PHY 712 Electrodynamics

12-12:50 AM MWF via video link:
https://wakeforest-university.zoom.us/my/natalie.holzwarth

Plan for Lecture 32:

Special Topics in Electrodynamics:

Cherenkov radiation

References: Jackson Chapter 13.4

Zangwill Chapter 23.7

Smith Chapter 6.4

In this lecture we will consider the phenomenon of Cherenkov radiation. The discussion follows the treatment of Zangwill and Smith.

21	Mon: 03/23/2020	Chap. 9	Radiation from localized oscillating sources	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 9	Radiation from oscillating sources	#18	03/27/2020
23	Fri: 03/27/2020	Chap. 9 and 10	Radiation from oscillating sources	#19	03/30/2020
24	Mon: 03/30/2020	Chap. 11	Special Theory of Relativity	#20	04/03/2020
25	Wed: 04/01/2020	Chap. 11	Special Theory of Relativity		
26	Fri: 04/03/2020	Chap. 11	Special Theory of Relativity	#21	04/06/2020
27	Mon: 04/06/2020	Chap. 14	Radiation from accelerating charged particles	#22	04/08/2020
28	Wed: 04/08/2020	Chap. 14	Synchrotron radiation		
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020	Chap. 14	Synchrotron radiation	#23	04/15/2020
30	Wed: 04/15/2020	Chap. 15	Radiation from collisions of charged particles	#24	04/17/2020
31	Fri: 04/17/2020	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/20/2020	Chap. 13	Cherenkov radiation		
33	Wed: 04/22/2020		Special topic: E & M aspects of superconductivity		
34	Fri: 04/24/2020		Special topic: Aspects of optical properties of materials		
35	Mon: 04/27/2020		Review		
36	Wed: 04/29/2020		Review		

04/20/2020 PHY 712 Spring 2020 -- Lecture 32 2

Reminder of the schedule. No new homework is assigned; leaving time for your work on your projects.

Cherenkov radiation

Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013. http://www.britannica.com/EBchecked/media/174732

This is a view of Cherenkov radiation with its typical blue glow.

The Nobel Prize in Physics 1958

Pavel A. Cherenkov
Il´ja M. Frank
Igor Y. Tamm

Affiliation at the time of the award: P.N. Lebedev Physical Institute, Moscow, USSR

Prize motivation: "for the discovery and the interpretation of the Cherenkov effect."

https://www.nobelprize.org/prizes/physics/1958/ceremony-speech/

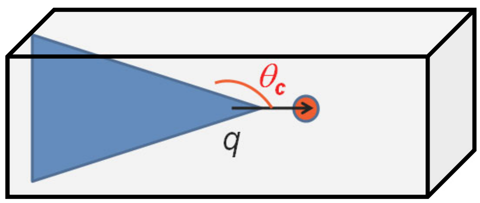
04/20/2020 PHY 712 Spring 2020 -- Lecture 32 4

A Nobel prize was awarded for the discovery and explanation of this phenomenon.

References for notes: Glenn S. Smith, *An Introduction to Electromagnetic Radiation* (Cambridge UP, 1997), Andrew Zangwill, Modern Electrodynamics (Cambridge UP, 2013)

Cherenkov radiation

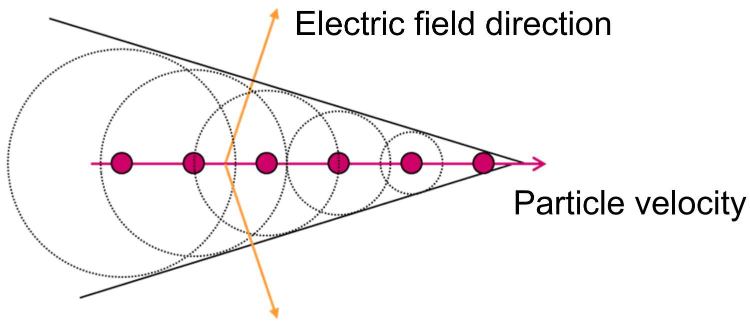
Discovered ~1930; bluish light emitted by energetic charged particles traveling within dielectric materials



Note that some treatments give the critical angle as θ_c-π/2.

A diagram describing the phenomenon.

From: http://large.stanford.edu/courses/2014/ph241/alaeian2/



Snapshots of the particle as it moves through the medium and of the wave fronts generated.

Maxwell's potential equations within a material having permittivity and permeability (Lorentz gauge; cgs Gaussian units)

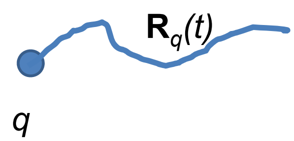
$$\nabla^2\Phi - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\rho$$

$$\nabla^2\mathbf{A} - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\mathbf{J}$$

Source: charged particle moving on trajectory $\mathbf{R}_q(t)$:

$$\rho(\mathbf{r},t) = q\delta\left(\mathbf{r} - \mathbf{R}_q(t)\right)$$

$$\mathbf{J}(\mathbf{r},t) = q\dot{\mathbf{R}}_q(t)\delta\left(\mathbf{r} - \mathbf{R}_q(t)\right)$$



Analysis of the scalar and vector potentials in the dielectric medium due to the particle of charge q.

Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon}\frac{1}{\left|R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r)\right|}$$

$$\mathbf{A}(\mathbf{r},t) = q\mu \frac{\boldsymbol{\beta}_n}{\left|R(t_r) - \boldsymbol{\beta}_n \cdot \mathbf{R}(t_r)\right|}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r)$$

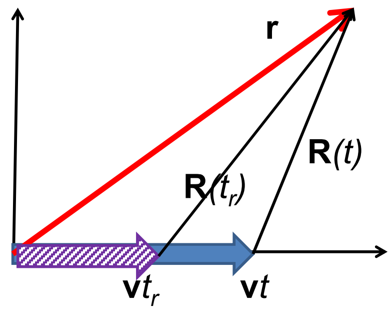
$$\boldsymbol{\beta}_n(t_r) \equiv \frac{\dot{\mathbf{R}}_q(t_r)}{c_n} \qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

$$t_r = t - \frac{R(t_r)}{c_n}$$

04/20/2020 PHY 712 Spring 2020 -- Lecture 32 8

Find the Lienard Wiechert solutions within the medium. Here, the major difference from previous solutions is that the wave speed c_n depends on the refractive index of the medium.

Consider a particle moving at constant velocity $\mathbf{v}$; $v > c_n$



Some algebra

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

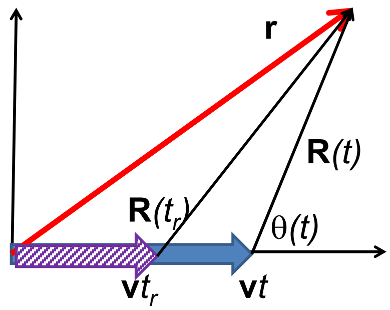
$$(t - t_r)c_n = R(t_r) = \left|\mathbf{R}(t) + \mathbf{v}(t - t_r)\right|$$

Quadratic equation for $(t - t_r)c_n$:

$$((t - t_r)c_n)^2 = R^2(t) + 2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t - t_r)c_n + \beta_n^{\,2}((t - t_r)c_n)^2$$

$$(t - t_r)c_n = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm \sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2 - (\beta_n^{\,2} - 1)R^2(t)}}{\beta_n^{\,2} - 1}$$

For the vacuum case v<c, but not it is possible for $v>c_n$. Here we can solve the quadratic equation for the variables of the problem. The physical solution must be positive.



$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r)$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n =$$

$$(t - t_r)c_n\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$= R(t_r)\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$R(t_r) = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm \sqrt{\left(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\right)^2 - \left(\beta_n^{\,2} - 1\right)R^2(t)}}{\beta_n^{\,2} - 1}$$

$$R(t_r) = \frac{R(t)}{\beta_n^{\,2} - 1}\left(-\beta_n\cos\theta \pm \sqrt{1 - \beta_n^{\,2}\sin^2\theta}\right) = (t - t_r)c_n$$

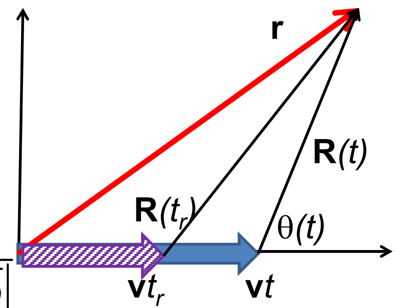
$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n = \mp R(t)\sqrt{1 - \beta_n^{\,2}\sin^2\theta}$$

Continuing the analysis for the variables needed to determine the scalar and vector potentials.

Liénard-Wiechert potentials for two solutions:

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon}\frac{1}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$

$$\mathbf{A}(\mathbf{r},t) = q\mu\frac{\boldsymbol{\beta}_n}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$



For $\beta_n > 1$, the range of θ is limited:

$$R(t_r) = \frac{R(t)}{\beta_n{}^2 - 1}\left(-\beta_n\cos\theta \pm \sqrt{1-\beta_n{}^2\sin^2\theta}\right) \geq 0$$

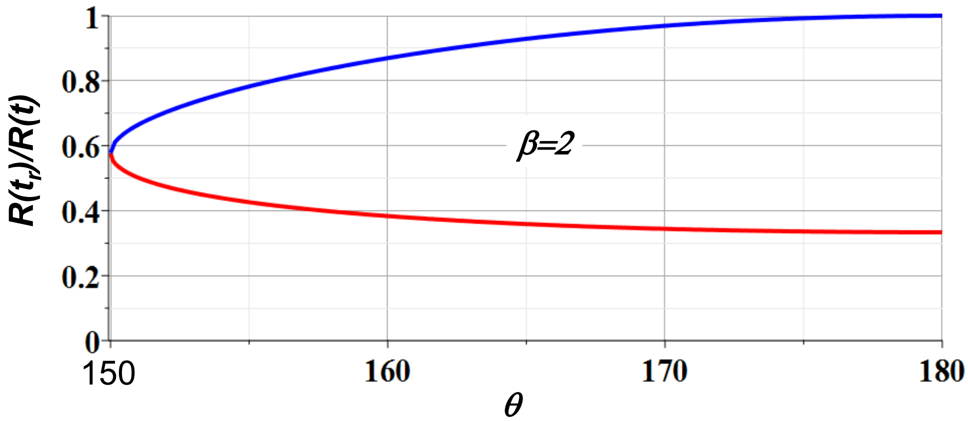
$$\Rightarrow |\sin\theta| \leq \frac{1}{\beta_n} \equiv |\sin\theta_c| \quad \text{and} \quad \pi \geq \theta_c \geq \pi/2$$

➔Diagram is not correct!

In this range, $\theta \geq \theta_c$

In order for the analysis to be consistent, cos(theta)<0 and sin(theta)<1/beta.

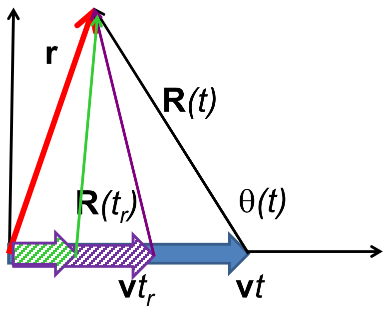
$$R(t_r) = \frac{R(t)}{\beta_n^2 - 1}\left(-\beta_n \cos\theta \pm \sqrt{1-\beta_n^2 \sin^2\theta}\right)$$



$$\theta_c = 150^o \text{ for this case}$$

Plot showing two solutions as a function of theta for a particular beta.

Physical fields for $\beta_n > 1$ -- two retarded solutions contribute



$$\theta \leq \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Define $\cos\theta_C \equiv -\sqrt{1-\frac{1}{\beta_n^2}}$

$$\Rightarrow \cos\theta \leq \cos\theta_C$$

Adding two solutions; in terms of Heaviside $\Theta(x)$:

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

Here we use the Heaviside step function to ensure that the angle theta is in the correct range.

Physical fields for β > 1

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon} \frac{1}{R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}} \Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu \frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}} \Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi - \frac{1}{c_n}\frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

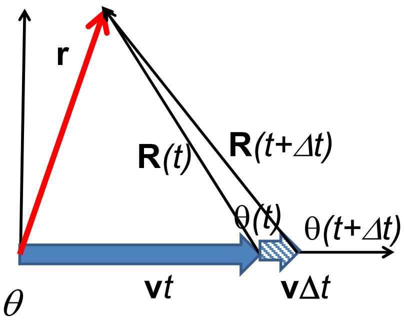
$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon} \frac{\hat{\mathbf{R}}}{\left(R(t)\right)^2 \sqrt{1-\beta_n^{\,2}\sin^2\theta}} \times$$

$$\left(\frac{\beta_n^{\,2}-1}{1-\beta_n^{\,2}\sin^2\theta} \Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^{\,2}-1}\ \delta(\cos\theta_C - \cos\theta(t)) \right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta \left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Summary of results for potentials and the corresponding electric and magnetic fields.

Intermediate steps:



$$\frac{d\theta}{dt} = \frac{v \sin\theta}{R} \qquad \frac{dR}{dt} = -v\cos\theta$$

Using instantaneous polar coordinates: $\nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial \theta}$

$$\nabla\Theta\left(\cos\theta_C - \cos\theta(t)\right) = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta\left(\cos\theta_C - \cos\theta(t)\right)}{\partial t} = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{v\sin^2\theta(t)}{R(t)}$$

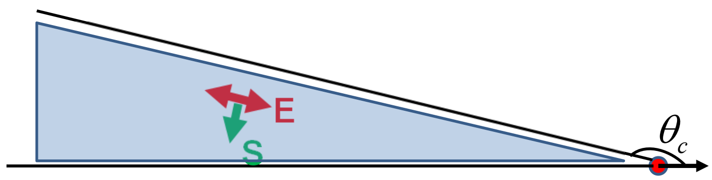
Some details for finding the fields.

Cherenkov radiation observed near the angle θ_c -- continued

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon} \frac{\hat{\mathbf{R}}}{(R(t))^2 \sqrt{1-\beta_n^{\,2}\sin^2\theta}} \times$$

$$\left(\frac{\beta_n^{\,2}-1}{1-\beta_n^{\,2}\sin^2\theta} \Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^{\,2}-1}\;\delta(\cos\theta_c - \cos\theta(t)) \right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta \left(\hat{\theta} \times \mathbf{E}(\mathbf{r},t) \right)$$



Frequency dependence of intensity:

$$\frac{dI}{d\omega} \approx \frac{q^2}{c^2}\omega\left(1 - \frac{1}{\beta^2\epsilon(\omega)}\right)$$

From this point, we need to calculate the intensity. When the dust clears, we find the intensity relationship mentioned above.

A few details --

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{\left(R(t)\right)^2\sqrt{1-\beta_n{}^2\sin^2\theta}} \quad \times$$

$$\left(\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n{}^2-1}\ \delta(\cos\theta_c - \cos\theta(t)\right.$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} dt\ e^{-i\omega t}\mathbf{E}(\mathbf{r},t) \qquad \tilde{\mathbf{B}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} dt\ e^{-i\omega t}\mathbf{B}(\mathbf{r},t)$$

$$\langle\mathbf{S}(\mathbf{r},\omega)\rangle = \frac{c}{8\pi\mu}\ \tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)$$

Frequency dependence of intensity:

$$\frac{dI}{d\omega} \approx \frac{q^2}{c^2}\omega\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)$$

04/20/2020 PHY 712 Spring 2020 -- Lecture 32 17

Why does this formula imply that the intensity is greatest for blue light?