PHY 712 Electrodynamics
12-12:50 AM MWF via video link:
https://wakeforest-university.zoom.us/my/natalie.holzwarth

Extra notes for Lecture 21:

Sources of radiation

Start reading Chap. 9

A. Electromagnetic waves due to specific sources

B. Dipole radiation patterns

03/23/2020 PHY 712 Spring 2020 -- Lecture 21-- extra notes 1

Welcome to the new version of PHY 712. As stated in the email, this new format offers us a wonderful opportunity to focus attention on our studies and advance our physics knowledge and skills quickly. We just have to figure out the best way to keep focused and motivated, filtering out the distractions at least for several hours each day..... In this new format, you are asked to carefully review the lecture and notes with time enough to pose questions before 10 AM each MWF. The online class time 12-12:50 will be devoted to discussion, initially starting with your questions.

Please start reading Chapter 9 in Jackson. We will cover most of the chapter, not quite in the order presented in Jackson. It turns out that radiation from the time harmonic sources considered in this chapter can evaluated exactly. From the exact expressions, we can see how the various approximations can be derived. Jackson's treatment is more focused on the approximations first.

Schedule as planned at the moment --

				Exam	
	Mon: 03/09/2020	No class	*Spring Break*		
	Wed: 03/11/2020	No class	*Spring Break*		
	Fri: 03/13/2020	No class	*Spring Break*		
	Mon: 03/16/2020	No class	*Classes Cancelled*		
	Wed: 03/18/2020	No class	*Classes Cancelled*		
	Fri: 03/20/2020	No class	*Classes Cancelled*		
21	Mon: 03/23/2020	Chap. 9	Radiation from localized oscillating sources	#17	03/25/2020
22	Wed: 03/25/2020	Chap. 9	Radiation from oscillating sources		
23	Fri: 03/27/2020	Chap. 9 and 10	Radiation from oscillating sources		
24	Mon: 03/30/2020	Chap. 11	Special Theory of Relativity		
25	Wed: 04/01/2020	Chap. 11	Special Theory of Relativity		
26	Fri: 04/03/2020	Chap. 11	Special Theory of Relativity		
27	Mon: 04/06/2020	Chap. 14	Radiation from accelerating charged particles		
28	Wed: 04/08/2020	Chap. 14	Synchrotron radiation		
	Fri: 04/10/2020	No class	*Good Friday*		
29	Mon: 04/13/2020	Chap. 14	Synchrotron radiation		
30	Wed: 04/15/2020	Chap. 15	Radiation from collisions of charged particles		
31	Fri: 04/17/2020	Chap. 13	Cherenkov radiation		
32	Mon: 04/20/2020		Special topic: E & M aspects of superconductivity		
33	Wed: 04/22/2020		Special topic: Aspects of optical properties of materials		
34	Fri: 04/24/2020				
35	Mon: 04/27/2020				
36	Wed: 04/29/2020		Review		

Here is the tentative new schedule. It differs from the original plan mainly in that the week we missed has pushed out the "presentations". As mentioned in the email, the projects will now presented in written form. At this time, you should have a reasonable idea of your project topics. Of course I am happy to discuss/advise/help in any way. Also note that the homework is now due by the next lecture. Hopefully with your cleared schedules this will be possible. Of course, I am always happy to consult via email or video conferencing.

Reminder about first of two Online colloquia on Wednesday –
https://www.physics.wfu.edu/events/rick-matthews-41-years/

Online Colloquium: "41 Years of Teaching and Technology"

Dr. Rick Matthews
Professor of Physics and
Director of Academic and Instructional Technology
Wake Forest University
Wednesday, March 25, 2020 at 3:00 PM
Video conference link: **https://wakeforest-university.zoom.us/my/matthews.rick**

Note: this is an online Zoom presentation. If you have not used Zoom recently, click on the above link to join about ten minutes early to be sure the necessary software installs itself.

This is a reminder that on Wednesday we will have the first of two online colloquia. Professor Matthews has been a leader in technology at WFU throughout his career. It will be very interesting to hear his perspective on how this has been put to good use in physics and physics education.

Questions:

From Laxman:

1. How is the integral over time solved to get the last equation in slide 9?
2. I don't understand why Phi, current density and charge density have to be changed into functions of (r,w) from (r,t).
3. I am not clear about last two equations in slide 15. Is it that we neglected the integral from r to infinity for r>>(extent of source)?
4. I think I cannot follow slides 23, 24, 25 very well. The expression of Power, that we will need to solve HW 17 is different in book (eq, 9.154) and in the slide. I hope my confusion will be clarified by questions from other friends too.

From Surya:

1. I am trying to figure out the expressions of **φlm (r, ω) and alm (r, ω)** but unable to get a clue. How can we derive these physical entity?
2. During the calculation of average-power radiated, we have to take the modulus squared of **p(r, t).** In doing so, the term exp(-iωt) seems to be passive. Can we treat problems of power radiation without considering this exponential term ? This terms gives us 1, Is this the effect of averaging or is simply due to an imaginary term?
3. Why dipole of oscillating spherical system with m=0 is not contributed by x- and y- dipole distributions?

Questions:

From Vincent:

1. Is the last eq. in EM slide 18 correct?
2. Why is it important we pick the Lorentz Gauge for radiation? Or What's the physical significances of the Lorentz Gauge?

Questions in order of slide presentation

Why Lorenz gauge?

Note: Please read pg. 294 (end of Chapter 5) in Jackson to learn about the two physicists Lorenz and Lorentz.

Expressing Maxwell's equations in terms of scalar and vector potentials:

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \Rightarrow \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \Rightarrow \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \Phi \Rightarrow \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

The source equations become:

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \quad \Rightarrow \quad \nabla^2 \Phi + \frac{\partial \nabla \cdot \mathbf{A}}{\partial t} = \rho / \varepsilon_0$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \Rightarrow \nabla \times (\nabla \times \mathbf{A}) + \frac{1}{c^2} \left(\frac{\partial \nabla \Phi}{\partial t} + \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) = \mu_0 \mathbf{J}$$

Continuing --

$$\nabla\cdot\mathbf{E}=\rho/\varepsilon_0 \qquad \Rightarrow\ \nabla^2\Phi+\frac{\partial\nabla\cdot\mathbf{A}}{\partial t}=\rho/\varepsilon_0$$

$$\nabla\times\mathbf{B}-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}=\mu_0\mathbf{J} \quad \Rightarrow\nabla\times\left(\nabla\times\mathbf{A}\right)+\frac{1}{c^2}\left(\frac{\partial\nabla\Phi}{\partial t}+\frac{\partial^2\mathbf{A}}{\partial t^2}\right)=\mu_0\mathbf{J}$$

$$\Rightarrow\nabla\left(\nabla\cdot\mathbf{A}\right)-\nabla^2\mathbf{A}+\frac{1}{c^2}\left(\frac{\partial\nabla\Phi}{\partial t}+\frac{\partial^2\mathbf{A}}{\partial t^2}\right)=\mu_0\mathbf{J}$$

$$\Rightarrow\left(\nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}\right)-\nabla\left(\nabla\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right)=\mu_0\mathbf{J}$$

There are multiple solutions of these equations; further conditions can be imposed to find physical solutions.

Continuing --

$$\nabla^2\Phi + \frac{\partial \nabla\cdot\mathbf{A}}{\partial t} = \rho/\varepsilon_0$$

$$\left(\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}\right) - \nabla\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) = \mu_0\mathbf{J}$$

Lorenz very cleverly suggested to impose $\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t} = 0$

This condition decouples the Φ and $\mathbf{A}$ equations:

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = \rho/\varepsilon_0 \qquad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \mu_0\mathbf{J}$$

As we will see later in the course, these equations are also convenient for use in the theory of special relativity.

Question: How is the integral over time solved to get the last equation in slide 9?

$$\Psi(\mathbf{r},t) = \Psi_{f=0}(\mathbf{r},t) +$$

$$\int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|} \delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) f(\mathbf{r}',t')$$

First integrate over t': $\Psi(\mathbf{r},t) = \Psi_{f=0}(\mathbf{r},t) + \int d^3r' \frac{1}{|\mathbf{r}-\mathbf{r}'|} f\left(\mathbf{r}', t' = \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right)$

$$\tilde{\Psi}(\mathbf{r},\omega)e^{-i\omega t} = \tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} +$$

$$\int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|} \delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) \tilde{f}(\mathbf{r}',\omega) e^{-i\omega t'}$$

$$= \tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{f}(\mathbf{r}',\omega) e^{-i\omega t}$$

Where $f(\mathbf{r},t) = \Re\left(\tilde{f}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Question: I don't understand why Phi, current density and charge density have to be changed into functions of (r,w) from (r,t).

Partial answer – Mathematically, separable expressions are much easier to analyze than interdependent functions (as we saw in dealing with the Lienard-Wiechert potentials). More generally, it can be shown that it is possible to take a Fourier transform of any [reasonable] function.

Question: I am trying to figure out the expressions of **φlm (r, ω) and alm (r, ω)** but unable to get a clue. How can we derive these physical entity?

Let us assume that the followin identity is correct:

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function: $j_l(kr)$

Spherical Hankel function: $h_l(kr) = j_l(kr) + in_l(kr)$

Previously we have shown:

$$\tilde{\Psi}(\mathbf{r},\omega)e^{-i\omega t} = \tilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{f}(\mathbf{r}',\omega)e^{-i\omega t}$$

Where $f(\mathbf{r},t) = \Re\left(\tilde{f}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Continued:

For scalar potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega),$$

where $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\tilde{\Phi}_0(\mathbf{r},\omega) = 0$

Substituting: $\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$

Rearranging terms:

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \sum_{lm}\tilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\phi}_{lm}(r,\omega) = \frac{ik}{\varepsilon_0}\int d^3r' \, \tilde{\rho}(\mathbf{r}',\omega) j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

Question: During the calculation of average-power radiated, we have to take the modulus squared of **p(r, t).** In doing so, the term exp(-iωt) seems to be passive. Can we treat problems of power radiation without considering this exponential term ? This terms gives us 1, Is this the effect of averaging or is simply due to an imaginary term?

Equations for time harmonic fields :

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)e^{-i\omega t}\right) \equiv \frac{1}{2}\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)e^{-i\omega t} + \tilde{\mathbf{E}}^*(\mathbf{r},\omega)e^{i\omega t}\right)$$

Poynting vector : $\mathbf{S}(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)$

$$\mathbf{S}(\mathbf{r},t) = \frac{1}{4}\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)e^{-i\omega t} + \tilde{\mathbf{E}}^*(\mathbf{r},\omega)e^{i\omega t}\right) \times \left(\tilde{\mathbf{H}}(\mathbf{r},\omega)e^{-i\omega t} + \tilde{\mathbf{H}}^*(\mathbf{r},\omega)e^{i\omega t}\right)$$

$$= \frac{1}{4}\left(\tilde{\mathbf{E}}(\mathbf{r},\omega) \times \tilde{\mathbf{H}}^*(\mathbf{r},\omega) + \tilde{\mathbf{E}}^*(\mathbf{r},\omega) \times \tilde{\mathbf{H}}(\mathbf{r},\omega)\right)$$

$$+ \frac{1}{4}\left(\tilde{\mathbf{E}}(\mathbf{r},\omega) \times \tilde{\mathbf{H}}(\mathbf{r},\omega)e^{-2i\omega t} + \tilde{\mathbf{E}}^*(\mathbf{r},\omega) \times \tilde{\mathbf{H}}^*(\mathbf{r},\omega)e^{2i\omega t}\right)$$

$$\left\langle \mathbf{S}(\mathbf{r},t) \right\rangle_{t\,\mathrm{avg}} = \Re\left(\frac{1}{2}\left(\tilde{\mathbf{E}}(\mathbf{r},\omega) \times \tilde{\mathbf{H}}^*(\mathbf{r},\omega)\right)\right)$$