**Extra-Credit Project**

worth up to 15 hw points; available only to those who attended 0 or 1 extra-credit talks

Conigli e Rettangoli, S.p.A. (Rabbits & Rectangles, Inc.)
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Math 112 Students
Suite 020, Manchester Hall
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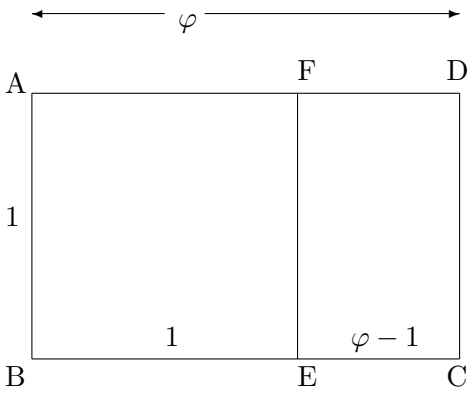
27 November 2009

Dear Math 112 Students:

Our small startup company, Rabbits and Rectangles, Inc., was founded in 2005 to leverage the myriad of connections between the golden ratio $\varphi$ and the Fibonacci numbers. As your expertise on sequences, such as the Fibonacci numbers, is of great acclaim, not only in North America but in Europe, we thank you for agreeing to consult with us. We ask that you complete the three tasks below. A timely response by Monday, December 7th is appreciated, as we need your answers for the Rabbits 2009 conference being held later that week in Rome. Your publicist, Professor J. Parsley, informs us that you will be unfortunately unable to attend our presentation in Rome on December 10th, due to some pressing calculus demands that day.

**Task 1: The golden rectangle.**
The **golden ratio** $\varphi$ is defined to be the length of a rectangle $(ABCD)$, which has width 1, so that when you remove a square $(ABEF)$ of width 1, you obtain a rectangle $(ECDF)$ of similar proportions. Here, $(ABCD)$ is sometimes called the **golden rectangle**.




The proportion we obtain is

$$\frac{\varphi}{1} = \frac{1}{\varphi - 1}$$

In this task, we ask you to solve this equation explicitly for the golden ratio $\varphi$. Express your answer both algebraically and as a decimal (to at least five decimal places). Also, express $1/\varphi$ and $\varphi^2$ in the same way. Please explain to us why there is a curious coincidence between their decimal representations.

**Task 2: The golden ratio in art & architecture.**
We understand the golden ratio appears often in art and architecture. Please investigate one such instance and discuss it in a well-written paragraph. Some examples you may want to investigate include the Parthenon in Athens, Notre Dame cathedral in Paris, da Vinci's "Vetruvian Man", but there are countless other works you may consider. You may want to include (properly cited) graphics in your response.

**Task 3: Fibonacci numbers and the golden ratio.**
As you know, the Fibonacci numbers are a sequence defined as $f_1 = 1$, $f_2 = 1$, and

$$f_n = f_{n-1} + f_{n-2} \quad \text{for } n \geq 2 \tag{1}$$

Make a table of the first 10 ratios $f_n/f_{n-1}$, that is for $n = 2$ to $n = 11$. (Your table should include these ratios both as fractions and as decimals to at least five places.) Make a conjecture about how $\varphi$ appears in the limit of $f_n/f_{n-1}$ as $n$ goes to infinity. Prove your conjecture by manipulating equation (1) algebraically.

*hint:* if $f_n/f_{n-1} \to L$, then $f_{n-1}/f_{n-2}$ also converges to $L$, and $f_{n-2}/f_{n-1}$ converges to $1/L$.

Make a table of the first 10 ratios $f_n/f_{n-2}$, that is for $n = 3$ to $n = 12$. Make a conjecture about how $\varphi$ appears in the limit of $f_n/f_{n-2}$ as $n$ goes to infinity.

In general, the ratio of $f_{n+k}/f_n$ converges to $\varphi^k$.

Again, we thank you for your time. Please keep us in mind for all of your needs when relating rabbits and rectangles.

Yours sincerely,
Lee O. Nardo
Chief Artistic Officer
Conigli e Rettangoli, S.p.A. (Rabbits & Rectangles, Inc.)

Re: **Written Requirements for Extra-Credit Project**
To: Math 112 students
From: Jason Parsley
Date: 27 October 2006

Mr. Nardo of Conigli e Rettangoli, S.p.A., has sent you a letter detailing your most recent consulting project. He has asked me to communicate the requirements that his company demands of all written reports.

### Requirements for Written Report

This is an individual project. Since this is an extra-credit project, and thus worth far less, the requirements are not as strict as for the other projects; this project is also significantly easier! You must attempt all three tasks in order to receive any credit

**Due date:** Monday, Dec. 7th, 5pm to my office or electronically

Tasks 1 and 3 may be neatly handwritten or may be typed. Task 2 must be typed. The proof in task 3 is similar to Stewart's problem 12.1.58, which appeared on an assignment.

Credit for the logo belongs to one of my research collaborators, Prof. Jason Cantarella at the University of Georgia. It is a computer-generated polyhedron of a rabbit first used for a geometry conference announcement.