

PHY 712 Electrodynamics

12-12:50 AM MWF via video link:

<https://wakeforest-university.zoom.us/my/natalie.holzwarth>

Extra notes for Lecture 30:

Finish reading Chap. 14 and start Chap. 15 –

Radiation from scattering charged particles

- 1. Thompson and Compton scattering**
- 2. Radiation from particle collisions**

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In this lecture we will discuss radiation due to collisions.

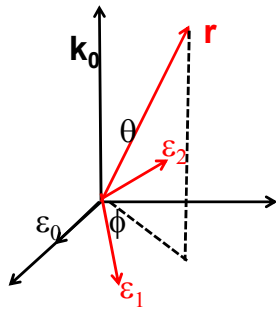
21	Mon: 03/23/2020	Chap. 9	Radiation from localized oscillating sources	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 9	Radiation from oscillating sources	#18	03/27/2020
23	Fri: 03/27/2020	Chap. 9 and 10	Radiation from oscillating sources	#19	03/30/2020
24	Mon: 03/30/2020	Chap. 11	Special Theory of Relativity	#20	04/03/2020
25	Wed: 04/01/2020	Chap. 11	Special Theory of Relativity		
26	Fri: 04/03/2020	Chap. 11	Special Theory of Relativity	#21	04/06/2020
27	Mon: 04/06/2020	Chap. 14	Radiation from accelerating charged particles	#22	04/08/2020
28	Wed: 04/08/2020	Chap. 14	Synchrotron radiation		
	Fri: 04/10/2020	No class	<i>Good Friday</i>		
29	Mon: 04/13/2020	Chap. 14	Synchrotron radiation	#23	04/15/2020
30	Wed: 04/15/2020	Chap. 15	Radiation from collisions of charged particles	#24	04/17/2020
31	Fri: 04/17/2020	Chap. 13	Cherenkov radiation		
32	Mon: 04/20/2020		Special topic: E & M aspects of superconductivity		
33	Wed: 04/22/2020		Special topic: Aspects of optical properties of materials		
34	Fri: 04/24/2020				
35	Mon: 04/27/2020				
36	Wed: 04/29/2020		Review		

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Last homework set for the semester.

Thompson scattering --

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



Incident electromagnetic wave:

\mathbf{k}_0 propagation direction

$\boldsymbol{\epsilon}_0$ polarization direction

$$\mathbf{E}(\mathbf{r}', t') = \Re(\boldsymbol{\epsilon}_0 E_0 e^{i\mathbf{k}_0 \cdot \mathbf{r}' - i\omega t'})$$

Scattered radiation:

\mathbf{r} observed position

$\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2$ polarization directions

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Consider the effects of a charged particle encountering an electromagnetic wave.

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \dot{\mathbf{v}})|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q)

is caused by an electric field: $\mathbf{E}(\mathbf{r}, t) = \Re(\boldsymbol{\epsilon}_0 E_0 e^{i\mathbf{k}_0 \cdot \mathbf{r} - i\omega t})$

$$\dot{\mathbf{v}} = \frac{q}{m_q} \Re(\boldsymbol{\epsilon}_0 E_0 e^{i\mathbf{k}_0 \cdot \mathbf{r} - i\omega t})$$

$$\text{Time averaged power : } \left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 |E_0|^2 |\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\epsilon}_0)|^2$$

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The power generated in the far field approximated by the non-relativistic limit, is given here. Now we consider that the particle acceleration is due to the oscillating electric field. The power of the radiation of the particle is then given here.

What assumptions are made to conclude that

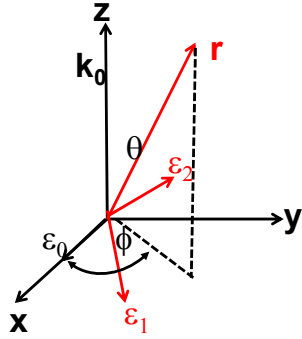
$$\dot{\mathbf{v}} = \frac{q}{m_q} \Re(\boldsymbol{\epsilon}_0 E_0 e^{i\mathbf{k}_0 \cdot \mathbf{r} - i\omega t}) \quad ?$$

Is it always true?

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 |E_0|^2 |\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0)|^2$

$$\hat{\mathbf{r}} = \sin \theta (\cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}}) + \cos \theta \hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\boldsymbol{\varepsilon}_1 = \cos \theta (\hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi) - \hat{\mathbf{z}} \sin \theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi$$

Are these polarizations unique?

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By evaluating the radiation geometry, we can determine the power distribution.

Note that we are associating the vector $\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \dot{\mathbf{v}})$ with the polarization of the light. Why?

Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \left(1 - \frac{v^2}{c^2}\right) + \left(\mathbf{R} \times \left\{ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \times \frac{\dot{\mathbf{v}}}{c^2} \right\}\right) \right]. \quad (19)$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}} \cdot \mathbf{R}}{c^2}\right) - \frac{\mathbf{R} \times \dot{\mathbf{v}}/c}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^2} \right]. \quad (20)$$

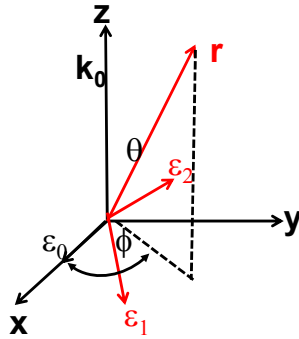
In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}. \quad (21)$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power:
$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 |E_0|^2 |\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0)|^2$$

$$\hat{\mathbf{r}} = \sin \theta (\cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}}) + \cos \theta \hat{\mathbf{z}}$$



Polarization of incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0) = \hat{\mathbf{r}} (\hat{\mathbf{r}} \cdot \boldsymbol{\varepsilon}_0) - \boldsymbol{\varepsilon}_0 \quad (\text{perpendicular to } \hat{\mathbf{r}})$$

denote scattered light polarization by $\boldsymbol{\varepsilon}^*$

$$\boldsymbol{\varepsilon}^* \cdot (\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0)) = -\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0$$

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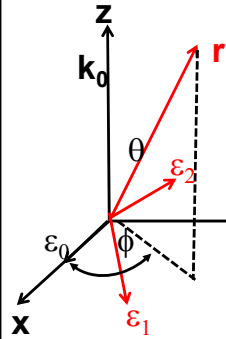
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The radiated field is polarized perpendicularly to the propagation direction so that the power depends on the dot product of radiation polarization and the incident polarization.

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 |E_0|^2 |\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0)|^2$

$$\hat{\mathbf{r}} = \sin \theta (\cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}}) + \cos \theta \hat{\mathbf{z}}$$



Incident light: $\boldsymbol{\varepsilon}_0 = \hat{\mathbf{x}}$

Polarization of scattered light: $\boldsymbol{\varepsilon}^*$

Linear combination of

$$\boldsymbol{\varepsilon}_1 = \cos \theta (\hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi) - \hat{\mathbf{z}} \sin \theta$$

$$\boldsymbol{\varepsilon}_2 = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi$$

$$\langle |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2 \rangle = \langle |\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0|^2 \rangle + \langle |\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0|^2 \rangle = \langle \cos^2 \theta \cos^2 \phi \rangle + \langle \sin^2 \phi \rangle$$

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Evaluating the dot products for this geometry.

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization $\boldsymbol{\epsilon}^*$:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \left(\frac{q^2}{m_q c^2} \right)^2 |E_0|^2 |\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0|^2$$

Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

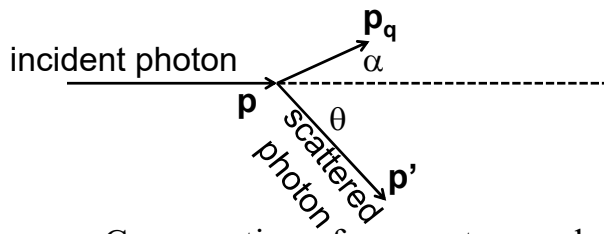
$$\left\langle |\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0|^2 \right\rangle_\phi = \left\langle |\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_0|^2 \right\rangle_\phi + \left\langle |\boldsymbol{\epsilon}_2 \cdot \boldsymbol{\epsilon}_0|^2 \right\rangle_\phi = \frac{1}{2} \cos^2 \theta + \frac{1}{2}$$

Averaged cross section:
$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \frac{1}{2} (1 + \cos^2 \theta)$$

This formula is appropriate in the X-ray scattering of electrons or soft γ -ray scattering of protons

Summary of results in the non-relativistic limit.

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy:

$$p = p' \cos \theta + p_q \cos \alpha \quad pc = \hbar \omega$$

$$0 = p' \sin \theta - p_q \sin \alpha \quad p' c = \hbar \omega'$$

$$\hbar \omega + m_q c^2 = \hbar \omega' + \sqrt{p_q'^2 c^2 + (m_q c^2)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar \omega}{m_q c^2} (1 - \cos \theta)}$$

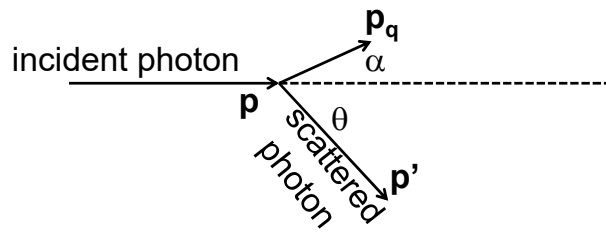
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When the photon energy is comparable to the rest mass energy of the charged particle, we have to consider momentum and energy conservation. This was first analyzed by Compton. Your homework asks you to examine the Compton scattering relationships.

Thompson scattering – relativistic and quantum modifications



Relativistic and quantum modifications to averaged cross section:

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2} (1 + \cos^2 \theta)$$

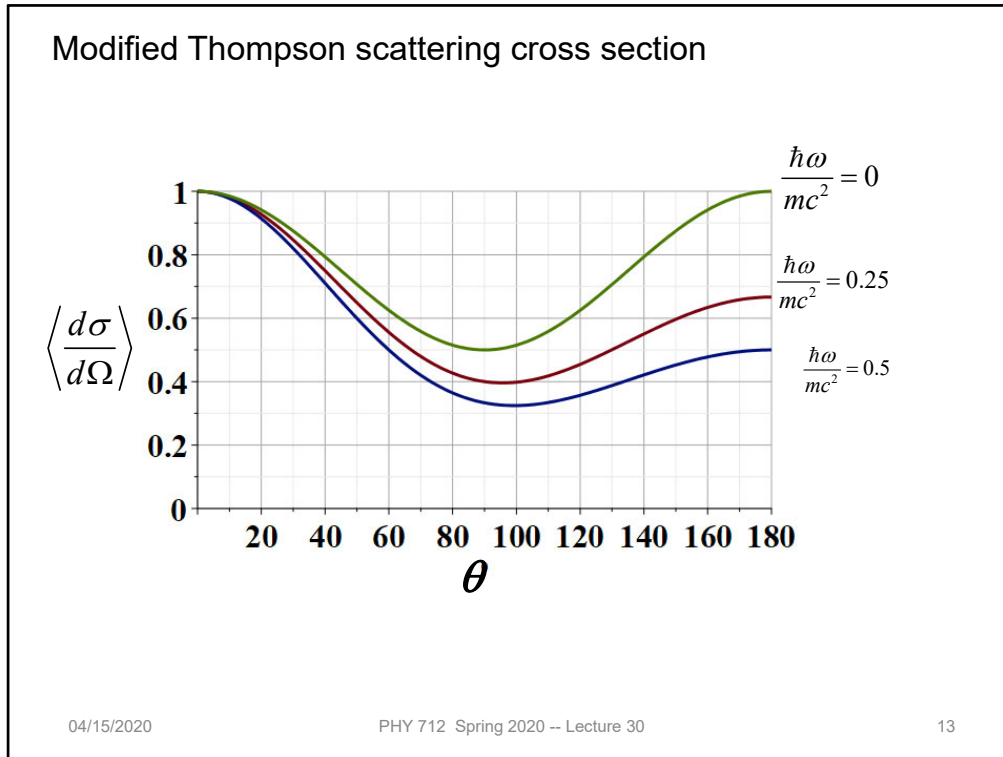
$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2} (1 - \cos\theta)}$$

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The effects of the Compton scattering change the cross section.



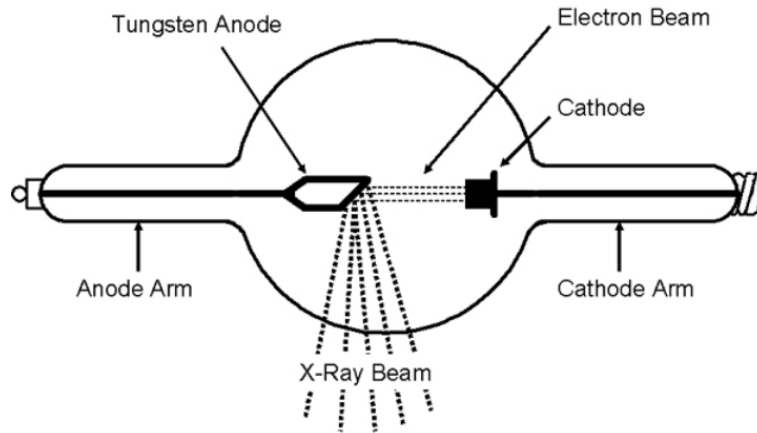
Plot of the changes to the Thompson cross section due to the Compton effect.

Up to now, we have been considering (re)radiation due to a charged particle interacting with an electromagnetic field. Next time we will consider radiation due to interactions (collisions) of charged particles themselves.

Radiation produced by collisions of charged particles

Generation of X-rays in a Coolidge tube

<https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm>



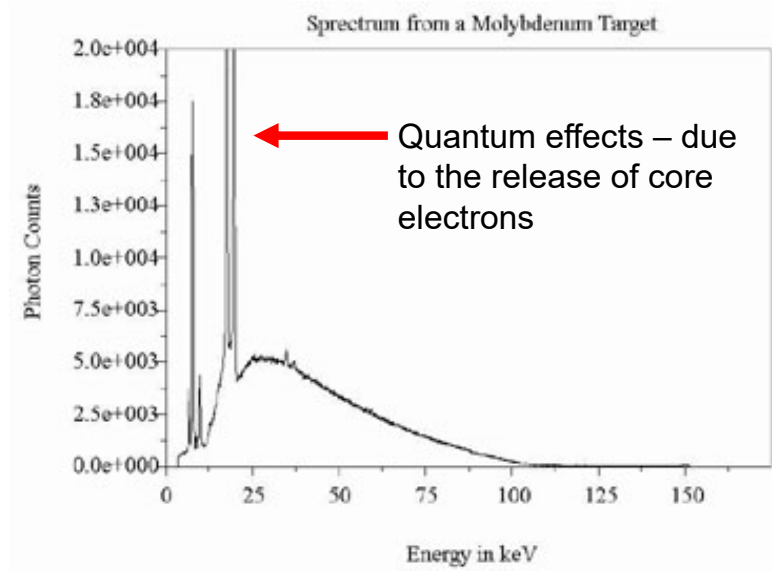
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Next time we will consider more generally the properties of radiation produced by charged particle collisions.

<http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm>



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Example of x-ray spectrum .