PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 32:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity

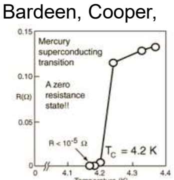
23	Fri: 03/22/2019	Chap. 9 and 10	Radiation from oscillating sources	#17	3/27/2019
24	Mon: 03/25/2019	Chap. 11	Special Theory of Relativity	Pick topic	3/29/2019
25	Wed: 03/27/2019	Chap. 11	Special Theory of Relativity	#18	4/01/2019
26	Fri: 03/29/2019	Chap. 11	Special Theory of Relativity	#19	4/03/2019
27	Mon: 04/01/2019	Chap. 14	Radiation from accelerating charged particles	#20	4/05/2019
28	Wed: 04/03/2019	Chap. 14	Synchrotron radiation		
29	Fri: 04/05/2019	Chap. 14	Synchrotron radiation	#21	4/10/2019
30	Mon: 04/08/2019	Chap. 15	Radiation from collisions of charged particles	#22	4/12/2019
31	Wed: 04/10/2019	Chap. 13	Cherenkov radiation		
32	Fri: 04/12/2019		Special topic: E & M aspects of superconductivity		
33	Mon: 04/15/2019		Special topic: Aspects of optical properties of materials		
34	Wed: 04/17/2019	Chap. 1-15	Review		
	Fri: 04/19/2019	No class	*Good Friday*		
35	Mon: 04/22/2019	Chap. 1-15	Review		
36	Wed: 04/24/2019	Chap. 1-15	Review		
	Fri: 04/26/2019		Presentations I		
	Mon: 04/29/2019		Presentations II		
	Wed: 05/01/2019		Presentations III		

Special topic: Electromagnetic properties of superconductors

Ref:D. Teplitz, editor, Electromagnetism – paths to research, Plenum Press (1982); Chapter 1 written by Brian Schwartz and Sonia Frota-Pessoa

History:

- 1908 H. Kamerlingh Onnes successfully liquified He
- 1911 H. Kamerlingh Onnes discovered that Hg at 4.2 K has vanishing resistance
- 1957 Theory of superconductivity by Bardeen, Cooper, and Schrieffer



Fritz London 1900-1954


Fritz London, 1947, photo: Lotte Meitner-Graf

Fritz London, one of the most distinguished scientists on the Duke University faculty, was an internationally recognized theorist in Chemistry, Physics and the Philosophy of Science. He was born in Breslau, Germany (now Wroclaw, Poland) in 1900. In 1933 he was

He immigrated to the United States in 1939, and came to Duke University, first as a Professor of Chemistry. In 1949 he received a joint appointment in Physics and Chemistry and became a James B. Duke Professor. In 1953 he became the 5th recipient of the Lorentz medal, awarded by the Royal Netherlands Academy of Sciences, and was the first American citizen to receive this honor. He died in Durham in 1954.

https://phy.duke.edu/about/history/historical-faculty/fritz-london

Some phenomenological theories < 1957 thanks to F. London

Drude model of conductivity in "normal" materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - m\frac{\mathbf{v}}{\tau}$$

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-t/\tau} - \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{J} = -ne\mathbf{v}; \qquad \text{for } t >> \tau \qquad \Rightarrow \qquad \mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} \equiv \sigma\mathbf{E}$$

Note: Equations are in cgs Gaussian units.

London model of conductivity in superconducting materials; $\tau \rightarrow \infty$

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{e\mathbf{E}}{m} \qquad \frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Some phenomenological theories < 1957

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \left(\nabla \times \mathbf{B}\right) = -\nabla^2\mathbf{B} = \frac{4\pi}{c}\nabla \times \mathbf{J} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

$$-\nabla^2\frac{\partial \mathbf{B}}{\partial t} = \frac{4\pi}{c}\nabla \times \frac{\partial \mathbf{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^3 \mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial \mathbf{B}}{\partial t} = \frac{4\pi ne^2}{mc}\nabla \times \mathbf{E} - \frac{1}{c^2}\frac{\partial^3 \mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial \mathbf{B}}{\partial t} = -\frac{4\pi ne^2}{mc^2}\frac{\partial \mathbf{B}}{\partial t} - \frac{1}{c^2}\frac{\partial^3 \mathbf{B}}{\partial t^3}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

London model – continued

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

For slowly varying solution:

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2}\right)\mathbf{B} = 0 \qquad \text{for } \frac{\partial \mathbf{B}}{\partial t} = \hat{\mathbf{z}}\frac{\partial B_z(x,t)}{\partial t}:$$

$$\Rightarrow \frac{\partial B_z(x,t)}{\partial t} = \frac{\partial B_z(0,t)}{\partial t}e^{-x/\lambda_L}$$

London leap: $B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Consistent results for current density:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \mathbf{J} = \hat{\mathbf{y}}J_y(x) \quad \Rightarrow J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$$

London model – continued

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$ Typically, $\lambda_L \approx 10^{-7}m$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Vector potential for $\mathbf{B} = \nabla\times\mathbf{A}$ and $\nabla\cdot\mathbf{A} = 0$:

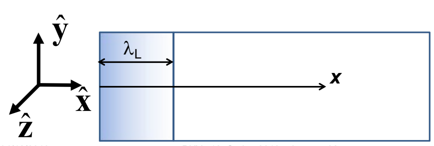
$\mathbf{A} = \hat{\mathbf{y}}A_y(x)$ $\qquad A_y(x) = -\lambda_L B_z(0)e^{-x/\lambda_L}$

Note that: $\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J}$

$-\nabla^2\mathbf{A} = \frac{4\pi}{c}\mathbf{J} \Rightarrow \nabla^2\mathbf{A} + \frac{4\pi}{c}\mathbf{J} = 0$

Recall form for current density: $J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$

$$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$$



Behavior of superconducting material – exclusion of magnetic field according to the London model

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

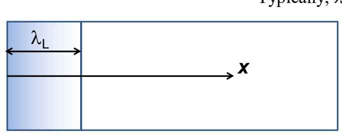
Vector potential for $\nabla\cdot\mathbf{A} = 0$:

$\mathbf{A} = \hat{\mathbf{y}}A_y(x)$ $\qquad A_y(x) = -\lambda_L B_z(0)e^{-x/\lambda_L}$

Current density: $J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$

$$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$$

Typically, $\lambda_L \approx 10^{-7}m$



Behavior of magnetic field lines near superconductor

normal state: superconducting state:

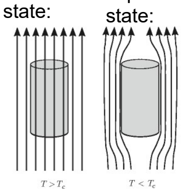
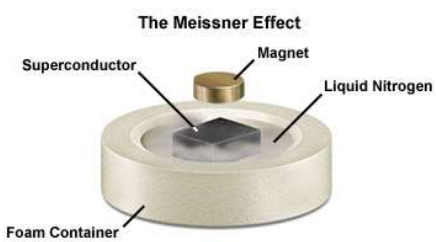


Figure 18.2 Exclusion of a weak external magnetic field from the interior of a superconductor.



Need to consider phase equilibria between “normal” and superconducting state as a function of temperature and applied magnetic fields.

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$$

Within the superconductor, if $\mathbf{B} = 0$

$$\text{then} \quad \mathbf{H} + 4\pi\mathbf{M} = 0 \quad \text{or} \quad \mathbf{M} = -\frac{\mathbf{H}}{4\pi}$$

Magnetization field

Treating London current in terms of corresponding magnetization field **M**:

$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$

$\Rightarrow$ For $x >> \lambda_L$, $\quad \mathbf{H}=-4\pi\mathbf{M}, \quad \mathbf{M}(\mathbf{H})=-\frac{\mathbf{H}}{4\pi}$

Gibbs free energy associated with magnetization for superconductor:

$$G_S(H_a)=G_S(H=0)-\int_0^{H_a} dH M(H)=G_S(0)-\int_0^{H_a} dH\left(\frac{-H}{4\pi}\right)=G_S(0)+\frac{1}{8\pi}H_a^2$$

This relation is true for an applied field $H_a \leq H_C$ when the superconducting and normal Gibbs free energies are equal:

$G_S(H_C)=G_N(H_C)\approx G_N(H=0)$

Condition at phase boundary between normal and superconducting states:

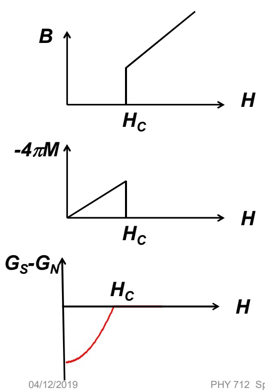
$$G_N(H_C)\approx G_N(0)=G_S(H_C)=G_S(0)+\frac{1}{8\pi}H_C^2$$

At T=0K

$$\Rightarrow G_S(0)-G_N(0)=-\frac{1}{8\pi}H_C^2$$

$$G_S(H_a)-G_N(H_a)=\begin{cases}-\frac{1}{8\pi}\left(H_C^2-H_a^2\right) & \text{for } H_a<H_C\\ 0 & \text{for } H_a>H_C\end{cases}$$

Magnetization field (for “type I” superconductor)



Inside superconductor

$\mathbf{B}=0=\mathbf{H}+4\pi\mathbf{M}$ for $H<H_C$

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. Bardeen, L. N. Cooper,† and J. R. Schrieffer‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

$$G_S(0)-G_N(0)=-\frac{H_C^2}{8\pi}\approx -2N(E_F)(\hbar\omega)^2 e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Temperature dependence of critical field

$$H_c(T) \approx H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

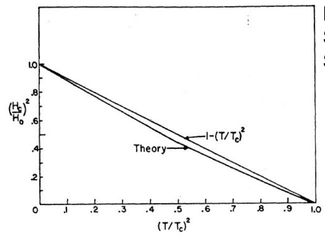


FIG. 2. Ratio of the critical field to its value at $T=0°K$ *vs* $(T/T_c)^2$. The upper curve is the $1-(T/T_c)^2$ law of the Gorter-Casimir theory and the lower curve is the law predicted by the theory in the weak-coupling limit. Experimental values generally lie between the two curves.

From PR **108**, 1175 (1957) Bardeen, Cooper, and Schrieffer, "Theory of Superconductivity"

$$T_c \approx \frac{\hbar\omega}{k} e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

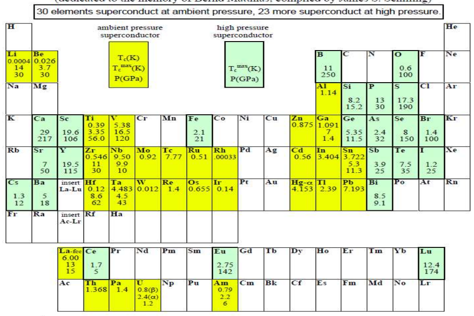
Type I elemental superconductors

http://wuphys.wustl.edu/~jss/NewPeriodicTable.pdf

Periodic Table of Superconductivity

(dedicated to the memory of Bernd Matthias; compiled by James S. Schilling)

30 elements superconduct at ambient pressure, 23 more superconduct at high pressure



M. Debessai, T. Matsuoka, J.J. Hamlin, W. Bi, Y. Meng, K. Shimizu, and J.S. Schilling, J. Phys.: Conf. Series **215**, 012034 (2010).
High pressure data for Ca and Be: K. Shimizu email from 9 Dec 2013.

Type I superconductors:

$$H_c(T) = H_c(0)\left(1-\frac{T^2}{T_c^2}\right)$$

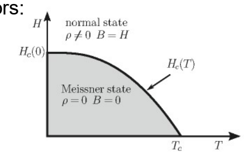


Figure 18.3 Schematic phase diagram illustrating normal and superconducting regions of a type-I superconductor.

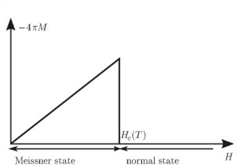


Figure 18.4 Magnetization versus applied field for type-I superconductors.

Type II superconductors

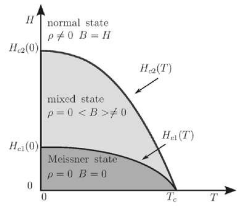


Figure 18.5 Schematic phase diagram illustrating normal, mixed and Meissner regions of a type-II superconductor (the vanishingly small resistivity of the mixed state occurs if flux lines are "pinned" by appropriate material defects); in the mixed state, $\langle B \rangle$ denotes the average magnetic field in the superconductor.

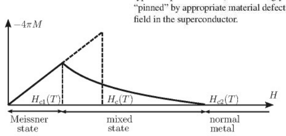


Figure 18.6 Magnetization versus applied field H for a type-II superconductor. The equivalent area construction of the thermodynamic field $H_c(T)$ is also illustrated.

Quantization of current flux associated with the superconducting state (Ref: Ashcroft and Mermin, ***Solid State Physics***)

From the London equations for the interior of the superconductor:

$$\left(m\mathbf{v} + \frac{e}{c}\mathbf{A} \right) = 0$$

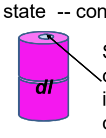
Now suppose that the current carrier is a pair of electrons characterized by a wavefunction of the form $\psi = |\psi| e^{i\phi}$

The quantum mechanical current associated with the electron pair is

$$\mathbf{j} = -\frac{e\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^* \right) - \frac{2e^2}{mc}\mathbf{A}|\psi|^2$$

$$= -\left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A} \right)|\psi|^2$$

Quantization of current flux associated with the superconducting state -- continued



Suppose a superconducting material has a cylindrical void. Evaluate the integral of the current in a closed path within the superconductor containing the void.

$$\oint \mathbf{j} \cdot d\mathbf{l} = 0 = -\oint \left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A} \right)|\psi|^2 \cdot d\mathbf{l}$$

$$\oint \mathbf{A} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \int \mathbf{B} \cdot d\mathbf{a} = \Phi \quad \text{magnetic flux}$$

$$\oint \nabla\phi \cdot d\mathbf{l} = 2\pi n \quad \text{for some integer } n$$

$$\Rightarrow \text{Quantization of flux in the void: } |\Phi| = n\frac{hc}{2e} \equiv n\Phi_0$$

Such "vortex" fields can exist within type II superconductors.

Table 18.1 Critical temperature of some selected superconductors, and zero-temperature critical field. For elemental materials, the thermodynamic critical field $H_c(0)$ is given in gauss. For the compounds, which are type-II superconductors, the upper critical field $H_{c2}(0)$ is given in Tesla ($1\ \mathrm{T} = 10^4\ \mathrm{G}$). The data for metallic elements and binary compounds of V and Nb are taken from G. Burns (1992). The data for MgB_2 and iron pnictide are taken from the references cited in the text, and refer to the two principal crystallographic axes. The data for the other compounds are taken from D. R. Harshman and A. P. Mills, Phys. Rev. B *45*, 10684 (1992)]. A more extensive list of data can be found in the mentioned references.

Metallic elements	$T_c(K)$	$H_c(0)$ (gauss)
Al	1.17	105
Sn	3.72	305
Pb	7.19	803
Hg	4.15	411
Nb	9.25	2060
V	5.40	1410
Binary compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
V_3Ga	16.5	27
V_3Si	17.1	25
Nb_3Al	20.3	34
Nb_3Ge	23.3	38
MgB_2	40	$\approx 5; \approx 20$
Other compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
UPt_3 (heavy fermion)	0.53	2.1
$PbMo_6S_8$ (Chevrel phase)	12	55
$\kappa-[BEDT-TTF]_2Cu[NCS]_2$ (organic phase)	10.5	≈ 10
Rb_2CsC_{60} (fullerene)	31.3	≈ 30
$NdFeAsO_{0.7}F_{0.3}$ (iron pnictide)	47	$\approx 30;\ \approx 50$
Cuprate oxides	$T_c(K)$	$H_{c2}(0)$ (Tesla)
$La_{2-x}Sr_xCuO_4$ ($x \approx 0.15$)	38	≈ 45
$YBa_2Cu_3O_7$	92	≈ 140
$Bi_2Sr_2CaCu_2O_8$	89	≈ 107
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125	≈ 75

Crystal structure of one of the high temperature superconductors

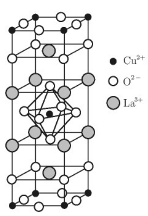


Figure 18.1 Crystal structure of the ceramic material La_2CuO_4. Appropriately doped, lanthanum-based cuprates opened the path to high-T_c superconductivity in 1986.

Some details of single vortex in type II superconductor

London equation without vortices:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \text{where} \quad \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

Equation for field with single quantum of vortex along z - axis:

$$\nabla^2\mathbf{B} - \frac{1}{\lambda_L^2}\mathbf{B} = -\frac{\Phi_0}{\lambda_L^2}\hat{\mathbf{z}}\delta(\mathbf{r}) \qquad \Phi_0 = \frac{hc}{2e} \qquad \mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$$

Solution: $\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}}\frac{\Phi_0}{2\pi\lambda_L^2}K_0\left(\frac{r}{\lambda_L}\right)$

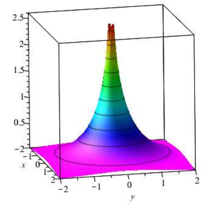
Check:

For $r > 0$ $\quad \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = 0$

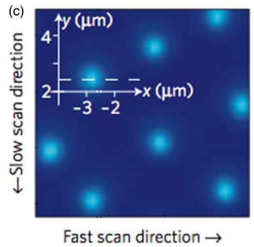
For $r \to 0$ $\quad 2\pi\int_0^r dr'\, r'\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = -2\pi$

Since $K_0(u) \underset{u\to 0}{\approx} -\ln u$

$$\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}} \frac{\Phi_0}{2\pi\lambda_L^2} K_0\left(\frac{r}{\lambda_L}\right)$$



Scanning probe images of vortices in YBCO at 22 K



IOP Publishing
Rep. Prog. Phys. 73 (2010) 126501 (36pp)
doi:10.1088/0034-4885/73/12/126501

Fundamental studies of superconductors using scanning magnetic imaging

J R Kirtley
Center for Probing the Nanoscale, Stanford University, Stanford, CA, USA