PHY 712 Electrodynamics

12-12:50 AM MWF via video link:

https://wakeforest-university.zoom.us/my/natalie.holzwarth

Plan for Lecture 31:

Start reading Chap. 15 –

Radiation from collisions of charged particles

1. Overview
2. X-ray tube
3. Radiation from Rutherford scattering
4. Other collision models

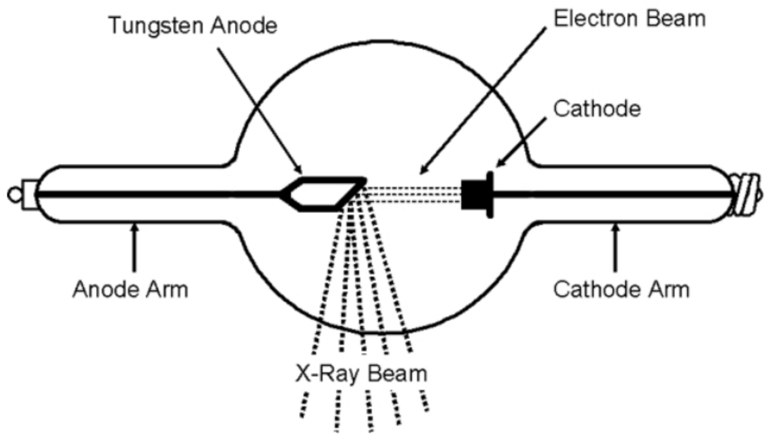
In this lecture we will discuss some examples of radiation due to charged particles colliding. It is a complicated topic which quite a few famous physicists have worked on.

21	Mon: 03/23/2020	Chap. 9	Radiation from localized oscillating sources	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 9	Radiation from oscillating sources	#18	03/27/2020
23	Fri: 03/27/2020	Chap. 9 and 10	Radiation from oscillating sources	#19	03/30/2020
24	Mon: 03/30/2020	Chap. 11	Special Theory of Relativity	#20	04/03/2020
25	Wed: 04/01/2020	Chap. 11	Special Theory of Relativity		
26	Fri: 04/03/2020	Chap. 11	Special Theory of Relativity	#21	04/06/2020
27	Mon: 04/06/2020	Chap. 14	Radiation from accelerating charged particles	#22	04/08/2020
28	Wed: 04/08/2020	Chap. 14	Synchrotron radiation		
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020	Chap. 14	Synchrotron radiation	#23	04/15/2020
30	Wed: 04/15/2020	Chap. 15	Radiation from collisions of charged particles	#24	04/17/2020
31	Fri: 04/17/2020	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/20/2020	Chap. 13	Cherenkov radiation		
33	Wed: 04/22/2020		Special topic: E & M aspects of superconductivity		
34	Fri: 04/24/2020		Special topic: Aspects of optical properties of materials		
35	Mon: 04/27/2020		Review		
36	Wed: 04/29/2020		Review		

This is the revised schedule, subject to your input.

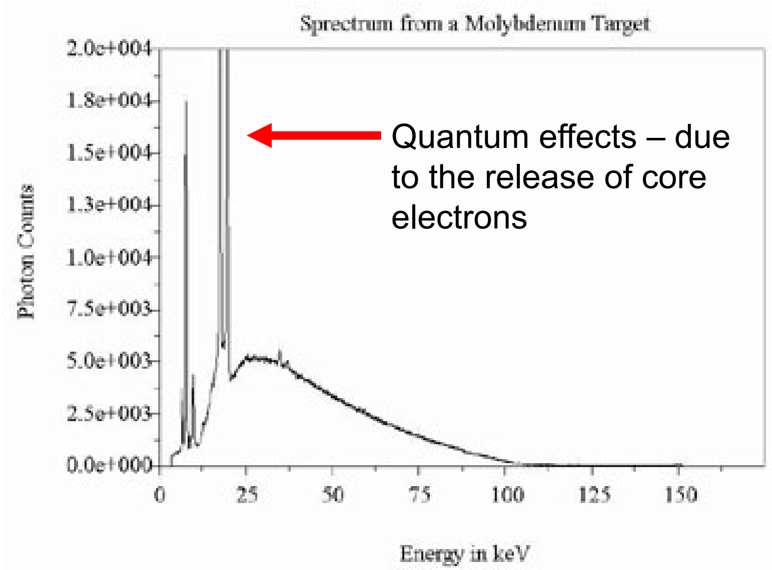
Generation of X-rays in a Coolidge tube

https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm

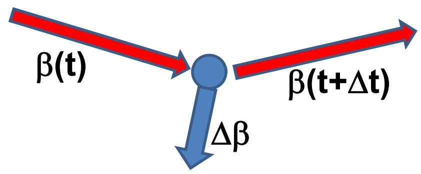


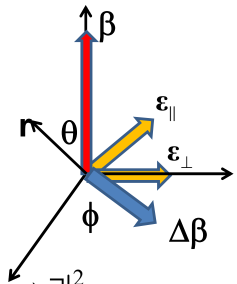
Invented in 1913. Associated with the German word "bremsstrahlung" – meaning breaking radiation.

http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Radiation during collisions





Intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\; e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)} \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

Note that $\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)=\hat{\mathbf{r}}\left(\hat{\mathbf{r}}\cdot\boldsymbol{\beta}\right)-\boldsymbol{\beta}=-\left(\boldsymbol{\varepsilon}_{\parallel}\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_{\parallel}-\left(\boldsymbol{\varepsilon}_{\perp}\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_{\perp}$

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega\rightarrow 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)}-\frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Starting from the intensity analysis for radiation due to a charged particle moving in a trajectory with beta representing its velocity/c. We will consider the velocity changing due to a collision process and analyze the radiation at small frequencies.

HN1

Radiation during collisions -- continued

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Non-relativistic limit:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot(\Delta\boldsymbol{\beta})\right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$$

Relativistic collision with small $|\Delta\boldsymbol{\beta}| \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$

For beta<<1, we can neglect the denominator of the expression and obtain the non-relativistic expression. It is also convenient to analyze the relativistic case when the change in velocity is small.

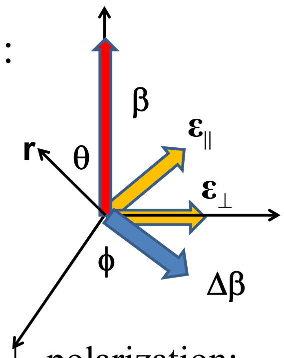
Slide 6

HN1 Holzwarth, Natalie, 4/16/2020

Radiation during collisions -- continued

Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta}+\hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$



Also assume $\Delta\boldsymbol{\beta}$ is perpendicular to $\boldsymbol{\beta}$ direction

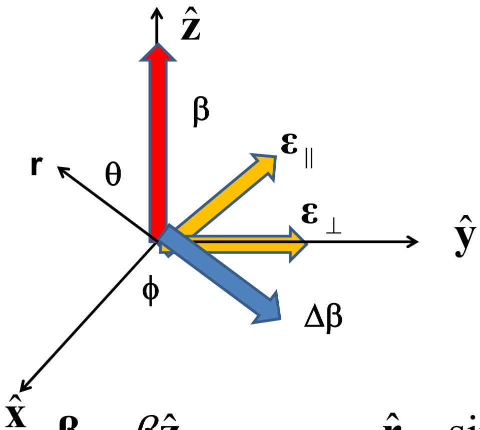
Expressions (averaging over φ) for || or $\perp$ polarization:

$$\frac{d^2I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\frac{(\beta-\cos\theta)^2}{(1-\beta\cos\theta)^4}$$ polarization in ***r*** and ***β*** plane

$$\frac{d^2I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2\frac{1}{(1-\beta\cos\theta)^2}$$ polarization perpendicular to ***r*** and ***β*** plane

It is convenient to consider two different polarizations of the radiation – parallel (meaning in the plane of the observation point r and the initial velocity of the particle) and perpendicular (meaning perpendicular to that plane).

Some details:



$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}} \qquad \hat{\mathbf{r}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{\parallel} = -\cos\theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{z}} \qquad \boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}}$$

$$\boldsymbol{\Delta\beta} = \Delta\beta\left(\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}\right)$$

04/17/2020 PHY 712 Spring 2020 -- Lecture 31 8

Showing the detailed geometry of the scattering process.

Some details -- continued:

$$\hat{\mathbf{r}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{||} = -\cos\ \theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}}$$

$$\Delta\boldsymbol{\beta} = \Delta\beta\left(\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}\right)$$

$$\Delta\boldsymbol{\beta} + \hat{\mathbf{r}} \times \left(\boldsymbol{\beta} \times \Delta\boldsymbol{\beta}\right) = \Delta\boldsymbol{\beta}\left(1 - \hat{\mathbf{r}} \cdot \boldsymbol{\beta}\right) + \boldsymbol{\beta}\left(\hat{\mathbf{r}} \cdot \Delta\boldsymbol{\beta}\right)$$

$$\boldsymbol{\varepsilon}_{\perp} \cdot \left(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}} \times \left(\boldsymbol{\beta} \times \Delta\boldsymbol{\beta}\right)\right) = \Delta\beta\sin\phi\left(1 - \beta\cos\theta\right)$$

$$\boldsymbol{\varepsilon}_{||} \cdot \left(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}} \times \left(\boldsymbol{\beta} \times \Delta\boldsymbol{\beta}\right)\right) = \Delta\beta\cos\phi\left(\beta - \cos\theta\right)$$

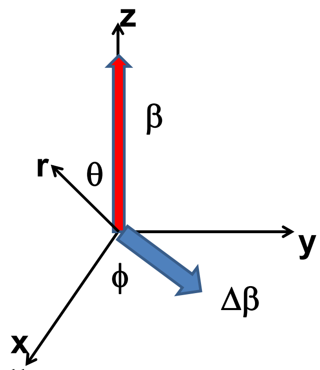
Evaluating the vectors.

Radiation during collisions -- continued

Intensity expressions:

$$\frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$



Relativistic collision at low ω and with small $|\Delta\boldsymbol{\beta}|$ and $\Delta\boldsymbol{\beta}$ perpendicular to plane of $\hat{\mathbf{r}}$ and $\boldsymbol{\beta}$, as a function of θ where $\hat{\mathbf{r}}\cdot\boldsymbol{\beta} = \beta\cos\theta$;

Integrating over solid angle:

$$\frac{dI}{d\omega} = \int d\Omega \left(\frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right) = \frac{2}{3\pi} \frac{q^2}{c} \gamma^2 \; |\Delta\boldsymbol{\beta}|^2$$

It is possible to analytically integrate over all solid angles.

Some details:

$$\int d\Omega \frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \, 2\pi \int_{-1}^{1} d\cos\theta \frac{(\beta - \cos\theta)^2}{(1 - \beta\cos\theta)^4}$$

$$= \frac{q^2}{4\pi c} |\Delta\boldsymbol{\beta}|^2 \frac{2}{3} \frac{1}{(1-\beta^2)}$$

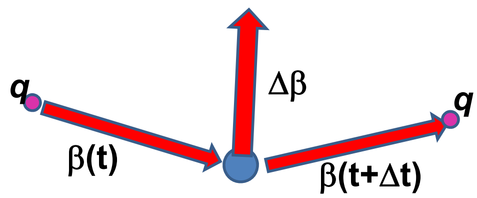
$$\int d\Omega \frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c} |\Delta\boldsymbol{\beta}|^2 \int_{-1}^{1} d\cos\theta \frac{1}{(1 - \beta\cos\theta)^2}$$

$$= \frac{q^2}{4\pi c} |\Delta\boldsymbol{\beta}| \frac{2}{(1-\beta^2)}$$

$$\frac{dI}{d\omega} = \int d\Omega \left(\frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right) = \frac{2}{3\pi} \frac{q^2}{c} \gamma^2 \; |\Delta\boldsymbol{\beta}|^2$$

Some details of the analysis. With all of these considerations, we still need to estimate delta beta.

Estimation of Δβ



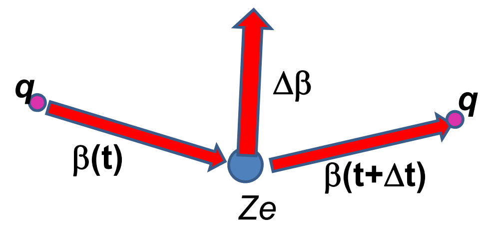
Momentum transfer:

$$Qc \equiv \left|\mathbf{p}(t+\tau) - \mathbf{p}(t)\right| c \approx \gamma M c^2 \left|\Delta\boldsymbol{\beta}\right|$$

mass of particle having charge *q*

$$\frac{dI}{d\omega} = \frac{2}{3\pi} \frac{q^2}{c} \gamma^2 \left|\Delta\boldsymbol{\beta}\right|^2 \approx \frac{2}{3\pi} \frac{q^2}{M^2 c^3} Q^2$$

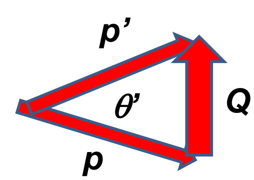
Estimation of $\Delta\beta$ or Q -- for the case of Rutherford scattering



Assume that target nucleus (charge Ze) has mass >>M;

Rutherford scattering cross-section in center of mass analysis:

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

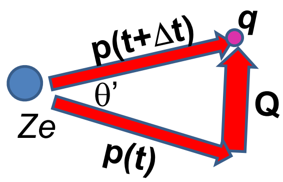


Assuming elastic scattering:

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2(1-\cos\theta')$$

Delta beta will depend on the particular system. As an example, consider the case of Rutherford scattering.. Here are some of the equations we used in classical mechanics class.

Case of Rutherford scattering -- continued



Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin\left(\theta'/2\right)\right)^4}$$

$$\frac{d\sigma}{dQ} = \int_{\varphi'} \frac{d\sigma}{d\Omega}\left|\frac{d\Omega}{dQ}\right|$$

$$d\Omega = d\varphi' d\cos\theta'$$

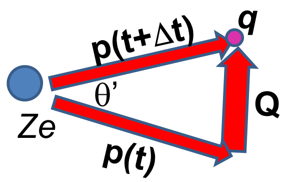
$$Q^2 = \left(2p\sin\left(\theta'/2\right)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

$$dQ = -\frac{p^2}{Q} d\cos\theta'$$

$$\Rightarrow \quad \frac{d\sigma}{dQ} = 8\pi\left(\frac{Zeq}{\beta c}\right)^2 \frac{1}{Q^3}$$

It is convenient to express the results in terms of the momentum transfer Q.

Case of Rutherford scattering -- continued



Differential radiation cross section:

$$\frac{d^2\chi}{d\omega dQ} = \frac{dI}{d\omega}\frac{d\sigma}{dQ} = \left(\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2\right)\left(8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}\right)$$

$$= \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\frac{1}{Q}$$

04/17/2020 PHY 712 Spring 2020 -- Lecture 31 15

It is of interest to estimate the probability of the radiation occurring which depends on the product of the radiation intensity for a given momentum transfer and the cross section as a function of momentum transfer.

Differential radiation cross section -- continued

Integrating over momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Comment on frequency dependence --

Original expression for radiation intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \int dt\, e^{i\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

In the previous derivations, we have assumed that $\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c) << 1.$

$$\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c) = \omega\left(t - \hat{\mathbf{r}}\cdot\int_0^t dt'\boldsymbol{\beta}(t')\right) \approx \omega\tau(1 - \hat{\mathbf{r}}\cdot\langle\boldsymbol{\beta}\rangle)$$

But we are not done. Thinking of the case of the charged particle moving through the target material, there will be a range of momentum transfers that should be integrated as indicated here. Note that we have assumed that the frequency of the radiation is very small. Here we consider how frequency might ener this analysis.

Differential radiation cross section -- continued

Radiation cross section in terms of momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Note that: $Q^2 = 2p^2(1-\cos\theta') \quad \Rightarrow Q_{\max} = 2p$

In general, $Q_{\min}$ is determined by the collision time condition $\omega\tau < 1 \quad \Rightarrow Q_{\min} \approx \frac{2Zeq\omega}{v^2}$

Radiation cross section for classical non-relativistic process

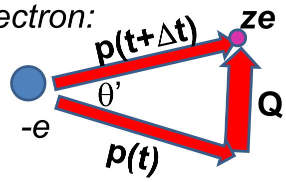
$$\frac{d\chi}{d\omega} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{\lambda M v^3}{Zeq\omega}\right)$$

λ= "fudge factor" of order unity

Hans Bethe considered this problem and also introduced a "correction" for quantum effects.

Electromagnetic effects in energy loss processes
(see Chap. 13 of Jackson)

Again consider Rutherford scattering – now of a nucleus (or alpha particle) *ze* incident on an electron *–e in rest frame of electron:*



Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{ze^2}{2pv}\right)^2 \frac{1}{\left(\sin\left(\theta'/2\right)\right)^4}$$

$$\frac{d\sigma}{dQ^2} = \int d\varphi' \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ^2}$$

$$Q^2 = \left(2p\sin\left(\theta'/2\right)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

$$\Rightarrow \quad \frac{d\sigma}{dQ^2} = 4\pi\left(\frac{ze^2}{\beta c Q^2}\right)^2$$

Electromagnetism affects energy loss processes more generally. In this case, there is not necessarily radiation involved. In Chapter 13 (which we will not cover in detail), energy loss by a charged particles moving through media was considered. In this case the charged particle could be a nucleus and the media effects are dominated by electrons. In this case, we can use the Rutherford model to derive the interaction cross section.

Energy loss continued

Let T represent energy loss due to electron of mass m:

$$T = Q^2 / 2m$$

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 e^4}{mc^2 \beta^2 T^2}$$

Estimate of energy loss per unit distance in the presence of NZ electrons per unit volume

$$\frac{dE}{dx} \approx NZ \int_{\varepsilon}^{T_{max}} dT\,T \frac{d\sigma}{dT}$$

minimum energy transfer

$$= 2\pi NZ \frac{z^2 e^4}{mc^2 \beta^2} \ln\left(\frac{2\gamma^2 \beta^2 mc^2}{\varepsilon}\right) + \text{(quantum effects)}$$

Here we estimate the energy loss per unit length traveled by the particle

Energy loss continued

Refining this result, Bethe and Fermi noticed that the analysis lacked consideration of the effects of electromagnetic fields. Representing the colliding electrons in terms of a dielectric function $\varepsilon(\omega)$ and the energetic particle of charge *ze* in terms of the charge and current density:

In Fourier space:

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\Phi(\mathbf{k},\omega) = \frac{4\pi}{\varepsilon(\omega)}\rho(\mathbf{k},\omega)$$

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\mathbf{A}(\mathbf{k},\omega) = \frac{4\pi}{c}\mathbf{J}(\mathbf{k},\omega)$$

$$\rho(\mathbf{k},\omega) = \frac{ze}{2\pi}\delta(\omega - \mathbf{v}\cdot\mathbf{k})$$

$$\mathbf{J}(\mathbf{k},\omega) = \mathbf{v}\rho(\mathbf{k},\omega)$$

Analysis by Fermi and Bethe.

Energy loss continued

$$\Phi(\mathbf{k},\omega) = \frac{2ze}{\varepsilon(\omega)} \frac{\delta(\omega - \mathbf{v}\cdot\mathbf{k})}{k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)}$$

$$\mathbf{A}(\mathbf{k},\omega) = \varepsilon(\omega)\frac{\mathbf{v}}{c}\Phi(\mathbf{k},\omega)$$

The energy loss will be calculated from the work on the electron by the field:

$$\Delta E = -e\int_{-\infty}^{\infty} dt\ \mathbf{v}\cdot\mathbf{E}(t) = 2e\Re\left(\int_0^{\infty} d\omega\ i\omega\mathbf{r}(\omega)\cdot\mathbf{E}^*(\omega)\right)$$

The resultant loss estimate is

$$\frac{dE}{dx} \approx \frac{z^2e^2\omega_P^2}{2c^2}\ln\left(\frac{2mc^2\varepsilon}{\hbar^2\omega_P^2}\right) \quad \text{where } \omega_P^2 \equiv \frac{4\pi NZe^2}{m}$$

Estimate of energy loss expressed in terms of the plasma frequency and other materials parameters.