

PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 27:

**Continue reading Chap. 14 –
Radiation by moving charges**

- 1. Motion in a line**
- 2. Motion in a circle**
- 3. Spectral analysis of radiation**

04/01/2019

PHY 712 Spring 2019 – Lecture 27

1

Wake Forest College & Graduate School of Arts and Sciences

WFU Physics | People | Events and News | Undergraduate | Graduate | Research | Resources

Events

Colloquium: "Emerson Corporation-A Global Manufacturer of Industrial and Residential Products" – Wednesday, April 3, 2019, at 4:00 PM
 Speaker: O. Leifels, PhD Wake Forest Alumni
 Retired CEO of Emerson Electric Company
 George P. Williams, Jr. Lecture Hall, (Olin 105)
 Wednesday, April 3, 2019, at 4:00 PM There will...

News

And... **Shubham**... with science museum to create exhibits

Randall Leiford receives Physics Department Outstanding Alumni Award

Nationally recognized for teaching excellence; internationally respected for research

04/01/2019 PHY 712 Spring 2019 – Lecture 27 2

Fri: 03/15/2019	No class	Spring Break		
21 Mon: 03/18/2019	Chap. 9	Radiation from localized oscillating sources	#15	3/22/2019
22 Wed: 03/20/2019	Chap. 9	Radiation from oscillating sources	#16	3/25/2019
23 Fri: 03/22/2019	Chap. 9 and 10	Radiation from oscillating sources	#17	3/27/2019
24 Mon: 03/25/2019	Chap. 11	Special Theory of Relativity	Pick topic	3/29/2019
25 Wed: 03/27/2019	Chap. 11	Special Theory of Relativity	#18	4/01/2019
26 Fri: 03/29/2019	Chap. 11	Special Theory of Relativity	#19	4/03/2019
27 Mon: 04/01/2019	Chap. 14	Radiation from accelerating charged particles	#20	4/05/2019
28 Wed: 04/03/2019				
29 Fri: 04/05/2019				
30 Mon: 04/08/2019				
31 Wed: 04/10/2019				
32 Fri: 04/12/2019				
33 Mon: 04/15/2019				
34 Wed: 04/17/2019				
Fri: 04/19/2019	No class	Good Friday		
35 Mon: 04/22/2019				
36 Wed: 04/24/2019				
Fri: 04/26/2019		Presentations I		
Mon: 04/29/2019		Presentations II		
Wed: 05/01/2019		Presentations III		

04/01/2019 PHY 712 Spring 2019 – Lecture 27 3

Radiation from a moving charged particle

Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 4

Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{(R - \mathbf{v} \cdot \mathbf{R})^3} \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \left(1 - \frac{v^2}{c^2} \right) + \left(\mathbf{R} \times \left\{ \left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right\} \right) \right]. \quad (19)$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[\frac{-\mathbf{R} \times \mathbf{v}}{(R - \mathbf{v} \cdot \mathbf{R})^3} \left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}} \cdot \mathbf{R}}{c^2} \right) - \frac{\mathbf{R} \times \dot{\mathbf{v}}/c}{(R - \mathbf{v} \cdot \mathbf{R})^2} \right]. \quad (20)$$

In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}. \quad (21)$$

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v} \quad \mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R} \quad \dot{\mathbf{v}} \equiv \frac{d^2\mathbf{R}_q(t_r)}{dt_r^2}$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 5

Electric field far from source:

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c} \right)^3} \left\{ \mathbf{R} \times \left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c} \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right] \right\}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}$$

Let $\hat{\mathbf{R}} \equiv \frac{\mathbf{R}}{R}$ $\boldsymbol{\beta} \equiv \frac{\mathbf{v}}{c}$ $\dot{\boldsymbol{\beta}} \equiv \frac{\dot{\mathbf{v}}}{c}$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{cR(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \left\{ \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right\}$$

$$\mathbf{B}(\mathbf{r}, t) = \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r}, t)$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 6

Poynting vector:

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{B})$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR(1-\boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \left\{ \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r},t)$$

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi} \hat{\mathbf{R}} |\mathbf{E}(\mathbf{r},t)|^2 = \frac{q^2}{4\pi c R^2} \hat{\mathbf{R}} \frac{\left| \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|^2}{(1-\boldsymbol{\beta} \cdot \hat{\mathbf{R}})^6}$$

Note: We have used the fact that $\hat{\mathbf{R}} \cdot \mathbf{E}(\mathbf{r},t) = 0$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 7

Power radiated

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi} \hat{\mathbf{R}} |\mathbf{E}(\mathbf{r},t)|^2 = \frac{q^2}{4\pi c R^2} \hat{\mathbf{R}} \frac{\left| \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|^2}{(1-\boldsymbol{\beta} \cdot \hat{\mathbf{R}})^6}$$

$$\frac{dP}{d\Omega} = \mathbf{S} \cdot \hat{\mathbf{R}} R^2 = \frac{q^2}{4\pi c} \frac{\left| \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|^2}{(1-\boldsymbol{\beta} \cdot \hat{\mathbf{R}})^6}$$

In the non-relativistic limit: $\beta \ll 1$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \left| \hat{\mathbf{R}} \times \left[\hat{\mathbf{R}} \times \dot{\boldsymbol{\beta}} \right] \right|^2 = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2 \Theta$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 8

Radiation from a moving charged particle

Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

When $v \ll c$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2 \Theta$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 9

Radiation power in non-relativistic case -- continued

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2 \Theta$$

$$P = \int d\Omega \frac{dP}{d\Omega} = \frac{2}{3} \frac{q^2}{c^3} |\dot{\mathbf{v}}|^2$$

04/01/2019

PHY 712 Spring 2019 -- Lecture 27

10

Radiation distribution in the relativistic case

$$\frac{dP}{d\Omega} = \mathbf{S} \cdot \hat{\mathbf{R}} R^2 = \frac{q^2}{4\pi c} \left| \hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}] \right|^2 \bigg|_{t_r = t - R/c}$$

This expression gives us the energy per unit field time t . We are often interested in the power per unit retarded time $t_r = t - R/c$:

$$\frac{dP(t)}{d\Omega} = \frac{dP_r(t_r)}{d\Omega} \frac{dt}{dt_r} \quad \frac{dt}{dt_r} = 1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}$$

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \left| \hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}] \right|^2 \bigg|_{t_r = t - R/c}$$

04/01/2019

PHY 712 Spring 2019 -- Lecture 27

11

Radiation distribution in the relativistic case -- continued

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \left| \hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}] \right|^2 \bigg|_{t_r = t - R/c}$$

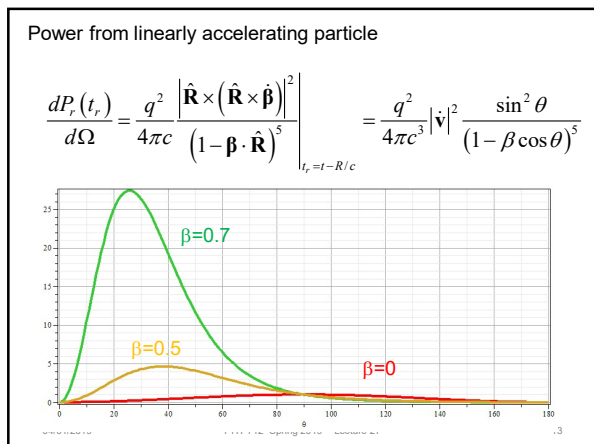
For linear acceleration: $\boldsymbol{\beta} \times \dot{\boldsymbol{\beta}} = 0$

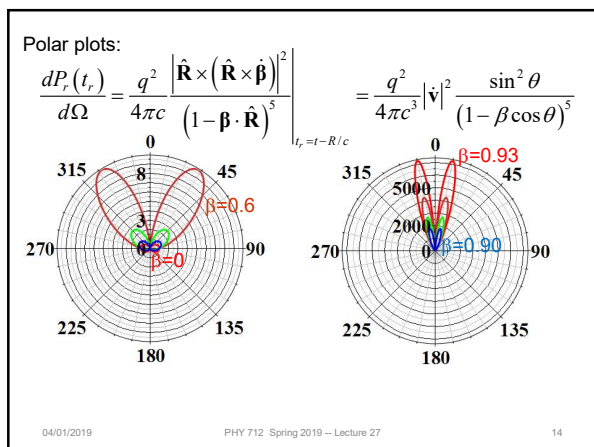
$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \left| \hat{\mathbf{R}} \times (\hat{\mathbf{R}} \times \dot{\boldsymbol{\beta}}) \right|^2 \bigg|_{t_r = t - R/c} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}$$

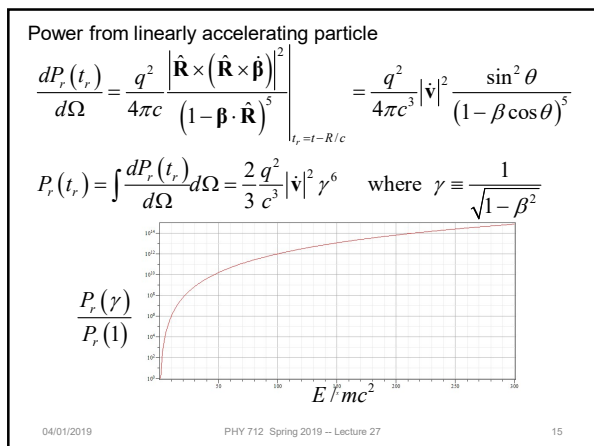
04/01/2019

PHY 712 Spring 2019 -- Lecture 27

12







Power distribution for linear acceleration -- continued

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \frac{|\hat{\mathbf{R}} \times (\hat{\mathbf{R}} \times \dot{\boldsymbol{\beta}})|^2}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^5} \bigg|_{t_r = t - R/c} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^3}$$

$$P_r(t_r) = \int \frac{dP_r(t_r)}{d\Omega} d\Omega = \frac{2}{3} \frac{q^2}{c^3} |\dot{\mathbf{v}}|^2 \gamma^6 \quad \text{where } \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$

04/01/2019 PHY 712 Spring 2019 -- Lecture 27 16

Power distribution for circular acceleration

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|^2}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^5} \bigg|_{t_r = t - R/c}$$

$$= \frac{q^2}{4\pi c} \frac{|\dot{\boldsymbol{\beta}}|^2 (1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^2 - (\hat{\mathbf{R}} \cdot \dot{\boldsymbol{\beta}})^2 (1 - \beta^2)}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^5} \bigg|_{t_r = t - R/c}$$

$$P_r(t_r) = \int d\Omega \frac{dP_r(t_r)}{d\Omega} = \frac{2}{3} \frac{q^2}{c^3} |\dot{\mathbf{v}}|^2 \gamma^4$$

04/01/2019 PHY 712 Spring 2019 -- Lecture 27 17

Power distribution for circular acceleration

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \frac{|\dot{\boldsymbol{\beta}}|^2 (1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^2 - (\hat{\mathbf{R}} \cdot \dot{\boldsymbol{\beta}})^2 (1 - \beta^2)}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^5} \bigg|_{t_r = t - R/c}$$

$$= \frac{q^2}{4\pi c^3} \frac{|\dot{\mathbf{v}}|^2}{(1 - \beta \cos(\theta))^3} \left(1 - \frac{\cos^2 \theta \sin^2 \phi}{\gamma^2 (1 - \beta \cos(\theta))^2} \right)$$

04/01/2019 PHY 712 Spring 2019 -- Lecture 27 18

Spectral composition of electromagnetic radiation
 Previously we determined the power distribution from a charged particle:

$$\frac{dP(t)}{d\Omega} = \mathbf{S} \cdot \hat{\mathbf{R}} R^2 = \frac{q^2}{4\pi c} \frac{\left| \hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta} \right) \times \dot{\boldsymbol{\beta}} \right] \right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^6} \Bigg|_{t_r = t - R/c}$$

$$\equiv |\mathbf{a}(t)|^2$$

where $\mathbf{a}(t) \equiv \sqrt{\frac{q^2}{4\pi c}} \frac{\left| \hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta} \right) \times \dot{\boldsymbol{\beta}} \right] \right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^3} \Bigg|_{t_r = t - R/c}$

Time integrated power per solid angle:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt |\mathbf{a}(t)|^2 = \int_{-\infty}^{\infty} d\omega |\tilde{\mathbf{a}}(\omega)|^2$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 19

Spectral composition of electromagnetic radiation – continued

Time integrated power per solid angle :

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt |\mathbf{a}(t)|^2 = \int_{-\infty}^{\infty} d\omega |\tilde{\mathbf{a}}(\omega)|^2$$

Fourier amplitude :

$$\tilde{\mathbf{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \mathbf{a}(t) e^{i\omega t} \quad \mathbf{a}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \tilde{\mathbf{a}}(\omega) e^{-i\omega t}$$

Parseval's theorem
Marc-Antoine Parseval des Chênes 1755-1836
<http://www-history.mcs.st-andrews.ac.uk/Biographies/Parseval.html>

04/01/2019 PHY 712 Spring 2019 – Lecture 27 20

Spectral composition of electromagnetic radiation – continued

Consequences of Parseval's analysis :

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt |\mathbf{a}(t)|^2 = \int_{-\infty}^{\infty} d\omega |\tilde{\mathbf{a}}(\omega)|^2$$

Note that : $\tilde{\mathbf{a}}(\omega) = \tilde{\mathbf{a}}^*(-\omega)$

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} d\omega |\tilde{\mathbf{a}}(\omega)|^2 = \int_0^{\infty} d\omega \left(|\tilde{\mathbf{a}}(\omega)|^2 + |\tilde{\mathbf{a}}(-\omega)|^2 \right) \equiv \int_0^{\infty} d\omega \frac{\partial^2 I}{\partial \Omega \partial \omega}$$

$$\frac{\partial^2 I}{\partial \Omega \partial \omega} \equiv 2 |\tilde{\mathbf{a}}(\omega)|^2$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 21

Spectral composition of electromagnetic radiation – continued

For our case:
$$\mathbf{a}(t) \equiv \sqrt{\frac{q^2}{4\pi c}} \left. \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c}$$

Fourier amplitude:

$$\begin{aligned} \tilde{\mathbf{a}}(\omega) &\equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt e^{i\omega t} \mathbf{a}(t) \\ &= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt e^{i\omega t} \left. \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c} \end{aligned}$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 22

Spectral composition of electromagnetic radiation – continued

Fourier amplitude :

$$\begin{aligned} \tilde{\mathbf{a}}(\omega) &\equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \mathbf{a}(t) e^{i\omega t} \\ &= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt \left. \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c} e^{i\omega t} \\ &= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \frac{dt}{dt_r} \left. \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)} \\ &= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \left. \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^2} \right|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)} \end{aligned}$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 23

Spectral composition of electromagnetic radiation – continued

Exact expression :

$$\tilde{\mathbf{a}}(\omega) = \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \left. \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^2} \right|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)}$$

Recall: $\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$ $\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$

For $r \gg R_q(t_r)$ $R(t_r) \approx r - \hat{\mathbf{r}} \cdot \mathbf{R}_q(t_r)$ where $\hat{\mathbf{r}} \equiv \frac{\mathbf{r}}{r}$

At the same level of approximation : $\hat{\mathbf{R}} \approx \hat{\mathbf{r}}$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 24

Spectral composition of electromagnetic radiation – continued

Exact expression:

$$\tilde{\mathbf{a}}(\omega) = \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^2} \Bigg|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)}$$

Approximate expression:

$$\tilde{\mathbf{a}}(\omega) = \sqrt{\frac{q^2}{8\pi^2 c}} e^{i\omega(r/c)} \int_{-\infty}^{\infty} dt_r \frac{\hat{\mathbf{r}} \times [(\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^2} \Bigg|_{t_r = t - R/c} e^{i\omega(t_r - \hat{\mathbf{r}} \cdot \mathbf{R}_q(t_r)/c)}$$

Resulting spectral intensity expression:

$$\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{q^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} dt_r \frac{\hat{\mathbf{r}} \times [(\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^2} \Bigg|_{t_r = t - R/c} e^{i\omega(t_r - \hat{\mathbf{r}} \cdot \mathbf{R}_q(t_r)/c)} \right|^2$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 25

Example – radiation from a collinear acceleration burst

$$\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{q^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} dt_r \frac{\hat{\mathbf{r}} \times [(\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^2} \Bigg|_{t_r = t - R/c} e^{i\omega(t_r - \hat{\mathbf{r}} \cdot \mathbf{R}_q(t_r)/c)} \right|^2$$

Suppose that $\dot{\boldsymbol{\beta}} = \begin{cases} \frac{\hat{\boldsymbol{\beta}} \Delta v}{c\tau} & 0 < t_r < \tau \\ 0 & \text{otherwise} \end{cases}$

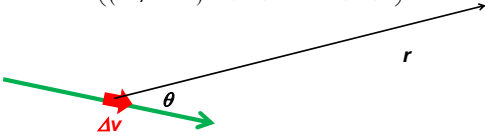
$$\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{q^2}{4\pi^2 c^3} \left| \frac{\hat{\mathbf{r}} \times [\hat{\mathbf{r}} \times \hat{\boldsymbol{\beta}}] \Delta v}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^2 \tau} \int_0^\tau dt_r e^{i\omega(t_r - \hat{\mathbf{r}} \cdot \boldsymbol{\beta} t_r)} \right|^2 \quad \text{Let } \boldsymbol{\beta} \cdot \hat{\mathbf{r}} = \beta \cos \theta$$

$$\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{q^2}{4\pi^2 c^3} \left(\frac{\Delta v \sin \theta}{(1 - \beta \cos \theta)^2} \frac{\sin(\omega\tau(1 - \beta \cos \theta)/2)}{(\omega\tau(1 - \beta \cos \theta)/2)} \right)^2$$

04/01/2019 PHY 712 Spring 2019 – Lecture 27 26

Example:

Suppose that $\dot{\boldsymbol{\beta}} = \begin{cases} \frac{\hat{\boldsymbol{\beta}} \Delta v}{c\tau} & 0 < t_r < \tau \\ 0 & \text{otherwise} \end{cases}$

$$\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{q^2}{4\pi^2 c^3} \left(\frac{\Delta v \sin \theta}{(1 - \beta \cos \theta)^2} \frac{\sin(\omega\tau(1 - \beta \cos \theta)/2)}{(\omega\tau(1 - \beta \cos \theta)/2)} \right)^2$$


Example: “Bremsstrahlung” radiation

04/01/2019 PHY 712 Spring 2019 – Lecture 27 27

Spectral composition of electromagnetic radiation -- continued

Alternative expression --

It can be shown that:

$$\frac{\hat{\mathbf{r}} \times [(\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^2} = \frac{d}{dt_r} \left(\frac{\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\beta})}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})} \right)$$

Integration by parts and assumptions about the integration limit behaviors shows that the spectral intensity depends on the following integral:

$$\frac{\partial^2 I}{\partial \omega \partial \Omega} = \frac{q^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} dt_r [\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\beta}(t_r))] e^{i\omega(t_r - \hat{\mathbf{r}} \cdot \mathbf{R}_r(t_r)/c)} \right|^2$$

04/01/2019

PHY 712 Spring 2019 -- Lecture 27

28
