PHY 712 Electrodynamics
12-12:50 AM Olin 103

Plan for Lecture 16:

Read Chapter 7

1. **Plane polarized electromagnetic waves**
2. **Reflectance and transmittance of electromagnetic waves – extension to anisotropy and complexity**

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11	Fri: 02/08/2019	Chap. 5	Magnetostatics	#8	02/11/2019
12	Mon: 02/11/2019	Chap. 5	Magnetic dipoles and hyperfine interaction	#9	02/13/2019
13	Wed: 02/13/2019	Chap. 5	Magnetic dipoles and dipolar fields	#10	02/15/2019
14	Fri: 02/15/2019	Chap. 6	Maxwell's Equations	#11	02/18/2019
15	Mon: 02/18/2019	Chap. 6	Electromagnetic energy and forces	#12	02/20/2019
16	Wed: 02/20/2019	Chap. 7	Electromagnetic plane waves	#13	02/22/2019
17	Fri: 02/22/2019				
18	Mon: 02/25/2019				
19	Wed: 02/27/2019				
20	Fri: 03/01/2019				
	Mon: 03/04/2019	No class	*APS March Meeting*	Take Home Exam	
	Wed: 03/06/2019	No class	*APS March Meeting*	Take Home Exam	
	Fri: 03/08/2019	No class	*APS March Meeting*	Take Home Exam	
	Mon: 03/11/2019	No class	*Spring Break*		
	Wed: 03/13/2019	No class	*Spring Break*		
	Fri: 03/15/2019	No class	*Spring Break*		
21	Mon: 03/18/2019				

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Colloquium: "Ink-Based Semiconductors: From In Situ Diagnostics to Autonomous Experimentation"

Dr. Aram Amassian
Associate Professor
Materials Science and Engineering
NC State University
Raleigh, NC
George P. Williams, Jr. Lecture Hall, (Olin 101)
Wednesday, February 19, 2020 at 3:00 PM

There will be a reception in the Olin Lounge at approximately 4 PM following the colloquium. All interested persons are cordially invited to attend.

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Maxwell's equations

For linear isotropic media and no sources: $\mathbf{D}=\varepsilon\mathbf{E}$; $\mathbf{B}=\mu\mathbf{H}$

Coulomb's law: $\nabla\cdot\mathbf{E}=0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t}=0$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

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Analysis of Maxwell's equations without sources -- continued:

Coulomb's law: $\nabla\cdot\mathbf{E}=0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t}=0$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

$$\nabla\times\left(\nabla\times\mathbf{B}-\mu\varepsilon\frac{\partial\mathbf{E}}{\partial t}\right)=-\nabla^2\mathbf{B}-\mu\varepsilon\frac{\partial(\nabla\times\mathbf{E})}{\partial t}$$

$$=-\nabla^2\mathbf{B}+\mu\varepsilon\frac{\partial^2\mathbf{B}}{\partial t^2}=0$$

$$\nabla\times\left(\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}\right)=-\nabla^2\mathbf{E}+\frac{\partial(\nabla\times\mathbf{B})}{\partial t}$$

$$=-\nabla^2\mathbf{E}+\mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2}=0$$

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Analysis of Maxwell's equations without sources -- continued:
Both E and B fields are solutions to a wave equation:

$$\nabla^2\mathbf{B}-\frac{1}{v^2}\frac{\partial^2\mathbf{B}}{\partial t^2}=0$$

$$\nabla^2\mathbf{E}-\frac{1}{v^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=0$$

where $v^2\equiv c^2\frac{\mu_0\varepsilon_0}{\mu\varepsilon}\equiv\frac{c^2}{n^2}$

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r},t)=\Re\left(\mathbf{B}_0e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)\qquad\mathbf{E}(\mathbf{r},t)=\Re\left(\mathbf{E}_0e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

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Analysis of Maxwell's equations without sources -- continued:

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \qquad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

Note: ε, μ, n, k can all be complex; for the moment we will assume that they are all real (no dissipation).

Note that $\mathbf{E}_0$ and $\mathbf{B}_0$ are not independent;

$$\mathbf{k} = n\frac{\omega}{c}\hat{\mathbf{k}}$$

from Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

$$\Rightarrow \mathbf{B}_0 = \frac{\mathbf{k}\times\mathbf{E}_0}{\omega} = \frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}$$

also note: $\hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$ and $\hat{\mathbf{k}}\cdot\mathbf{B}_0 = 0$

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Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

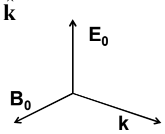
$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$$

Poynting vector and energy density:

$$\langle \mathbf{S} \rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\,\hat{\mathbf{k}}$$

$$\langle u \rangle_{avg} = \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$



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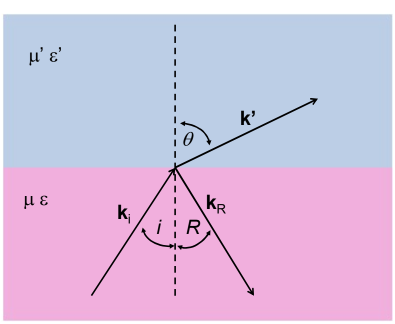
Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



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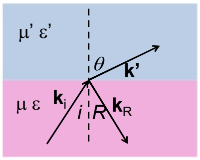
Reflection and refraction -- continued



In medium $\mu'\varepsilon'$:

$$\mathbf{E}'(\mathbf{r},t) = \Re\left(\mathbf{E}'_0\, e^{i\frac{\omega}{c}\left(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}'(\mathbf{r},t) = \frac{n'}{c}\hat{\mathbf{k}}'\times\mathbf{E}'(\mathbf{r},t) = \sqrt{\mu'\varepsilon'}\,\hat{\mathbf{k}}'\times\mathbf{E}(\mathbf{r},t)$$

In medium $\mu\varepsilon$:

$$\mathbf{E}_i(\mathbf{r},t) = \Re\left(\mathbf{E}_{0i}\, e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_i(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t) = \sqrt{\mu\varepsilon}\,\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t)$$

$$\mathbf{E}_R(\mathbf{r},t) = \Re\left(\mathbf{E}_{0R}\, e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_R(\mathbf{r},t) = \frac{n}{c}\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t) = \sqrt{\mu\varepsilon}\,\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t)$$

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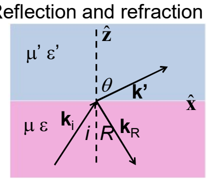
Reflection and refraction -- continued



Snell's law – matching phase factors at boundary plane *z=0*.

$$e^{i\frac{\omega}{c}\left(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct\right)}\Big|_{z=0} = e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct\right)}\Big|_{z=0} = e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct\right)}\Big|_{z=0}$$

matching plane: $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$

$\hat{\mathbf{k}}'\cdot\mathbf{r} = x\sin\theta$

$\hat{\mathbf{k}}_i\cdot\mathbf{r} = x\sin i = \hat{\mathbf{k}}_R\cdot\mathbf{r} = x\sin R \quad \Rightarrow i = R$

$n'\hat{\mathbf{k}}'\cdot\mathbf{r} = n\hat{\mathbf{k}}_i\cdot\mathbf{r} \quad \Rightarrow n'x\sin\theta = nx\sin i$

Snell's law: $n'\sin\theta = n\sin i$

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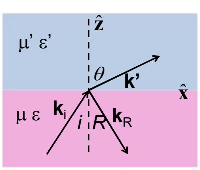
Reflection and refraction -- continued



Continuity equations at boundary with no sources:

$\nabla\cdot\mathbf{D} = 0 \qquad \nabla\cdot\mathbf{B} = 0$

$\nabla\times\mathbf{H} - \frac{\partial\mathbf{D}}{\partial t} = 0 \qquad \nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0$

Matching field amplitudes at boundary plane:

$\mathbf{D}\cdot\hat{\mathbf{z}},\ \mathbf{B}\cdot\hat{\mathbf{z}}$ continuous

$\mathbf{H}\times\hat{\mathbf{z}},\ \mathbf{E}\times\hat{\mathbf{z}}$ continuous

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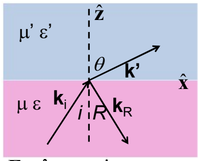
Reflection and refraction -- continued



Matching field amplitudes at boundary plane:

$\mathbf{D}\cdot\hat{\mathbf{z}}$ continuous:

$$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

$\mathbf{B}\cdot\hat{\mathbf{z}}$ continuous:

$$n\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=n'\hat{\mathbf{k}}'\times\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

$\mathbf{E}\times\hat{\mathbf{z}}$ continuous:

$$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

Known: $\mathbf{E}_{0i},\hat{\mathbf{k}}_i$

Unknown: $\mathbf{E}'_0,\mathbf{E}_{0R},\hat{\mathbf{k}}'$

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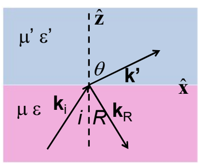
Reflection and refraction -- continued



s-polarization – **E** field "polarized" perpendicular to plane of incidence

$\mathbf{E}\times\hat{\mathbf{z}}$ continuous:

$$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{n\cos i-\frac{\mu}{\mu'}n'\cos\theta}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta}\qquad\frac{E'_0}{E_{0i}}=\frac{2n\cos i}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta}$$

$$\text{Note that: } n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

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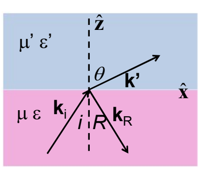
Reflection and refraction -- continued



p-polarization – **E** field "polarized" parallel to plane of incidence

$\mathbf{D}\cdot\hat{\mathbf{z}}$ continuous:

$$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{\frac{\mu}{\mu'}n'\cos i-n\cos\theta}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta}\qquad\frac{E'_0}{E_{0i}}=\frac{2n\cos i}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta}$$

$$\text{Note that: } n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

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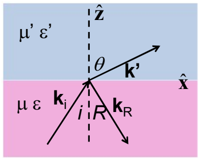
Reflection and refraction -- continued



Intensity in terms of Poynting vector:

$$\langle \mathbf{S} \rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\hat{\mathbf{k}}$$

Reflectance, transmittance:

$$R = \left|\frac{\mathbf{S}_R\cdot\hat{\mathbf{z}}}{\mathbf{S}_i\cdot\hat{\mathbf{z}}}\right| = \left|\frac{E_{0R}}{E_{0i}}\right|^2 \qquad T = \frac{\mathbf{S}'\cdot\hat{\mathbf{z}}}{\mathbf{S}_i\cdot\hat{\mathbf{z}}} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i}$$

Note that $R + T = 1$

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For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'}n'\cos\theta}{n\cos i + \frac{\mu}{\mu'}n'\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'}n'\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

$$R = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{n\cos i - \frac{\mu}{\mu'}n'\cos\theta}{n\cos i + \frac{\mu}{\mu'}n'\cos\theta}\right|^2$$

$$T = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i} = \frac{4nn'\cos i\cos\theta}{\left|n\cos i + \frac{\mu}{\mu'}n'\cos\theta\right|^2}\frac{\mu}{\mu'}$$

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For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n'\cos i - n\cos\theta}{\frac{\mu}{\mu'}n'\cos i + n\cos\theta} \qquad \frac{E'_0}{E_{0i}} = \frac{2n\cos i}{\frac{\mu}{\mu'}n'\cos i + n\cos\theta}$$

Note that: $n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$

$$R = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'\cos i - n\cos\theta}{\frac{\mu}{\mu'}n'\cos i + n\cos\theta}\right|^2$$

$$T = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}\frac{\cos\theta}{\cos i} = \frac{4nn'\cos i\cos\theta}{\left|\frac{\mu}{\mu'}n'\cos i + n\cos\theta\right|^2}\frac{\mu}{\mu'}$$

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Special case: normal incidence (i=0, θ=0)

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n} \qquad \frac{E'_0}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'}n' + n}$$

Reflectance, transmittance:

$$\mathrm{R} = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n' - n}{\frac{\mu}{\mu'}n' + n}\right|^2$$

$$\mathrm{T} = \left|\frac{E'_0}{E_{0i}}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'} = \left|\frac{2n}{\frac{\mu}{\mu'}n' + n}\right|^2 \frac{n'}{n}\frac{\mu}{\mu'}$$

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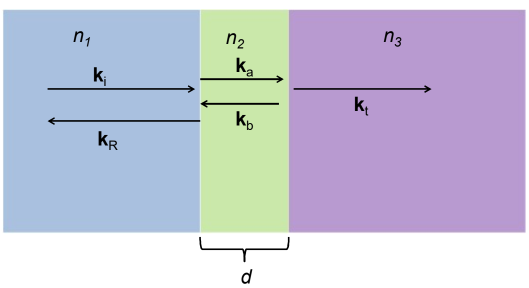
Multilayer dielectrics (Problem #7.2)



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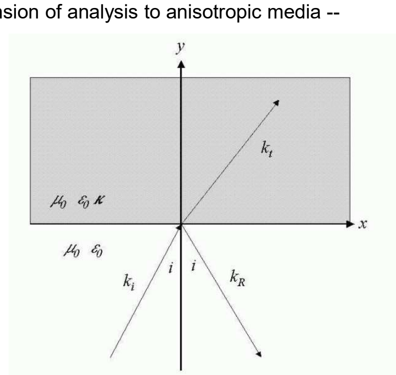
Extension of analysis to anisotropic media --



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Consider the problem of determining the reflectance from an anisotropic medium with isotropic permeability μ_0 and anisotropic permittivity $\varepsilon_0 \boldsymbol{\kappa}$ where:

$$\boldsymbol{\kappa} \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}$$

By assumption, the wave vector in the medium is confined to the *x-y* plane and will be denoted by

$$\mathbf{k}_t \equiv \frac{\omega}{c}(n_x \hat{\mathbf{x}} + n_y \hat{\mathbf{y}}), \text{where } n_x \text{ and } n_y \text{ are to be determined.}$$

The electric field inside the medium is given by:

$$\mathbf{E} = (E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{z}}) \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

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Inside the anisotropic medium, Maxwell's equations are:

$$\nabla \cdot \mathbf{H} = 0 \qquad \nabla \cdot \boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} - i\omega\mu_0 \mathbf{H} = 0 \qquad \nabla \times \mathbf{H} + i\omega\epsilon_0 \boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

After some algebra, the equation for **E** is:

$$\begin{pmatrix} \kappa_{xx} - n_y^2 & n_x n_y & 0 \\ n_x n_y & \kappa_{yy} - n_x^2 & 0 \\ 0 & 0 & \kappa_{zz} - (n_x^2 + n_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0.$$

From **E, H** can be determined from

$$\mathbf{H} = \frac{1}{\mu_0 c} \left\{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) + (E_y n_x - E_x n_y) \hat{\mathbf{z}} \right\} \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

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The fields for the incident and reflected waves are the same as for the isotropic case.

$$\mathbf{k}_i = \frac{\omega}{c}(\sin i \hat{\mathbf{x}} + \cos i \hat{\mathbf{y}}),$$

$$\mathbf{k}_R = \frac{\omega}{c}(\sin i \hat{\mathbf{x}} - \cos i \hat{\mathbf{y}}).$$

Note that, consistent with Snell's law: $n_x = \sin i$

Continuity conditions at the *y=0* plane must be applied for the following fields:

$$\mathbf{H}(x,0,z,t), \; E_x(x,0,z,t), \; E_z(x,0,z,t), \text{ and } D_y(x,0,z,t).$$

There will be two different solutions, depending of the polarization of the incident field.

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Solution for s-polarization

$$E_x = E_y = 0 \quad \Rightarrow n_y^2 = \kappa_{zz} - n_x^2$$

$$\mathbf{E} = E_z \hat{\mathbf{z}} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t} \qquad \mathbf{H} = \frac{1}{\mu_0 c}\left\{E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}})\right\} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}$$

E_z must be determined from the continuity conditions:

$$E_0 + E_0'' = E_z \qquad (E_0 - E_0'')\cos i = E_z n_y \qquad (E_0 + E_0'')\sin i = E_z n_x$$

$$\frac{E_0''}{E_0} = \frac{\cos i - n_y}{\cos i + n_y}.$$

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Solution for p-polarization

$$E_z = 0 \quad \Rightarrow n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}}(\kappa_{yy} - n_x^2).$$

$$\mathbf{E} = E_x \left(\hat{\mathbf{x}} - \frac{\kappa_{xx} n_x}{\kappa_{yy} n_y}\hat{\mathbf{y}}\right) e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

$$\mathbf{H} = -\frac{E_x}{\mu_0 c}\frac{\kappa_{xx}}{n_y}\hat{\mathbf{z}} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}.$$

E_x must be determined from the continuity conditions:

$$(E_0 - E_0'')\cos i = E_x \qquad (E_0 + E_0'') = \frac{\kappa_{xx}}{n_y} E_x \qquad (E_0 + E_0'')\sin i = \frac{\kappa_{xx} n_x}{n_y} E_x.$$

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx}\cos i - n_y}{\kappa_{xx}\cos i + n_y}.$$

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Extension of analysis to complex dielectric functions

For simplicity assume that $\mu = \mu_0$

Suppose the dielectric function is complex:

$$\varepsilon = \varepsilon_R + i\varepsilon_I \qquad \frac{\varepsilon}{\varepsilon_0} = (n_R + i n_I)^2 \equiv \alpha + i\beta$$

$$n_R = \left(\frac{\sqrt{\alpha^2 + \beta^2} + \alpha}{2}\right)^{1/2} \qquad n_I = \left(\frac{\sqrt{\alpha^2 + \beta^2} - \alpha}{2}\right)^{1/2}$$

$$\mathbf{E}(\mathbf{r}, t) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}(n\hat{\mathbf{k}}\cdot\mathbf{r} - ct)}\right) = \Re\left(\mathbf{E}_0 e^{i\frac{\omega}{c}(n_R\hat{\mathbf{k}}\cdot\mathbf{r} - ct)}\right) e^{-\frac{\omega}{c} n_I \hat{\mathbf{k}}\cdot\mathbf{r}}$$

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