as described above. Furthermore, the final projects of math 631 students will be held to a higher standard of both mathematics and presentation than those of the math 331 students.

If you have a disability which may require an accomodation for taking this course, please contact the Learning Assistance Center (758 5929), then contact me, within the first 2 weeks of the semester.

Course material:

We will cover most of Greenberg's text in varying levels of thoroughness. Our emphasis will be towards understanding hyperbolic geometry and how it differs from standard Euclidean geometry.

Background

- Chapter 1: *Euclid's Geometry* (1 week)
  introduce the axiomatic method & Euclid's 5 postulates
- Chapter 2: *Logic* (1 lecture) don't get scared by the text here
- Chapter 2: *Incidence Geometry* (1 week)
  we encounter our first 3 axioms and discuss what kinds of geometries can obey these; we finish with a section on the projective plane.
- Chapter 3: *Hilbert's Axioms* (2-3 weeks)
  - betweenness
  - congruence
  - continuity
  - parallelism
- Chapter 4: *Neutral Geometry* (2 weeks)
  What do we get if we assume all of the axioms except the parallel postulate? It's rather exciting already!

Discovery!

- Chapter 5: *History of the Parallel Postulate* (1 week or less)
  attempts to prove the parallel postulate and what mathematicians learned from them
- Chapter 6: *Discovery of Non-Euclidean Geometry* (2-3 weeks) Bolyai's startling discoveries and how Gauss deflates his enthusiasm. Others pick up the pieces and lead us to hyperbolic geometry.

Hyperbolic Geometry

- Chapter 7: *Independence of the Parallel Postulate* (2-3 weeks)
  different models of hyperbolic geometry
- Chapter 8: *Philosophical Implications* (1 week or less)
  What does the universe look like? Dr. Jeff Weeks will give our Gentry Lectures in October on this topic. How has non-Euclidean geometry changed the world? (One section is entitled 'The Mess'!)
- Chapter 9: *Geometric Transformations* (2-3 weeks)
  learning how to do the analogue of high-school geometry in hyperbolic space
- Chapter 10: *More hyperbolic geometry* (we likely won't have time)
  So what's the Pythagorean Theorem for hyperbolic geometry? Cool results that didn't fit in chapter 9.