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RESEARCH ARTICLE

Understanding Geometry in the Dance Studio

Jason Parsley[a*] and Christina Tsoules Soriano[b]

[a] *Mathematics Dept., Box 7388*; [b] *Theatre and Dance Dept., Box 7264*
Wake Forest University, Winston-Salem, NC 27109
www.wfu.edu/∼parslerj/geometry-dance/

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This paper details the collaboration between a math and a dance professor, which begins with the choreography of a geometrically inspired work. It charts a shared pedagogical experiment, undertaken in two successive semesters, which joins dance students alongside students in a liberal arts math course. Together they experience Platonic solids both in a mathematics classroom and later in a dance studio. We collect feedback from the experience and describe its background and generalizations. This paper concludes with two open questions in human graph theory.

Keywords: dance, Platonic solids, interdisciplinary pedagogy

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1 Introduction

Most geometers can trace their spatial intuition back to some experiential learning in their childhood. One author, Parsley, recollects a childhood spent obsessively studying maps and sledding down hills, the topography of his favorite ones still firmly etched in memory. The other author, Soriano, a choreographer and a dance professor, argues that choreographers are active practitioners of three-dimensional geometry. Many shapes familiar to mathematicians arise naturally in the dance studio during rehearsals. Perspective, similarity, and dimension are vital concepts which choreographers must utilize in their aesthetic design.

Our collaboration together began in December 2006 when Soriano was creating a dance (*The Trace of a Moving Point*) about potential deconstructions of a line for 14 dancers, and sought Parsley's assistance with applying geometry to moving bodies in space. Through rehearsals and discussions, we considered mathematical ideas such as self-similarity, the Cantor set, Pascal's triangle, and reflections within an ellipse for this piece. Once it was complete, our partnership shifted in a pedagogical direction: the connection between dance and mathematics was symbiotic and something our students could benefit from recognizing.

We were both teaching courses that are particularly ripe environments for interdisciplinary innovation. *Explorations in Mathematics*, a liberal arts math course, steps away from the oft comfortable realm of theorem-proof discourse and finds diverse entries into mathematics. Similarly, *Dance Composition* teaches about the process of making a dance, one free from immutable rules or principles.

Choreographers and mathematicians are more similar than one might initially believe. Creating a dance and constructing a proof are both journeys, where missteps along the way are frequently

*Corresponding author. Email: parslerj@wfu.edu
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instructive and where inspiration often arrives from unexpected sources. Choreographers and mathematicians both search for patterns, even beyond the obvious primary ones, and examine why such patterns exist. Both disciplines hope students arrive with a deep curiosity and an adventurous heart.

In this paper, we describe our endeavor joining math and dance students to learn and interpret Platonic solids. By using the kinetic, three-dimensional body as a dynamic construction, math students may peer into a choreographer's toolbox and discover how dance borrows from geometry; dance students may recognize how geometric shapes are omnipresent in the studio. After describing our exercise and its goals below, we examine students' feedback. This endeavour seeks, among other things, to improve their spatial reasoning and help them understand duality and the students claim it was a success. After a brief view of established connections between dance and math, we look forward and pose two open questions. But first, let us begin with the Platonic solids.

2 Platonic Solids

Recall that a Platonic solid is a regular polyhedron; its congruent faces are comprised of the same regular polygon. Their namesake, Plato, was fascinated by these solids and pondered them in his treatise *Timaeus*. There are precisely five different Platonic solids: the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron.[1] A straightforward argument, relying on either topology or geometry, shows that these are the only regular polyhedra; proofs of this fact date back to Euclid's *Elements* (Book XIII, Proposition 18). Euclid also provided constructions for each solid in Propositions 13-17.

We define the dual of a Platonic solid as an inscribed polyhedron in the following way. Begin by placing a vertex at the center of each face of the original solid. If two faces of the original solid share an edge, connect the corresponding new vertices via an edge. Finally, add a new face to the dual solid whenever the edges that bound it correspond to edges in the original solid that all meet at a vertex. Thus, original faces correspond to dual vertices, original vertices correspond to dual faces, and original edges correspond to dual edges. Since this construction preserves regularity, the dual polyhedron is itself a Platonic solid. Hence, the dual of a dual solid is a smaller copy of the original Platonic solid,[2] and the five solids pair off via duality:

- The cube and the octahedron are dual.
- The dodecahedron and the icosahedron are dual.
- The tetrahedron is dual to itself.

3 Motivation and Goals

Our interdisciplinary endeavor uses human motion to teach math students about geometry and uses geometry to teach dance students about human motion. We begin with dance students attending math class to learn about Platonic solids. Using pipe cleaners and straws (or toothpicks and gumdrops), the students build models and discover for themselves that there are only five types of Platonic solids. A few days later, students in both disciplines gather in the dance studio and are asked to interpret the Platonic solids. Through this physical learning exercise, we seek to permanently etch their geometric properties into the students' memories. Our collaboration seeks to develop understanding via

[1]Though 'tetrahedron' refers in general to any 4-faced polyhedron, in this paper we only refer to regular tetrahedra; we omit the adjective 'regular' for all of these solids.

[2]While this version of duality does not produce a congruent copy of the original solid, it has the advantage of being more easily described and visualized than versions of duality which do. Furthermore, it produces a solid which contains a sequence of smaller copies of itself.

- *spatial reasoning skills* – such skills are vital for both dance and geometry students.
- *experiential learning* – math students, especially in a liberal arts math course, can benefit from and enjoy active learning approaches, which are naturally more common for dance students.
- *duality and self-similarity* – Platonic solids provide a concrete example of duality. By viewing for example the dual of the dual to a cube, students obtain a sequence of cubes containing smaller cubes. Such a construction motivates discussions of scale and self-similarity, important concepts in both dance and mathematics.
- *regularity* – what is the significance of equal lengths and angles? From an aesthetic or a mathematical perspective, what are the consequences of breaking regularity?
- *the interconnectedness of math and art* – through this exploration, dance students might better utilize geometric ideas in future projects, and math students might better understand mathematics in a broader, real-world context.
- *human structural forms* – for a choreographer, Platonic solids propose a challenging creative idea; what static and dynamic spatial realities result?

4 Our Interdisciplinary Classroom

During two different semesters in 2007, our dance and math courses met together in the dance studio to interpret Platonic solids after they had first constructed models of these polyhedra in math class. Both semesters, the math students were from a liberal arts math course. In April, *Dance Composition* students participated; Soriano's teaching schedule dictated that *Beginning Modern Dance Technique* students took part in September. Provided with toy models and a handout, students from both disciplines were tasked to form a cube, an octahedron, a dodecahedron, and an icosahedron, along with a dual cube-octahedron pair. Each shape was then locomoted and shown from different perspectives. In April, the students also dynamically embodied reflections within an ellipse using rope, mirrors, and each other; split into two groups, they next choreographed a dance illustrating these reflections. Below we describe their creations; most examples and figures are taken from the September collaboration, which primarily focused on the Platonic solids.

Below we present a few images from our class; more photos and short videos, along with supplementary materials may be found at `www.wfu.edu/∼parslerj/geometry-dance/`. All photos are copyright Jason Parsley, 2007.

4.1 *Cube*

Beginning with the most recognizable shape, the cube, the students split into four groups of 7 and one group of 8. The group of 8 had the easiest task; each person formed a triad, see Figure **??**; each torso became a trivalent vertex. Half were seated with each leg forming half an edge, 90° apart; the other half were standing with their arms 90° apart as half-edges. By joining hands or feet appropriately, they completed each horizontal edge.

The groups of 7 faced a more challenging task; seven triads do not form a cube. Their constructions required different uses for their bodies; some bodies became edges, some were faces, while some people remained as triads. Each group constructed their cube differently; even within a group different members represented different objects. Often certain vertices, edges, and/or faces were merely implied, not embodied. The floor's edge was a natural choice as the bottom face of the cube, until it was put in motion. Once their design was finished, each group put their shape into motion; they had to show the cube having just one edge on the ground and then just one vertex. For example, one group first had to expand in height and width, while maintaining their form of edges and vertices, before rotating. These 'performances' allowed ideas to cross-pollinate.

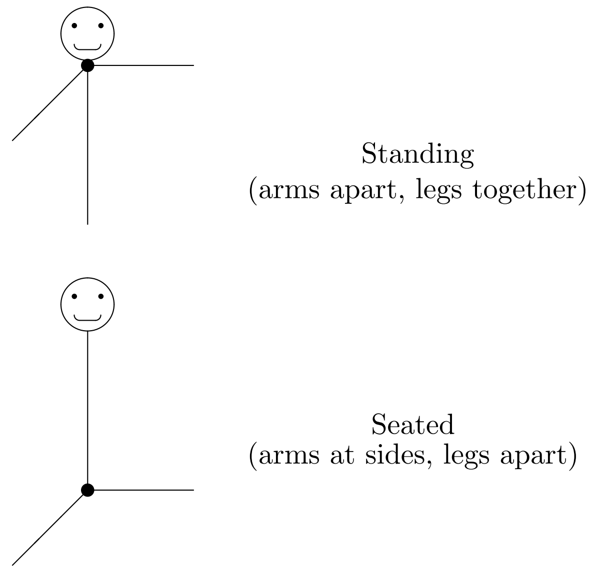

Figure 1. Triads, three edges concurrent at a vertex, represent stick figures who are forming a cube

Figure 2. First group's octahedron (left) and second group's octahedron (right).

Figure 3. The third and fourth groups' octahedra; the third group (left) features counterbalancing and issues with regularity.

4.2 *Octahedron*

Next four groups of 9 or 10 each formed an octahedron; see Figures **??** and **??**. Our first group of 10 utilized a central axis connecting top and bottom vertices through reaching arms upward in space while eight others, some seated, some leaning with outstretched arms toward the top, formed faces around her. Offering a similar construction, a second group gathered eight people around a central axis figure, who clapped to commence the rotational motion of octahedron. Since all figures were standing, rotating their figure was easy; laying it on a face or edge was more challenging. Four members of this group braced onto one another's elbows to display, from any perspective, the right angle formed by non-adjacent edges meeting at a vertex.

Another group embodied octahedron exploited gravity and shared weight. By bending at their hips, four students form the eight faces of the octahedron in Figure **??**. In opposite pairs, these students counterbalance by linking hands; this added support allows each pelvis to jut out in space at a pronounced angle, approximating the obtuse dihedral angle of the octahedron. By sharing weight, this group had several dynamic options for locomoting their shape. At the top vertex, four other students of vastly different sizes struggled to form a realistic framework.

The fourth octahedron group chose to kneel and lean back towards their heels and against the supportive, standing 'upper faces'. Both dancers and math students animatedly worked together negotiating weight, height, and codependency of movement in order to maintain a challenging shape. While the other groups were able to rotate, and in some cases roll their figure, this group's construction struggled with motion.

Each group represented the faces of the octahedra in slightly different ways. Most used portions of their torsos; for some it was from shoulders to waist, for others shoulders to knees; the third group had two faces per body – one from head to waist, one from waist to feet. One dance student had choreographed movement challenge where individual bodies peeled away from the

Figure 4. Two different dual cube/octahedron pairs

Figure 5. The first group constructs a dodecahedron. Five seated figures (shown in blue on the diagram) lean back and form edges with their torsos; each of their arms forms one edge. Five standing figures (show in green on the diagram) lean in and form faces with their torsos; their lower arms are together to form an edge; their upper arms with their shoulder form one edge.

Figure 6. The second group's dodecahedron also features five seated figures (shown in blue on the diagram), each of whom represents four edges. There are 10 people standing; the outer five (in green) represent upper edges of the solid, while the inner five (in red) outstretch their arms to form the top pentagon.

octahedron while other people noticed the absence and tried to replace the missing edges or vertices.

4.3 *Dual Cube/Octahedron*

The next task was to build a dual cube and octahedron pair. Two groups of 20 both chose to build a cube containing a smaller octahedron; see Figure **??**. They drew directly upon previous constructions: all bottom vertices began as seated 'L-shapes' with their legs extended. All top vertices were either triads or 'L-shapes' formed by extended arms. One octahedron followed the third group's weight counterbalancing approach, while the other knelt and leaned like the fourth group's did.

4.4 *Dodecahedron*

The same two groups then attempted the more challenging construction of dodecahedra. More vertices, edges, and faces led to more confusion; leaders for each group quickly emerged. Also, our students were simply not as familiar with regular pentagons as they were with equilateral triangles and squares. One immediate challenge was which body parts to use in making a pentagon. Some pentagons had two people's arms, two torsos and one person's extended legs. Others had two torsos, two arms, and used the floor to imply the fifth edge. Others used half an arm for an edge. An upper pentagonal face was formed by another group of five students in the middle with arms extended at shoulder length and approximating 108 degree angles with each other. A number of other body combinations achieved pentagons as well; see Figures **??** and **??**. Though successful at constructing a dodecahedron, our groups learned the difficulties of putting it into motion.

4.5 *Icosahedron*

In neither semester was there adequate time to construct an icosahedron. In September, we allowed students a few minutes to grasp how complicated an object with 20 faces can be. We were delighted when one student suggested to her peers that they use the dodecahedron and swap the roles of faces and vertices; implementing this duality construction was a challenge as fatigue set in and time ran out.

5 Feedback and Evaluations

We concluded the class period with a discussion and later surveyed our students in order to measure the impacts of this exercise. Although subjective, our survey results indicated that we had satisfied, to varying extents, our goals from Section **??**. Our first goal was to increase our

Figure 7. A student examines duality

students' spatial reasoning abilities; 21 out of 22 respondents mentioned that their visualization skills had increased. One student wrote, "I really got a feel for how 3D these figures are. Dealing with bodies and gravity while trying to construct these figures really showed all of their dimensions and how they can be rotated in space." Another dance student saw future benefits: "The 3D image we created with bodies is now something that will come to mind anytime I think of a Platonic solid. Attempting to move through space in these shapes was an interesting inspiration for possible spatial patterns to be used in dance choreography."

Extending this reasoning to human forms, many students were shocked by the utility and necessity of leaning on and supporting each other, both at rest and in motion. Figures **??** and **??** each show two pairs of students counterbalancing each other to form an octahedron. One student noted, "My favorite part was seeing how to use other bodies and gravity to make different shapes, to support, and to stretch."

Duality, a concept transcending both disciplines, became less abstract for our participants, who were fascinated by the idea of an infinite pattern of solids within solids. One wrote, it is "easier to remember what different shapes are after this class. Duality is much more understandable once I was physically able to see it."

Regularity was the most frequently mentioned topic in our discussion. Students appreciated how symmetrical the solids were and struggled to achieve that symmetry in their constructions. A varsity football player in the math class (featured in Figure **??**) mentioned the physical flexibility challenges of trying to pair with someone 14 inches shorter as shown in Figure **??**. One student wrote, "I learned how important it is for the angles and sides of the regular polygon to be the same. Our shapes were distorted because not all our bodies were the same." Another commented, "Each person has a different height, so making the shapes perfect is almost impossible. You also have to physically rely on one another and be aware of what everyone's doing around you."

The surveys illuminate another important point: our students thoroughly enjoyed this experiential learning activity. Thirteen respondents mentioned this specifically; for instance, "I thought this was a really fun activity, especially the more complicated solids." "Do it again! It's fun to apply math in different ways." One benefit of nontraditional pedagogical approaches is to incite enthusiasm among disaffected students, who often disproportionately populate a liberal arts math class.

As further feedback, our first collaboration in the spring encouraged enrollment in the fall *Explorations in Mathematics* class. Over one-quarter of the students (10 of 38) described themselves as having a moderate or strong dance background. One math student taught dance in the summer at Governor's School, for talented high school students across disciplines, and mentioned our exercise. Furthermore, learning about the aims of this math course motivated a spring dance student to take it in the fall, even though she had finished her math requirements; she, so far, is the only one to have participated from both 'sides' of our exercise.

6 Connections Between Dance and Geometry

Many others have explored the crossroads of geometry and modern dance; see the detailed bibliography in [**?**]. Mathematicians sarah-marie belcastro and Karl Schaffer have both explored the mathematical underpinnings of dance; see [**?**] for a profile of Schaffer. Together they gave a well-attended presentation at the 2008 Joint Meetings on *Dancing mathematics and the mathematics of dance.*

Geometry's inherent connection to the moving body has also been studied by several dance and design scholars. Most important among them are two German artists: Oskar Schlemmer, a Bauhaus influenced choreographer, artist, architect and costume designer, and Rudolph von

Laban, founder of the most widely used notation system in dance: Laban Movement Analysis – a system of documenting a dance with symbols or descriptions based on the dance's effort, time and space. Schlemmer and Laban both kept geometric ideas, and Platonic solids in particular, at the core of their movement and design philosophies.

The kinesphere, as defined by Laban, represents "the sphere around the human body whose periphery is reachable by easily extended limbs" [**?** , p. 10]. Not all points inside this sphere are physically attainable, so according to Laban, an individual's kinesphere more accurately forms an icosahedron. He also describes how one natural subset of these reaches forms an octahedron, while another subset represents a cube. Following Laban, we developed one activity for our next collaboration: students should trace out, individually or in pairs, their own kinespheres, find an icosahedron within it and explore the golden rectangles which span it.

Oskar Schlemmer [**?**] utilized costume design as a geometric transformation of the human body; his sketches recognize common mathematical symbols inherent in the human form, such as the infinity sign made by folded arms. His designs are instructed by what he calls the "laws of cubical space in planimetric and stereometric relationships" [**?** , p. 332].

Geometric shapes embodied in human form are commonplace in early childhood and K-12 dance education. Dance education pioneer Anne Green Gilbert dedicates an entire section of [**?**] to using dance as a tool to understand mathematical problems, with one chapter specifically dedicated to physically constructing geometric shapes. Schaffer's book aims to simultaneously teach dance and mathematics to students in grades 4-12; "when we choreograph a new dance or investigate a mathematical problem we are doing much the same thing: creatively exploring patterns in space and time with an eye towards aesthetic potential" [**?** , p. 5]. We concur that such an interdisciplinary approach works well "when a concept needs to be understood mentally, physically, and emotionally" [**?** , p. 5].

7 Future Directions

Geometry and dance are fundamentally connected, and our collaboration regarding this connection strives to both enrich our students and further geometric concepts in dance composition. By providing a working model, we seek to convince others that interdisciplinary collaborations can bring together math, sciences, and the arts in fruitful, meaningful ways. To accomplish this, we are presenting our experience together in a variety of forums: math conferences, dance panels, internal teaching fairs. Our collaboration is ongoing and might expand to other courses that we teach and possibly to future choreographed works.

To construct faces, vertices, and edges, our students utilized body parts in many different ways, many more than we present herein. Two natural open questions arise:

Question 7.1 What polyhedral shapes can one person accurately[1] depict with her body? In particular, if restricted to the 1-skeleton (vertices and edges only), what graphs are humanly realizable by one person? By two people?

Question 7.2 Again restricting to the 1-skeleton, what is the fewest number of people necessary to accurately construct the vertices and edges of each Platonic solid? Here, bodies are not allowed to represent faces.

A complete answer to either question must document the arrangement of limbs and torsos. Though two people can build a cube if their torsos are allowed to be faces, we believe it takes at least four people to accurately depict the 1-skeleton of the cube. The same questions can also be raised for hand and finger constructions, with presumably smaller answers; [**?** , §8.4] investigates 'hand polyhedra' and describes a two-person cube and tetrahedron.

[1]'Accurately' is purposefully left undefined. One interpretation is that each edge must be physically realized, not implied; the regularity of the solid could be compromised to some degree.