PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 30:

Start reading Chap. 15 –

Radiation from collisions of charged particles

1. Overview
2. X-ray tube
3. Radiation from Rutherford scattering
4. Other collision models

23	Fri: 03/22/2019	Chap. 9 and 10	Radiation from oscillating sources	#17	3/27/2019
24	Mon: 03/25/2019	Chap. 11	Special Theory of Relativity	Pick topic	3/29/2019
25	Wed: 03/27/2019	Chap. 11	Special Theory of Relativity	#18	4/01/2019
26	Fri: 03/29/2019	Chap. 11	Special Theory of Relativity	#19	4/03/2019
27	Mon: 04/01/2019	Chap. 14	Radiation from accelerating charged particles	#20	4/05/2019
28	Wed: 04/03/2019	Chap. 14	Synchrotron radiation		
29	Fri: 04/05/2019	Chap. 14	Synchrotron radiation	#21	4/10/2019
30	Mon: 04/08/2019	Chap. 15	Radiation from collisions of charged particles	#22	4/12/2019
31	Wed: 04/10/2019	Chap. 13	Cherenkov radiation		
32	Fri: 04/12/2019		Special topic: E & M aspects of superconductivity		
33	Mon: 04/15/2019		Special topic: Aspects of optical properties of materials		
34	Wed: 04/17/2019	Chap. 1-15	Review		
	Fri: 04/19/2019	No class	*Good Friday*		
35	Mon: 04/22/2019	Chap. 1-15	Review		
36	Wed: 04/24/2019	Chap. 1-15	Review		
	Fri: 04/26/2019		Presentations I		
	Mon: 04/29/2019		Presentations II		
	Wed: 05/01/2019		Presentations III		

Schedule for PHY 712 presentations

Friday, April 26, 2019

	Name	Topic
9:00-9:16	Ian Newsome	Current density from particle production in classical background E-field coupled to quantum scalar field. :)
9:17-9:31	Dizhou Wu	Ewald
9:32-9:48	Leda Gao	Hyperfine Hamiltonian

Monday, April 29, 2019

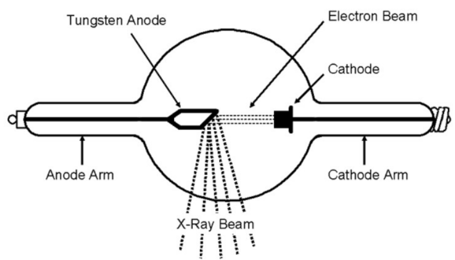
	Name	Topic
9:00-9:16	Shohreh Gholizadeh	3D FEM
9:17-9:31	Lindsey Gray	
9:32-9:48		

Wednesday, May 1, 2019

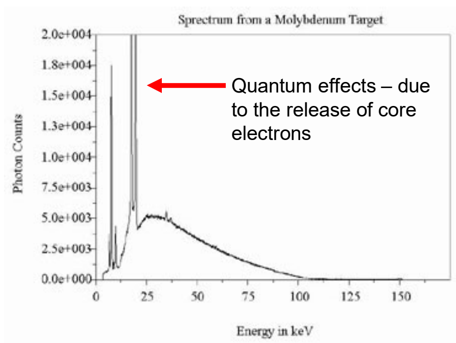
	Name	Topic
9:00-9:16	Eric	TBD
9:17-9:31	Ryan	Planetary Magnetism?
9:32-9:48		

Generation of X-rays in a Coolidge tube

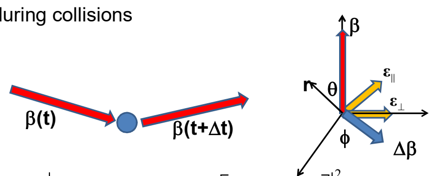
https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm



http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Radiation during collisions



Intensity:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\, e^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

Note that $\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}) = \hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\boldsymbol{\beta}) - \boldsymbol{\beta} = -(\boldsymbol{\varepsilon}_{\parallel}\cdot\boldsymbol{\beta})\boldsymbol{\varepsilon}_{\parallel} - (\boldsymbol{\varepsilon}_{\perp}\cdot\boldsymbol{\beta})\boldsymbol{\varepsilon}_{\perp}$

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \rightarrow 0$:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Radiation during collisions -- continued

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Non-relativistic limit:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot(\Delta\boldsymbol{\beta})\right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$$

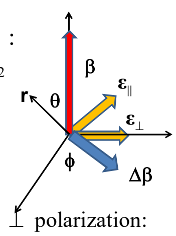
Relativistic collision with small $|\Delta\boldsymbol{\beta}| \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$

Radiation during collisions -- continued

Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$



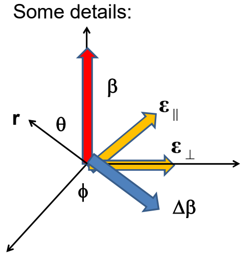
Also assume $\Delta\boldsymbol{\beta}$ is perpendicular to $\boldsymbol{\beta}$ direction

Expressions (averaging over φ) for $\|$ or $\perp$ polarization:

$$\frac{d^2 I_{\|}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$ polarization in ***r*** and ***β*** plane

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$ polarization perpendicular to ***r*** and ***β*** plane

Some details:



$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}} \qquad \hat{\mathbf{r}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{\|} = -\cos\theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{z}} \qquad \boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}}$$

$$\Delta\boldsymbol{\beta} = \Delta\beta(\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}})$$

Some details -- continued:

$$\hat{\mathbf{r}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_\perp = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_\parallel = -\cos\,\theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}}$$

$$\Delta\boldsymbol{\beta} = \Delta\beta\left(\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}}\right)$$

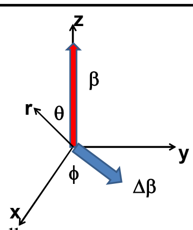
$$\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times\left(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta}\right) = \Delta\boldsymbol{\beta}\left(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}\right) + \boldsymbol{\beta}\left(\hat{\mathbf{r}}\cdot\Delta\boldsymbol{\beta}\right)$$

$$\boldsymbol{\varepsilon}_\perp\cdot\left(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times\left(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta}\right)\right) = \Delta\beta\sin\phi\left(1-\beta\cos\theta\right)$$

$$\boldsymbol{\varepsilon}_\parallel\cdot\left(\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times\left(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta}\right)\right) = \Delta\beta\cos\phi\left(\beta-\cos\theta\right)$$

Radiation during collisions -- continued

Intensity expressions:



$$\frac{d^2I_\parallel}{d\omega d\Omega} = \frac{q^2}{8\pi^2c}\left|\Delta\boldsymbol{\beta}\right|^2\frac{\left(\beta-\cos\theta\right)^2}{\left(1-\beta\cos\theta\right)^4}$$

$$\frac{d^2I_\perp}{d\omega d\Omega} = \frac{q^2}{8\pi^2c}\left|\Delta\boldsymbol{\beta}\right|^2\frac{1}{\left(1-\beta\cos\theta\right)^2}$$

Relativistic collision at low ω and with small $\left|\Delta\boldsymbol{\beta}\right|$ and $\Delta\boldsymbol{\beta}$ perpendicular to plane of $\hat{\mathbf{r}}$ and $\boldsymbol{\beta}$, as a function of θ where $\hat{\mathbf{r}}\cdot\boldsymbol{\beta} = \beta\cos\theta$;

Integrating over solid angle:

$$\frac{dI}{d\omega} = \int d\Omega\left(\frac{d^2I_\parallel}{d\omega d\Omega} + \frac{d^2I_\perp}{d\omega d\Omega}\right) = \frac{2}{3\pi}\frac{q^2}{c}\gamma^2\ \left|\Delta\boldsymbol{\beta}\right|^2$$

Some details:

$$\int d\Omega\frac{d^2I_\parallel}{d\omega d\Omega} = \frac{q^2}{8\pi^2c}\left|\Delta\boldsymbol{\beta}\right|^2 2\pi\int_{-1}^{1}d\cos\theta\frac{\left(\beta-\cos\theta\right)^2}{\left(1-\beta\cos\theta\right)^4}$$

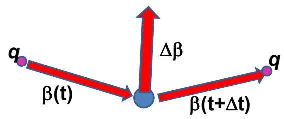
$$= \frac{q^2}{4\pi c}\left|\Delta\boldsymbol{\beta}\right|^2\frac{2}{3}\frac{1}{\left(1-\beta^2\right)}$$

$$\int d\Omega\frac{d^2I_\perp}{d\omega d\Omega} = \frac{q^2}{8\pi^2c}\left|\Delta\boldsymbol{\beta}\right|^2\int_{-1}^{1}d\cos\theta\frac{1}{\left(1-\beta\cos\theta\right)^2}$$

$$= \frac{q^2}{4\pi c}\left|\Delta\boldsymbol{\beta}\right|\frac{2}{\left(1-\beta^2\right)}$$

$$\frac{dI}{d\omega} = \int d\Omega\left(\frac{d^2I_\parallel}{d\omega d\Omega} + \frac{d^2I_\perp}{d\omega d\Omega}\right) = \frac{2}{3\pi}\frac{q^2}{c}\gamma^2\ \left|\Delta\boldsymbol{\beta}\right|^2$$

Estimation of $\Delta\beta$



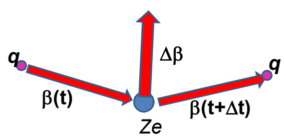
Momentum transfer:

$$Qc \equiv \left|\mathbf{p}(t+\tau) - \mathbf{p}(t)\right| c \approx \gamma M c^2 \left|\boldsymbol{\Delta\beta}\right|$$

mass of particle having charge *q*

$$\frac{dI}{d\omega} = \frac{2}{3\pi}\frac{q^2}{c}\gamma^2\left|\boldsymbol{\Delta\beta}\right|^2 \approx \frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2$$

Estimation of $\Delta\beta$ -- for the case of Rutherford scattering

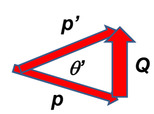


Assume that target nucleus (charge *Ze*) has mass >>M;
Rutherford scattering cross-section in center of mass analysis:

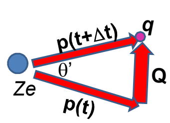
$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

Assuming elastic scattering:

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$



Case of Rutherford scattering -- continued



Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

$$\frac{d\sigma}{dQ} = \int_{\varphi'} \frac{d\sigma}{d\Omega}\left|\frac{d\Omega}{dQ}\right|$$

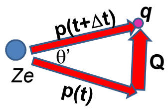
$$d\Omega = d\varphi' d\cos\theta'$$

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

$$dQ = -\frac{p^2}{Q}d\cos\theta'$$

$$\Rightarrow \quad \frac{d\sigma}{dQ} = 8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}$$

Case of Rutherford scattering -- continued



Differential radiation cross section :

$$\frac{d^2\chi}{d\omega dQ} = \frac{dI}{d\omega}\frac{d\sigma}{dQ} = \left(\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2\right)\left(8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}\right)$$

$$= \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\frac{1}{Q}$$

Differential radiation cross section -- continued

Integrating over momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Comment on frequency dependence --

Original expression for radiation intensity :

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\, e^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)}\frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

In the previous derivations, we have assumed that

$\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c) << 1.$

$$\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c) = \omega\left(t-\hat{\mathbf{r}}\cdot\int_0^t dt'\boldsymbol{\beta}(t')\right) \approx \omega\tau(1-\hat{\mathbf{r}}\cdot\langle\boldsymbol{\beta}\rangle)$$

Differential radiation cross section -- continued

Radiation cross section in terms of momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Note that: $Q^2 = 2p^2(1-\cos\theta') \quad \Rightarrow Q_{\max} = 2p$

In general, $Q_{\min}$ is determined by the collision time

condition $\omega\tau < 1 \quad \Rightarrow Q_{\min} \approx \frac{2Zeq\omega}{v^2}$

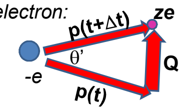
Radiation cross section for classical non-relativistic process

$$\frac{d\chi}{d\omega} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{\lambda M v^3}{Zeq\omega}\right)$$

λ= "fudge factor" of order unity

Electromagnetic effects in energy loss processes
(see Chap. 13 of Jackson)

Again consider Rutherford scattering – now of a nucleus (or alpha particle *ze* incident on an electron *–e in rest frame of electron:*



Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{ze^2}{2pv}\right)^2 \frac{1}{\left(\sin(\theta'/2)\right)^4}$$

$$\frac{d\sigma}{dQ^2} = \int d\varphi' \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ^2}$$

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2(1-\cos\theta')$$

$$\Rightarrow \quad \frac{d\sigma}{dQ^2} = 4\pi\left(\frac{ze^2}{\beta c Q^2}\right)^2$$

Energy loss continued

Let T represent energy loss due to electron of mass m:

$$T = Q^2/2m$$

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 e^4}{mc^2\beta^2 T^2}$$

Estimate of energy loss per unit distance in the presence of NZ electrons per unit volume

$$\frac{dE}{dx} \approx NZ \int_{\varepsilon}^{T_{max}} dT\, T \frac{d\sigma}{dT}$$

minimum energy transfer

$$= 2\pi NZ \frac{z^2 e^4}{mc^2\beta^2} \ln\left(\frac{2\gamma^2\beta^2 mc^2}{\varepsilon}\right) + (\text{quantum effects})$$

Energy loss continued

Refining this result, Bethe and Fermi noticed that the analysis lacked consideration of the effects of electromagnetic fields. Representing the colliding electrons in terms of a dielectric function $\varepsilon(\omega)$ and the energetic particle of charge *ze* in terms of the charge and current density:

In Fourier space:

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\Phi(\mathbf{k},\omega) = \frac{4\pi}{\varepsilon(\omega)}\rho(\mathbf{k},\omega)$$

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\mathbf{A}(\mathbf{k},\omega) = \frac{4\pi}{c}\mathbf{J}(\mathbf{k},\omega)$$

$$\rho(\mathbf{k},\omega) = \frac{ze}{2\pi}\delta(\omega - \mathbf{v}\cdot\mathbf{k})$$

$$\mathbf{J}(\mathbf{k},\omega) = \mathbf{v}\rho(\mathbf{k},\omega)$$

Energy loss continued

$$\Phi(\mathbf{k},\omega) = \frac{2ze}{\varepsilon(\omega)} \frac{\delta(\omega - \mathbf{v}\cdot\mathbf{k})}{k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)}$$

$$\mathbf{A}(\mathbf{k},\omega) = \varepsilon(\omega)\frac{\mathbf{v}}{c}\Phi(\mathbf{k},\omega)$$

The energy loss will be calculated from the work on the electron by the field:

$$\Delta E = -e\int_{-\infty}^{\infty} dt\ \mathbf{v}\cdot\mathbf{E}(t) = 2e\Re\left(\int_0^{\infty} d\omega\ i\omega\mathbf{r}(\omega)\cdot\mathbf{E}^*(\omega)\right)$$

The resultant loss estimate is

$$\frac{dE}{dx} \approx \frac{z^2e^2\omega_P^2}{2c^2}\ln\left(\frac{2mc^2\varepsilon}{\hbar^2\omega_P^2}\right) \quad \text{where } \omega_P^2 \equiv \frac{4\pi NZe^2}{m}$$