PHY 712 Electrodynamics
12-12:50 AM MWF Olin 103

Plan for Lecture 12:

Continue reading Chapter 5

A. Examples of magnetostatic fields

B. Magnetic dipoles

C. Hyperfine interaction

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Course schedule for Spring 2020

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/13/2020	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/17/2020
2	Wed: 01/15/2020	Chap. 1	Electrostatic energy calculations	#2	01/22/2020
3	Fri: 01/17/2020	Chap. 1	Electrostatic potentials and fields	#3	01/24/2020
	Mon: 01/20/2020	No class	Martin Luther King Holiday		
4	Wed: 01/22/2020	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions	#4	01/27/2020
5	Fri: 01/24/2020	Chap. 1 - 3	Brief introduction to numerical methods	#5	01/31/2020
6	Mon: 01/27/2020	Chap. 2 & 3	Image charge constructions	#6	02/03/2020
7	Wed: 01/29/2020	Chap. 2 & 3	Cylindrical and spherical geometries	#7	02/05/2020
8	Fri: 01/31/2020	Chap. 3 & 4	Spherical geometry and multipole moments	#8	02/07/2020
9	Mon: 02/03/2020	Chap. 4	Dipoles and Dielectrics	#9	02/10/2020
10	Wed: 02/05/2020	Chap. 4	Polarization and Dielectrics		
11	Fri: 02/07/2020	Chap. 5	Magnetostatics	#10	02/12/2020
12	Mon: 02/10/2020	Chap. 5	Magnetic dipoles and hyperfine interaction	#11	02/14/2020
13	Wed: 02/12/2020	Chap. 5	Magnetic dipoles and dipolar fields		
14	Fri: 02/14/2020	Chap. 6	Maxwell's Equations		
15	Mon: 02/17/2020	Chap. 6	Electromagnetic energy and forces		
16	Wed: 02/19/2020	Chap. 7	Electromagnetic plane waves		
17	Fri: 02/21/2020	Chap. 7	Electromagnetic plane waves		
18	Mon: 02/24/2020	Chap. 7	Refractive index		
19	Wed: 02/26/2020	Chap. 8	EM waves in wave guides		
20	Fri: 02/28/2020	Chap. 1-8	Review		

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Various forms of Ampere's law :

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

$$\text{Vector potential: } \mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

$$\text{For Coulomb gauge: } \nabla \cdot \mathbf{A}(\mathbf{r}) = 0$$

$$\Rightarrow \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r})$$

For confined current density :

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

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Other examples of current density sources:

Quantum mechanical expression for current density

for a particle of mass M and charge e and of probability amplitude $\Psi(\mathbf{r})$:

$$\mathbf{J}(\mathbf{r}) = -\frac{e\hbar}{2Mi}\left(\Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r}) - \Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r})\right)$$

For an electron in a spherical potential (such as in an atom):

$$\Psi(\mathbf{r}) \equiv \Psi_{nlm_l}(\mathbf{r}) = R_{nl}(r)Y_{lm_l}(\hat{\mathbf{r}})$$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{2Mi}\left|R_{nl}(r)\right|^2 \frac{1}{r\sin\theta}\left(Y_{lm_l}^*(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}(\hat{\mathbf{r}})}{\partial\varphi} - Y_{lm_l}(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}^*(\hat{\mathbf{r}})}{\partial\varphi}\right)\hat{\boldsymbol{\varphi}}$$

$$= \frac{e\hbar}{M}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2\hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}} = \frac{\hat{\mathbf{z}}\times\mathbf{r}}{r\sin\theta}$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{M}\frac{m_l}{r^2\sin^2\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2(\hat{\mathbf{z}}\times\mathbf{r})$$

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Details of the electron orbital magnetic dipole moment

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{m_e}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2\hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}}$

Magnetic dipole moment:

$$\mathbf{m} = \frac{1}{2}\int d^3r'\mathbf{r}'\times\mathbf{J}(\mathbf{r}') = -\frac{e\hbar m_l}{2m_e}\int d^3r'\frac{\mathbf{r}'\times\hat{\boldsymbol{\varphi}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2$$

$$= -\frac{e\hbar m_l}{2m_e}\int d^3r'\frac{-r'\hat{\boldsymbol{\theta}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2$$

Note that: $\hat{\boldsymbol{\theta}} = \cos\theta\cos\varphi\,\hat{\mathbf{x}} + \cos\theta\sin\varphi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$

$$\mathbf{m} = -\frac{e\hbar m_l\hat{\mathbf{z}}}{2m_e}\int d^3r'\left|\Psi_{nlm_l}(\mathbf{r})\right|^2$$

$$= -\frac{e\hbar m_l}{2m_e}\hat{\mathbf{z}}$$

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Summary of magnetic field generated by point magnetic dipole moment:

$$\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\hat{\mathbf{r}}(\boldsymbol{\mu}_e\cdot\hat{\mathbf{r}}) - \boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_e\delta(\mathbf{r})\right)$$

Magnetic field near nucleus due to orbiting electron:

$$\mathbf{B}_O(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{e}{m_e}L_z\hat{\mathbf{z}}\left\langle\frac{1}{r^3}\right\rangle$$

"Hyperfine" interaction energy:

$$\mathcal{H}_{HF} = -\boldsymbol{\mu}_N\cdot\left(\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r}) + \mathbf{B}_O(\mathbf{r})\right)$$

$$= \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{\mu}_N\cdot\hat{\mathbf{r}})(\boldsymbol{\mu}_e\cdot\hat{\mathbf{r}}) - \boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e\delta(\mathbf{r}) + \frac{e}{m_e}\left\langle\frac{\mathbf{L}\cdot\boldsymbol{\mu}_N}{r^3}\right\rangle\right)$$

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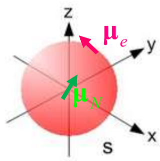
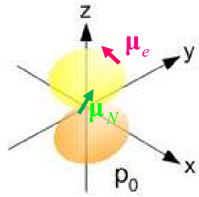
$$\mathcal{H}_{HF} = \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{\mu}_N\cdot\hat{\mathbf{r}})(\boldsymbol{\mu}_e\cdot\hat{\mathbf{r}})-\boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e}{r^3}+\frac{8\pi}{3}\boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e\delta(\mathbf{r})+\frac{e}{m_e}\left\langle\frac{\mathbf{L}\cdot\boldsymbol{\mu}_N}{r^3}\right\rangle\right)$$





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