PHY 712 Electrodynamics

12-12:50 AM MWF via video link:

https://wakeforest-university.zoom.us/my/natalie.holzwarth

Extra notes for Lecture 32:

Special Topics in Electrodynamics:

Cherenkov radiation

References: Jackson Chapter 13.4

Zangwill Chapter 23.7

Smith Chapter 6.4

In this lecture we will consider the phenomenon of Cherenkov radiation. The discussion follows the treatment of Zangwill and Smith.

21	Mon: 03/23/2020	Chap. 9	Radiation from localized oscillating sources	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 9	Radiation from oscillating sources	#18	03/27/2020
23	Fri: 03/27/2020	Chap. 9 and 10	Radiation from oscillating sources	#19	03/30/2020
24	Mon: 03/30/2020	Chap. 11	Special Theory of Relativity	#20	04/03/2020
25	Wed: 04/01/2020	Chap. 11	Special Theory of Relativity		
26	Fri: 04/03/2020	Chap. 11	Special Theory of Relativity	#21	04/06/2020
27	Mon: 04/06/2020	Chap. 14	Radiation from accelerating charged particles	#22	04/08/2020
28	Wed: 04/08/2020	Chap. 14	Synchrotron radiation		
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020	Chap. 14	Synchrotron radiation	#23	04/15/2020
30	Wed: 04/15/2020	Chap. 15	Radiation from collisions of charged particles	#24	04/17/2020
31	Fri: 04/17/2020	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/20/2020	Chap. 13	Cherenkov radiation		
33	Wed: 04/22/2020		Special topic: E & M aspects of superconductivity		
34	Fri: 04/24/2020		Special topic: Aspects of optical properties of materials		
35	Mon: 04/27/2020		Review		
36	Wed: 04/29/2020		Review		

Reminder of the schedule. No new homework is assigned; leaving time for your work on your projects.

Your questions –

From Trevor

1. If we were to calculate the intensity of Cherenkov radiation, would it be done in the same way as is shown for the synchrotron radiation? Would it require us to find the power distribution, and then apply Parceval's theorem to get from the time integrated power per solid angle to the intensity per solid angle per frequency, or is there a faster way to get there?
2. On slide 16, the equation for the frequency dependent intensity introduces what appears to be a function of angular frequency, epsilon(omega). Is this a special function, or is it just a simplified way of writing something that comes out of the details for that derivation?

From Surya

When a charged particle travels in the dielectric medium, its velocity decreases, so does light. The definition of Cherenkov radiation implies that charged particles have greater velocity than the phase velocity of light in the dielectric medium. Does this necessarily mean that Cherenkov particles have velocities greater than light in vacuum as well? What are the applications of Cherenkov radiation?

Comments – There were some omissions/shortcuts in the derivation; more comments made in context ..

Cherenkov radiation

Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013. http://www.britannica.com/EBchecked/media/174732

This is a view of Cherenkov radiation with its typical blue glow.

The Nobel Prize in Physics 1958

Pavel A. Cherenkov
Il´ja M. Frank
Igor Y. Tamm

Affiliation at the time of the award: P.N. Lebedev Physical Institute, Moscow, USSR

Prize motivation: "for the discovery and the interpretation of the Cherenkov effect."

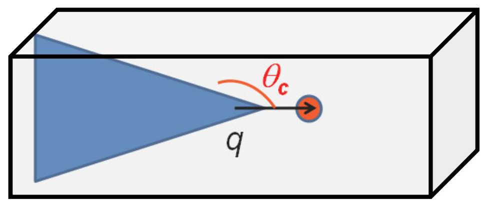
https://www.nobelprize.org/prizes/physics/1958/ceremony-speech/

A Nobel prize was awarded for the discovery and explanation of this phenomenon.

References for notes: Glenn S. Smith, *An Introduction to Electromagnetic Radiation* (Cambridge UP, 1997), Andrew Zangwill, Modern Electrodynamics (Cambridge UP, 2013)

Cherenkov radiation

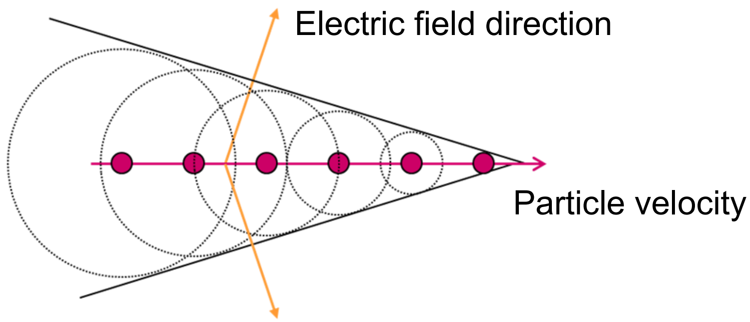
Discovered ~1930; bluish light emitted by energetic charged particles traveling within dielectric materials



Note that some treatments give the critical angle as $\theta_c-\pi/2$.

A diagram describing the phenomenon.

From: http://large.stanford.edu/courses/2014/ph241/alaeian2/



Snapshots of the particle as it moves through the medium and of the wave fronts generated.

Maxwell's potential equations within a material having permittivity and permeability (Lorentz gauge; cgs Gaussian units)

$$\nabla^2\Phi - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\rho$$

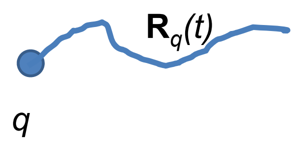
$$\nabla^2\mathbf{A} - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\mathbf{J}$$

Here the values of μ and ε depend on the material and on frequency.

Source: charged particle moving on trajectory $\mathbf{R}_q(t)$:

$$\rho(\mathbf{r},t) = q\delta\left(\mathbf{r} - \mathbf{R}_q(t)\right)$$

$$\mathbf{J}(\mathbf{r},t) = q\dot{\mathbf{R}}_q(t)\delta\left(\mathbf{r} - \mathbf{R}_q(t)\right)$$



Analysis of the scalar and vector potentials in the dielectric medium due to the particle of charge q.

Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t)=\frac{q}{\varepsilon}\frac{1}{\left|R(t_r)-\boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)\right|}$$

$$\mathbf{A}(\mathbf{r},t)=q\mu\frac{\boldsymbol{\beta}_n}{\left|R(t_r)-\boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)\right|}$$

$$\mathbf{R}(t_r)\equiv\mathbf{r}-\mathbf{R}_q(t_r)$$

$$\boldsymbol{\beta}_n(t_r)\equiv\frac{\dot{\mathbf{R}}_q(t_r)}{c_n} \qquad c_n\equiv\frac{c}{\sqrt{\mu\varepsilon}}\equiv\frac{c}{n}$$

$$t_r=t-\frac{R(t_r)}{c_n}$$

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Find the Lienard Wiechert solutions within the medium. Here, the major difference from previous solutions is that the wave speed c_n depends on the refractive index of the medium.

Example --

$$\beta_n \equiv \frac{v}{c_n} \qquad c_n \equiv \frac{c}{\sqrt{\mu\varepsilon}} \equiv \frac{c}{n}$$

Consider water with $n \approx 1.3$

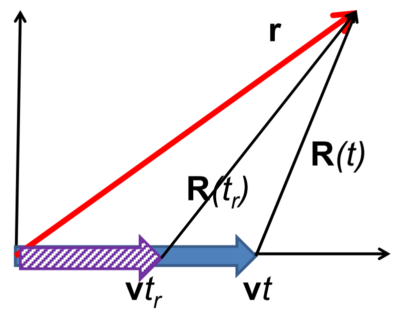
Which of these particles could produce Cherenkov radiation?

1. A neutron with speed c?
2. An electron with speed 0.6c?
3. A proton with speed 0.6c?
4. An electron with speed 0.8c?
5. An alpha particle with speed 0.8c?
6. None of these?

Further comment –

As Surya noted and as discussed particularly in Chap. 13 of Jackson, a particle moving within a medium is likely to be slowed down so that the Cherenkov effect will only happen while $\beta_n>1$.

Consider a particle moving at constant velocity **v**; $v > c_n$



Some algebra

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r) = \left|\mathbf{R}(t) + \mathbf{v}(t - t_r)\right|$$

Quadratic equation for $(t - t_r)c_n$:

$$((t - t_r)c_n)^2 = R^2(t) + 2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t - t_r)c_n + \beta_n^{\,2}((t - t_r)c_n)^2$$

$$(t - t_r)c_n = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm \sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2 - (\beta_n^{\,2} - 1)R^2(t)}}{\beta_n^{\,2} - 1}$$

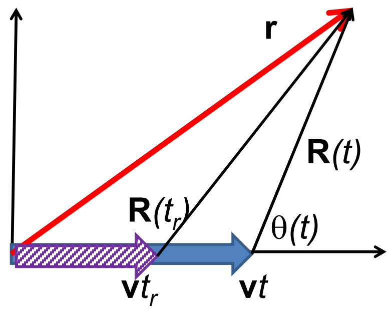
For the vacuum case v<c, but not it is possible for v>c_n. Here we can solve the quadratic equation for the variables of the problem. The physical solution must be positive.

Consider the result:

$$\left(t-t_r\right)c_n = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm \sqrt{\left(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\right)^2 - \left(\beta_n^{\,2}-1\right)R^2(t)}}{\beta_n^{\,2}-1}$$

How can this result be physical when $\beta_n>1$?

1. It is always physical.
2. It cannot be physical.
3. It can be physical for only for the + solution
4. It can be physical for only the – solution
5. It makes no sense.



$$\mathbf{R}(t_r) = \mathbf{r} - \mathbf{v}t_r = \mathbf{R}(t) + \mathbf{v}(t - t_r)$$

$$(t - t_r)c_n = R(t_r)$$

$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n = (t - t_r)c_n\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n = R(t_r)\left(1 - \beta_n^2\right) - \mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$R(t_r) = \frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n \pm \sqrt{\left(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\right)^2 - \left(\beta_n^{\,2} - 1\right)R^2(t)}}{\beta_n^{\,2} - 1}$$

$$R(t_r) = \frac{R(t)}{\beta_n^{\,2} - 1}\left(-\beta_n \cos\theta \pm \sqrt{1 - \beta_n^{\,2}\sin^2\theta}\right) = (t - t_r)c_n$$

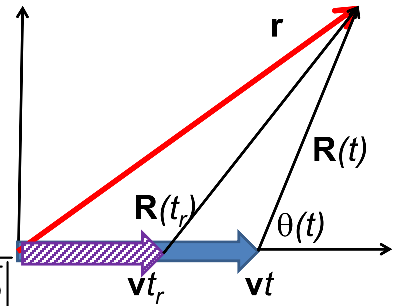
$$R(t_r) - \mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n = \mp R(t)\sqrt{1 - \beta_n^{\,2}\sin^2\theta}$$

Continuing the analysis for the variables needed to determine the scalar and vector potentials.

Liénard-Wiechert potentials for two solutions:

$$\Phi(\mathbf{r},t)=\frac{q}{\varepsilon}\frac{1}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$

$$\mathbf{A}(\mathbf{r},t)=q\mu\frac{\boldsymbol{\beta}_n}{\left|\mp R(t)\sqrt{1-\beta_n{}^2\sin^2\theta}\right|}$$



For $\beta_n>1$, the range of θ is limited:

$$R(t_r)=\frac{R(t)}{\beta_n{}^2-1}\left(-\beta_n\cos\theta\pm\sqrt{1-\beta_n{}^2\sin^2\theta}\right)\geq 0$$

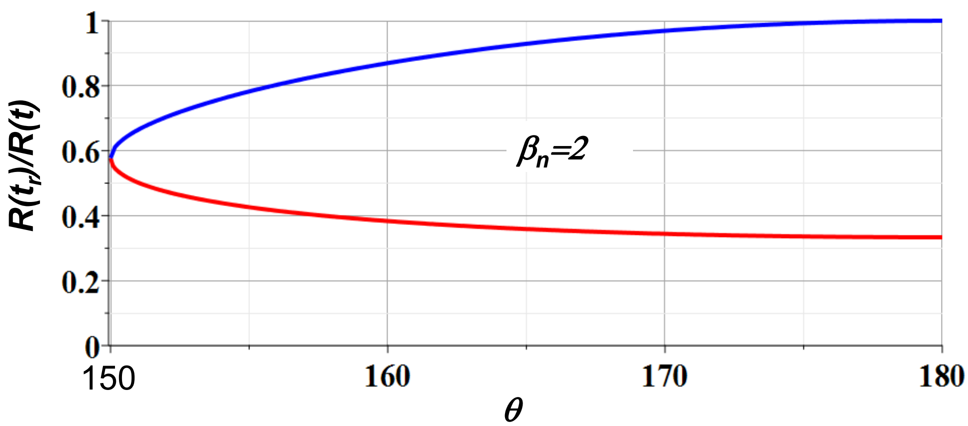
$$\Rightarrow\left|\sin\theta\right|\leq\frac{1}{\beta_n}\equiv\left|\sin\theta_c\right| \quad \text{and} \quad \pi\geq\theta_c\geq\pi/2$$

In this range, $\theta\geq\theta_c$

➔Diagram is not correct!

In order for the analysis to be consistent, cos(theta)<0 and sin(theta)<1/beta.

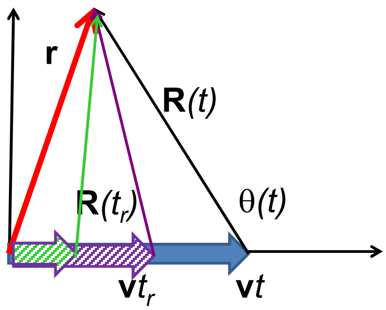
$$R(t_r) = \frac{R(t)}{\beta_n^{\,2} - 1}\left(-\beta_n \cos\theta \pm \sqrt{1 - \beta_n^{\,2} \sin^2\theta}\right)$$



$$\theta_c = 150^o \text{ for this case}$$

Plot showing two solutions as a function of theta for a particular beta.

Physical fields for $\beta_n > 1$ -- two retarded solutions contribute



$$\theta \leq \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Define $\cos\theta_C \equiv -\sqrt{1-\frac{1}{\beta_n^{\,2}}}$

$$\Rightarrow \cos\theta \leq \cos\theta_C$$

Adding two solutions; in terms of Heaviside $\Theta(x)$:

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

Here we use the Heaviside step function to ensure that the angle theta is in the correct range.

Physical fields for β > 1

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi - \frac{1}{c_n}\frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$

$$\left(\frac{\beta_n^2-1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\ \delta(\cos\theta_C - \cos\theta(t))\right)$$

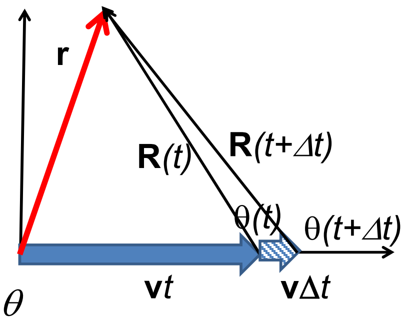
$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Summary of results for potentials and the corresponding electric and magnetic fields.

How are these results different from the analysis of Chapter 14 such as synchrotron radiation?

1. Very similar
2. Equations are similar
3. Total different case

Intermediate steps:



$$\frac{d\theta}{dt} = \frac{v\sin\theta}{R} \qquad \frac{dR}{dt} = -v\cos\theta$$

Using instantaneous polar coordinates: $\nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial \theta}$

$$\nabla\Theta\left(\cos\theta_C - \cos\theta(t)\right) = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta\left(\cos\theta_C - \cos\theta(t)\right)}{\partial t} = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{v\sin^2\theta(t)}{R(t)}$$

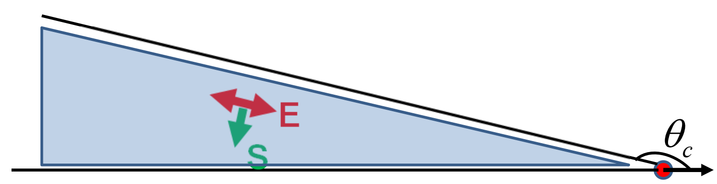
Some details for finding the fields.

Cherenkov radiation observed near the angle θ_c -- continued

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n{}^2\sin^2\theta}} \times$$

$$\left(\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C-\cos\theta(t))+\sqrt{\beta_n{}^2-1}\;\delta(\cos\theta_c-\cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$



Frequency dependence of intensity:

$$\frac{dI}{d\omega} \approx \frac{q^2}{c^2}\omega\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)$$

Note that the permittivity ε function is written as a function of ω

From this point, we need to calculate the intensity. When the dust clears, we find the intensity relationship mentioned above.

A few details --

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n{}^2\sin^2\theta}} \quad \times$$

$$\left(\frac{\beta_n{}^2-1}{1-\beta_n{}^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n{}^2-1}\ \delta(\cos\theta_c - \cos\theta(t)\right.$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} dt\ e^{-i\omega t}\mathbf{E}(\mathbf{r},t) \qquad \tilde{\mathbf{B}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} dt\ e^{-i\omega t}\mathbf{B}(\mathbf{r},t)$$

$$\langle\mathbf{S}(\mathbf{r},\omega)\rangle = \frac{c}{8\pi\mu}\ \tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)$$

In order to evaluate the integrals, further approximations must be made, particularly recognizing that the dominating signal occurs very close to the time the particle passes. Details are discussed by Smith and by Zangwill.

Why does this formula imply that the intensity is greatest for blue light?

When the dust clears --

Frequency dependence of intensity:

$$\frac{dI}{d\omega} \approx \frac{q^2}{c^2}\omega\left(1 - \frac{1}{\beta^2\epsilon(\omega)}\right)$$

From this expression, how would you explain that Cherenkov radiation is typically observed as a blue glow?

1. It is still a mystery.
2. It is obvious from the result.

Why is this useful?