Kraft E. Coyote
1998 Birdseed Lane
Santa Fe, NM 87501

Math 112 Students
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Wake Forest University
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Dear Math 112 Students:

Please help me!!! For the last two months, I have had these recurring nightmares that are about to drive me crazy. When I went looking for help, the folks at Acme referred me to your professor, who said you would be perfect for the job.

The nightmare scenario is nearly always the same. I am standing at the road that is 1 kilometer long, and there at the other end is that infernal Roadrunner. Perhaps you have heard of my father, Wile E., and the torments this bird inflicted upon him.

Anyway, the Roadrunner is just standing there, sticking his tongue out, saying "Beep, beep!" I start to go after him, but I can only run in slow motion, at $1 \mathrm{~m} / \mathrm{s}$. Surreally, after one second, the road stretches uniformly and instantaneously by 1 kilometer, so now that pesky fowl is 1998 meters away, since some of the stretch happens behind me. I try to speed up, but I'm still moving in slow motion, at $1 \mathrm{~m} / \mathrm{s}$. After another second, I've moved another meter, and then the road stretches again, uniformly and instantaneously, by 1 kilometer. This just keeps on happening. Over and over, again and again. Then I wake up, frustrated and hungry, my pillowcase wet with drool.

I've gotta know: Do I ever get to the Roadrunner? Is there a chance? If I do get there, how long does it take? Should I take a snack to eat along the way? If I can, please give me some reference for the time, since every second chasing that tasty bird seems like an eternity.
Also, would running faster help? I'm faster than my father ever was, and I'm sure with positive thinking, I could run faster in my dream. Can you suggest a minimum speed (please tell me in both $m / s$ and $m p h$ ) at which I could run in my dream that would guarantee that I'd catch the Roadrunner before I come out of the dream cycle 90 minutes later? Is it possible for me to go that fast?

You've gotta help me! Please let me know by December 2nd, so that I can make my plans for Christmas dinner. You're invited to drop by - dinner will be BYOB (bring your own bird).

Carnivorously yours,

Kraft E. Coyote

## Comments \& Requirements for Written Report

This project is an individual one. You have the choice of completing Mr. Coyote's request, or to complete the Applied Project on page 793 of the text on blackbody radiation. (I recommend the Coyote's, unless you're willing to read ahead.)
Your findings should be detailed in a typed technical report of 3-8 pages. The hungry Coyote will accept (neatly) hand-drawn calculations and figures; professional graphics, generated from Maple, might help your report, but are certainly not required. No scratch work should be submitted.
The report should be written in a clear, concise manner using proper grammar. You are free to use any references you like. You should cite any references that you utilize, except for the course lecture and book. I encourage you to work very little with other people; please cite anyone who has helped you with your project. Dr. Parsley is available for consultation, either in office hours or by appointment.
The report is due at the start of class on Wednesday, December 2nd. You should submit an electronic copy of this project on Sakai, and submit a printed copy in class.

## Grade Determination

Your compensation ('grade', if you prefer such language) will be calculated in four categories as given below.

| Category | Worth |
| :--- | :---: |
| Mathematical Description of the Problem | 20 pts |
| Correct Solution | 30 pts |
| Explanation of Reasoning for Solutions | 35 pts |
| Style and Grammar | 15 pts |
| Total | 100 pts |

## Hints

1. First, make sure you understand why the Roadrunner is 1998 meters away after the first stretching of the road.
2. Next, set up a sequence $\left\{d_{n}\right\}$ where $d_{n}$ represents the distance in meters between Kraft E. and the roadrunner after $n$ seconds, but before the road does its stretch. For instance, $d_{0}=1000 \mathrm{~m}$ and $d_{1}=999 \mathrm{~m}$. Then, write

$$
\begin{aligned}
d_{1} & =1 *\left(\text { some expression involving } d_{0}\right) \\
d_{2} & =2 *\left(\text { some expression involving } d_{1}\right) \\
d_{3} & =3 *\left(\text { some expression involving } d_{2}\right)
\end{aligned}
$$

Now convert your expressions for $d_{2}$ and $d_{3}$ so that they only involve $d_{0}$. Use this to find a general expression for $d_{n}$ in terms of $d_{0}$. What is the behavior of $\left\{d_{n}\right\}$ ?
3. The question about a faster speed involves applying some similar reasoning to step 2 , but with proper adjustments. You may need to look back at some of our estimates and error bounds.
4. Good luck, this is one hungry coyote we're dealing with. I don't recommend getting on his bad side (e.g., by missing his deadline).
5. You might be able to use a fact about the integral test that we mentioned in class: the $k$ th partial sum $s_{k}$ of a series is bounded by

$$
\int_{2}^{k+1} f(x) d x \leq s_{k}=\sum_{n=1}^{k} a_{n} \leq \int_{1}^{k} f(x) d x
$$

where $f$ is a postive, decreasing, continuous function with $f(n)=a_{n}$.

