PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 29:

Finish reading Chap. 14 –

Radiation from charged particles

1. **Review of synchrotron radiation**
2. **Free electron laser**
3. **Thompson and Compton scattering**

21	Mon: 03/18/2019	Chap. 9	Radiation from localized oscillating sources	#15	3/22/2019
22	Wed: 03/20/2019	Chap. 9	Radiation from oscillating sources	#16	3/25/2019
23	Fri: 03/22/2019	Chap. 9 and 10	Radiation from oscillating sources	#17	3/27/2019
24	Mon: 03/25/2019	Chap. 11	Special Theory of Relativity	Pick topic	3/29/2019
25	Wed: 03/27/2019	Chap. 11	Special Theory of Relativity	#18	4/01/2019
26	Fri: 03/29/2019	Chap. 11	Special Theory of Relativity	#19	4/03/2019
27	Mon: 04/01/2019	Chap. 14	Radiation from accelerating charged particles	#20	4/05/2019
28	Wed: 04/03/2019	Chap. 14	Synchrotron radiation		
29	Fri: 04/05/2019	Chap. 14	Synchrotron radiation	#21	4/10/2019
30	Mon: 04/08/2019				
31	Wed: 04/10/2019				
32	Fri: 04/12/2019				
33	Mon: 04/15/2019				
34	Wed: 04/17/2019				
	Fri: 04/19/2019	No class	*Good Friday*		
35	Mon: 04/22/2019				
36	Wed: 04/24/2019				
	Fri: 04/26/2019		Presentations I		
	Mon: 04/29/2019		Presentations II		
	Wed: 05/01/2019		Presentations III		

Schedule for PHY 712 presentations
Friday, April 26, 2019

	Name	Topic
9:00-9:16		
9:17-9:31		
9:32-9:48		

Monday, April 29, 2019

	Name	Topic
9:00-9:16		
9:17-9:31		
9:32-9:48		

Wednesday, May 1, 2019

	Name	Topic
9:00-9:16		
9:17-9:31		
9:32-9:48		

Advice about remaining topics to cover

- Cherenkov radiation
- Electrodynamics of superconductivity
- Radiation from collisions of charged particles
- Optical properties of materials

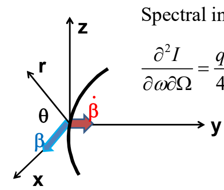
Spectral composition of electromagnetic radiation -- continued

The spectral intensity per unit solid angle Ω and frequency ω for a particle of charge q, with trajectory $\mathbf{R}_q(t)$ and velocity $\frac{d\mathbf{R}_q(t)}{dt} \equiv \boldsymbol{\beta}(t)c$, depends on the following integral:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt\; e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t)\right)\right]\right|^2$$

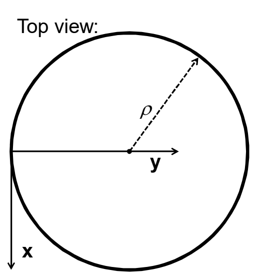
Recall that the spectral intensity is related to the time integrated power:

$$\int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} d\omega \frac{\partial^2 I}{\partial\omega\partial\Omega}$$



Spectral intensity relationship:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt\; e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t)\right)\right]\right|^2$$

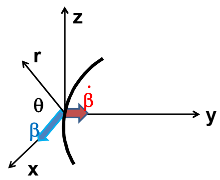


$$\mathbf{R}_q(t) = \rho\hat{\mathbf{x}}\sin(vt/\rho) + \rho\hat{\mathbf{y}}\left(1-\cos(vt/\rho)\right)$$

$$\boldsymbol{\beta}(t) = \beta\left(\hat{\mathbf{x}}\cos(vt/\rho) + \hat{\mathbf{y}}\sin(vt/\rho)\right)$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



$$\mathbf{R}_q(t) = \rho\hat{\mathbf{x}}\sin(vt/\rho) + \rho\hat{\mathbf{y}}\left(1-\cos(vt/\rho)\right)$$

$$\boldsymbol{\beta}(t) = \beta\left(\hat{\mathbf{x}}\cos(vt/\rho) + \hat{\mathbf{y}}\sin(vt/\rho)\right)$$

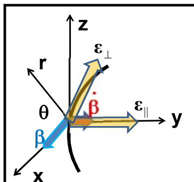
For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Note that we have previous shown that in the radiation zone, the Poynting vector is in the $\hat{\mathbf{r}}$ direction; we can then choose to analyze two orthogonal polarization directions:

$$\boldsymbol{\varepsilon}_{\|} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right) = \beta\left(-\boldsymbol{\varepsilon}_{\|}\sin(vt/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt/\rho)\right)$$



$$\boldsymbol{\varepsilon}_{\|} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right) = \beta\left(-\boldsymbol{\varepsilon}_{\|}\sin(vt/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt/\rho)\right)$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})e^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)}dt\right|^2$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\|}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\} \qquad \text{where} \quad \beta \equiv \frac{v}{c}$$

$$C_{\|}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho)e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho)e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

PHYSICAL REVIEW VOLUME 75, NUMBER 12 JUNE 15, 1949

On the Classical Radiation of Accelerated Electrons

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

(Received March 8, 1949)

This paper is concerned with the properties of the radiation from a high energy accelerated electron, as recently observed in the General Electric synchrotron. An elementary derivation of the total rate of radiation is first presented, based on Larmor's formula for a slowly moving electron, and arguments of relativistic invariance. We then construct an expression for the instantaneous power radiated by an electron moving along an arbitrary, prescribed path. By casting this result into various forms, one obtains the angular distribution, the spectral distribution, or the combined angular and spectral distributions of the radiation. The method is based on an examination of the rate at which the electron irreversibly transfers energy to the electromagnetic field, as determined by half the difference of retarded and advanced electric field intensities. Formulas are obtained for an arbitrary charge-current distribution and then specialized to a point charge. The total radiated power and its angular distribution are obtained for an arbitrary trajectory. It is found that the direction of motion is a strongly preferred direction of emission at high energies. The spectral distribution of the radiation depends upon the detailed motion over a time interval large compared to the period of the radiation. However, the narrow cone of radiation generated by an energetic electron indicates that only a small part of the trajectory is effective in producing radiation observed in a given direction, which also implies that very high frequencies are emitted. Accordingly, we evaluate the spectral and angular distributions of the high frequency radiation by an energetic electron, in their dependence upon the parameters characterizing the instantaneous orbit. The average spectral distribution, as observed in the synchrotron measurements, is obtained by averaging the electron energy over an acceleration cycle. The entire spectrum emitted by an electron moving with constant speed in a circular path is also discussed. Finally, it is observed that quantum effects will modify the classical results here obtained only at extraordinarily large energies.

EARLY in 1945, much attention was focused on the design of accelerators for the production of very high energy electrons and other charged particles.[1] In connection with this activity, the author investigated in some detail the limitations to the

is instantaneously at rest is

$$P = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{d\mathbf{v}}{dt}\right)^2 = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{d\mathbf{p}}{dt}\right)^2. \qquad \text{(I.1)}$$

Evaluation of integrals representing distant charged particles moving in a circular trajectory such that the spectrum represents a superposition of light generated over many complete circles. In this case, there is an interference effect which results in the spectrum consisting of discrete multiples of v/ρ.

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\}$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

Useful identity involving Bessel functions

$$e^{-iA\sin\alpha} = \sum_{m=-\infty}^{\infty} J_m(A) e^{-im\alpha} \qquad \text{Here } J_m(A) \text{ is a}$$

Bessel function of integer order m.

In our case, $A = \frac{\omega\rho}{c}\cos\theta$ and $\alpha = \frac{vt}{\rho}$.

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= \frac{c}{-i\omega\rho}\frac{\partial}{\partial\cos\theta}\int_{-\infty}^{\infty} dt e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= \frac{c}{-i\omega\rho}\frac{\partial}{\partial\cos\theta}\sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right) 2\pi\delta(\omega - m\frac{v}{\rho}).$$

Astronomical synchrotron radiation -- continued:

Note that:

$$\int_{-\infty}^{\infty} dt e^{i(\omega - m\frac{v}{\rho})t} = 2\pi\delta(\omega - m\frac{v}{\rho}).$$

$$\Rightarrow C_{\parallel}(\omega) = 2\pi i \sum_{m=-\infty}^{\infty} J_m'\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}),$$

where $J_m'(A) \equiv \frac{dJ_m(A)}{dA}$

Similarly:

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= 2\pi\frac{\tan\theta}{v/c}\sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}).$$

Astronomical synchrotron radiation -- continued:

In both of the expressions, the sum over *m* includes both negative and positive values. However, only the positive values of ω and therefore positive values of *m* are of interest. Using the identity: $J_{-m}(A) = (-1)^m J_m(A)$, the result becomes:

$$\frac{d^2 I}{d\omega d\Omega} =$$

$$\frac{q^2\omega^2\beta^2}{c}\sum_{m=0}^{\infty}\delta(\omega - m\frac{v}{\rho})\left\{\left[J_m'\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2 + \frac{\tan^2\theta}{v^2/c^2}\left[J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2\right\}.$$

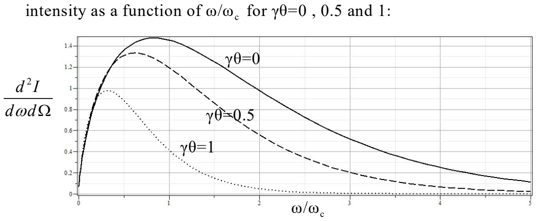
These results were derived by Julian Schwinger (Phys. Rev. **75**, 1912-1925 (1949)). The discrete case is similar to the result quoted in Problem 14.15 in Jackson's text.

Recall the previous result for man-made synchrotrons used as light sources. In this case, the light is produced by short bursts of electrons moving close to the speed of light ($v \approx c(1 - 1/(2\gamma^2))$) passing a beam line port. In addition, because of the design of the radiation ports, $\theta \approx 0$, and the relevant integration times t are close to $t \approx 0$. This results in the form shown in Eq. 14.79 of your text. It is convenient to rewrite this form in terms of a critical frequency $\omega_c \equiv \frac{3c\gamma^3}{2\rho}$. In that range, the differential intensity takes the form:

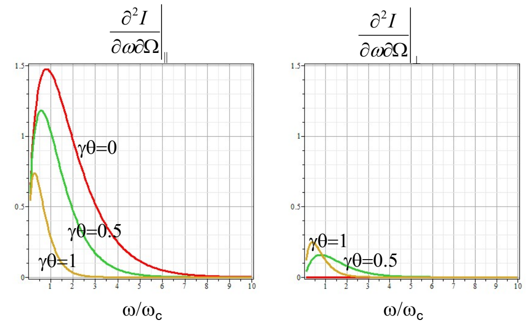
$$\frac{d^2 I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2\right\}$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2\right\}$$

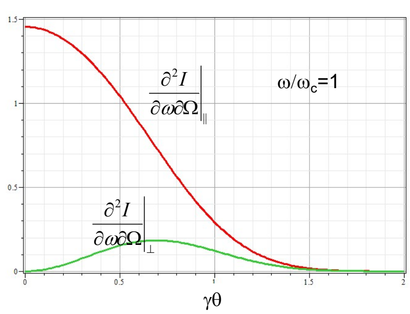
By plotting the intensity as a function of ω, we see that the intensity is largest near $\omega \approx \omega_c$. The plot below shows the intensity as a function of ω/ω_c for $\gamma\theta$=0 , 0.5 and 1:



Plots of the intensity function for synchrotron radiation

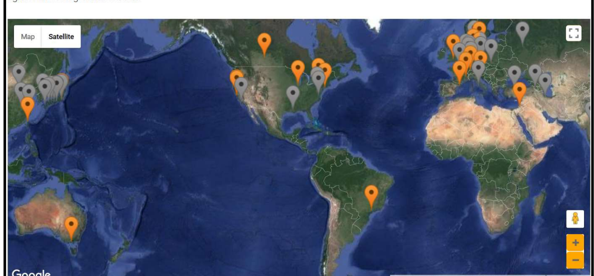


Plots of the intensity function for synchrotron radiation



Synchrotron facilities worldwide
https://lightsources.org/

There are more than 50 light sources in the world (operational, or under construction). This page lists all the members of the lightsources.org collaboration.



Free electron laser
Reference:

Classical Theory of Free-Electron Lasers

A text for students and researchers

Eric B Szarmes

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ISBN 978-1-6270-5572-7 (print)
ISBN 978-1-6270-5680-9 (mobi)

DOI 10.1088/978-1-6270-5573-4

1.1 The free-electron laser

A free-electron laser (FEL) is a laser source that produces spatially and temporally coherent optical radiation by stimulated emission, where in place of an atomic or molecular medium to provide amplification the gain medium is comprised of a beam of relativistic electrons traveling in a vacuum through a periodic magnetic field. The basic components common to all FELs are a relativistic electron beam, a periodic magnetic structure (an undulator or wiggler magnet of spatial period λ_w), and an optical resonator providing feedback and amplification. (X-ray FELs such as the Linac Coherent Light Source at Stanford omit the optical resonator by necessity and achieve the required gain on a single pass.) The features that make FELs particularly useful as research devices are the unique combination of continuous and broadband tunability, high peak and average power, and spatial and temporal coherence.

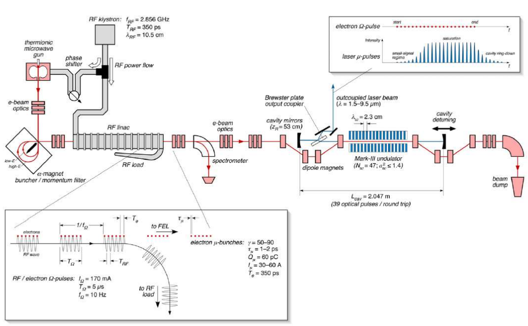
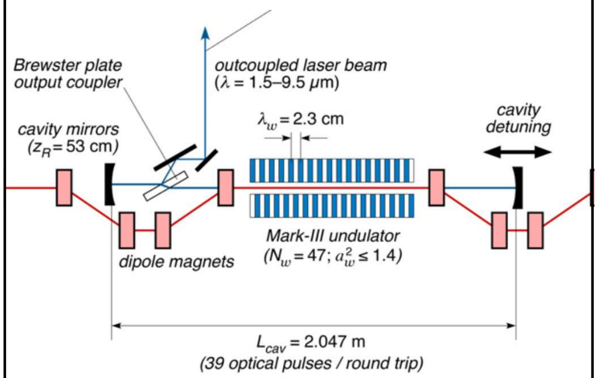


Figure 1.1. The MkIII RF-linac FEL.



Electron emission in periodic magnet

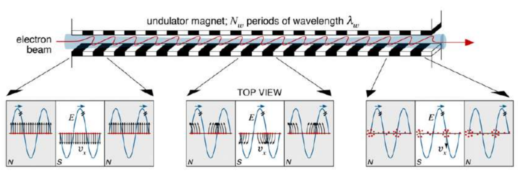


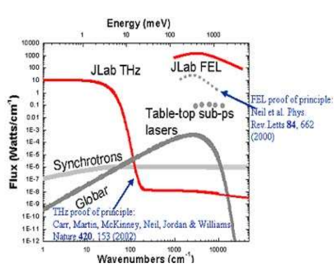
Figure 1.3. Conceptual illustration of the bunching mechanism in an FEL.

Because of Doppler shift and Thompson scattering, effective

wavelength is: $\lambda = \dfrac{\lambda_w}{2\gamma_z^{\,2}}$

frequency is: $\omega = \dfrac{4\pi}{\lambda_w}\gamma_z^{\,2} c$

Spectral output for FEL at Jefferson Lab (in Virginia)



SCIENCE'S COMPASS • REVIEW

REVIEW: LASER TECHNOLOGY

Free-Electron Lasers: Status and Applications

Patrick G. O'Shea and Henry P. Freund

SCIENCE VOL 292 8 JUNE 2001

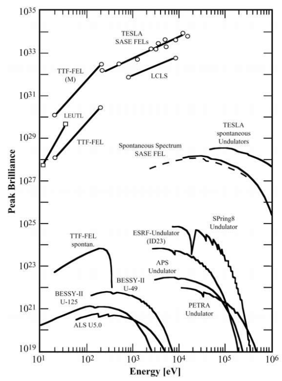
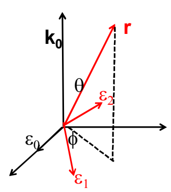


Fig. 3. Peak brilliance of x-ray FELs and undulators for spontaneous radiation at the TESLA Test Facility, in comparison with synchrotron radiation sources. Brilliance is expressed as photons s^{-1} $mrad^{-2}$ mm^{-2} per 0.1% bandwidth. For comparison, the spontaneous spectrum of x-ray FEL undulators is also shown. The label TTF-FEL indicates design values for the FEL at the TESLA Test Facility, with (M) for the planned seeded version (28).

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



$$\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\dot{\mathbf{v}}\right)\right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q) is caused by an electric field : $\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$

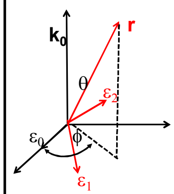
$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Time averaged power : $\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 \left|E_0\right|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0\right)\right|^2$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\varepsilon_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\varepsilon_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\varepsilon_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization ε^*:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2$$

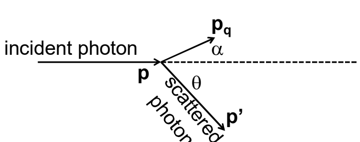
Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

$$\left\langle |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \left\langle |\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi + \left\langle |\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \frac{1}{2}\cos^2\theta + \frac{1}{2}$$

Averaged cross section: $\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \frac{1}{2}\left(1 + \cos^2\theta\right)$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy:

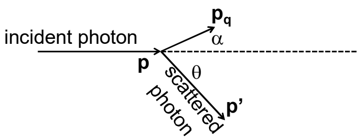
$$p = p'\cos\theta + p_q\cos\alpha \qquad pc = \hbar\omega$$

$$0 = p'\sin\theta - p_q\sin\alpha \qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p_q^2 c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$

Thompson scattering – relativistic and quantum modifications



Relativistic and quantum modifications to averaged cross section :

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

$$\frac{p'}{p} = \frac{1}{1+\frac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$

04/5/2019 PHY 712 Spring 2019 -- Lecture 29 31