PHY 712 Electrodynamics
9-9:50 AM Olin 105

Plan for Lecture 20:

Review Chapter 1-8 in Jackson

1. **Math identities and tricks**
2. **Maxwell's equations**
3. **Examples and homework problems**

14	Fri: 02/15/2019	Chap. 6	Maxwell's Equations	#11	02/18/2019
15	Mon: 02/18/2019	Chap. 6	Electromagnetic energy and forces	#12	02/20/2019
16	Wed: 02/20/2019	Chap. 7	Electromagnetic plane waves	#13	02/22/2019
17	Fri: 02/22/2019	Chap. 7	Electromagnetic plane waves	#14	02/25/2019
18	Mon: 02/25/2019	Chap. 7	Refractive index		
19	Wed: 02/27/2019	Chap. 8	EM waves in wave guides		
20	Fri: 03/01/2019	Chap. 1-8	Review		
	Mon: 03/04/2019	No class	*APS March Meeting*	Take Home Exam	
	Wed: 03/06/2019	No class	*APS March Meeting*	Take Home Exam	
	Fri: 03/08/2019	No class	*APS March Meeting*	Take Home Exam	
	Mon: 03/11/2019	No class	*Spring Break*		
	Wed: 03/13/2019	No class	*Spring Break*		
	Fri: 03/15/2019	No class	*Spring Break*		
21	Mon: 03/18/2019				
22	Wed: 03/20/2019				

Events next week --

Career advising event –

- Wed. Mar. 6, 2019 — 12:00PM, Dr. Charles W. Miller, Consultant in Nuclear and Radiological Environmental Health, Olin Lounge

Colloquium: "Can You Survive the Detonation of an Improvised Nuclear Device in a Major American City?" – Wednesday, March 6, 2019, at 4:00 PM

Charles W. Miller, PhD,
Consultant in Nuclear and Radiological Environmental Health
George P. Williams, Jr. Lecture Hall, (Olin 101)
Wednesday, March 6, 2019, at 4:00 PM

There will be a reception with refreshments at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

Review of mathematical relationships

Some useful identities for vectors and vector operators

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$

$$(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d}) = (\mathbf{a}\cdot\mathbf{c})(\mathbf{b}\cdot\mathbf{d}) - (\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c})$$

$$\nabla\times\nabla\psi = 0$$

$$\nabla\cdot(\nabla\times\mathbf{a}) = 0$$

$$\nabla\times(\nabla\times\mathbf{a}) = \nabla(\nabla\cdot\mathbf{a}) - \nabla^2\mathbf{a}$$

$$\nabla\cdot(\psi\mathbf{a}) = \mathbf{a}\cdot\nabla\psi + \psi\nabla\cdot\mathbf{a}$$

$$\nabla\times(\psi\mathbf{a}) = \nabla\psi\times\mathbf{a} + \psi\nabla\times\mathbf{a}$$

$$\nabla(\mathbf{a}\cdot\mathbf{b}) = (\mathbf{a}\cdot\nabla)\mathbf{b} + (\mathbf{b}\cdot\nabla)\mathbf{a} + \mathbf{a}\times(\nabla\times\mathbf{b}) + \mathbf{b}\times(\nabla\times\mathbf{a})$$

$$\nabla\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\nabla\times\mathbf{a}) - \mathbf{a}\cdot(\nabla\times\mathbf{b})$$

$$\nabla\times(\mathbf{a}\times\mathbf{b}) = \mathbf{a}(\nabla\cdot\mathbf{b}) - \mathbf{b}(\nabla\cdot\mathbf{a}) + (\mathbf{b}\cdot\nabla)\mathbf{a} - (\mathbf{a}\cdot\nabla)\mathbf{b}$$

Vector relations for spherical polar coordinates

$$\nabla\psi = \hat{\mathbf{r}}\frac{\partial\psi}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial\psi}{\partial\theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \hat{\mathbf{r}}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta A_\phi\right) - \frac{\partial A_\theta}{\partial\phi}\right] + \hat{\boldsymbol{\theta}}\left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}\left(rA_\phi\right)\right] + \hat{\boldsymbol{\phi}}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial\theta}\right]$$

$$\hat{\mathbf{x}} = \hat{\mathbf{r}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$$

$$\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$$

$$\hat{\mathbf{z}} = \hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$$

$$\frac{\partial}{\partial x} = \sin\theta\cos\phi\frac{\partial}{\partial r} + \cos\theta\cos\phi\frac{1}{r}\frac{\partial}{\partial\theta} - \frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi}$$

$$\frac{\partial}{\partial y} = \sin\theta\sin\phi\frac{\partial}{\partial r} + \cos\theta\sin\phi\frac{1}{r}\frac{\partial}{\partial\theta} + \frac{\cos\phi}{r\sin\theta}\frac{\partial}{\partial\phi}$$

$$\frac{\partial}{\partial z} = \cos\theta\frac{\partial}{\partial r} - \sin\theta\frac{\partial}{\partial\theta}$$

Some properties of a delta function

In one-dimension:

Note that for any function $F(x)$:

$$\int_{-\infty}^{\infty} F(x)\delta(x-x_0)dx = F(x_0)$$

Now consider a function $p(x)$, for which $p(x_i)=0$ for $i=1,2,\cdots$

$$\int_{-\infty}^{\infty} F(x)\delta(p(x))dx = \int_{-\infty}^{\infty} F(x)\left(\sum_i \delta\left((x-x_i)\left.\frac{dp}{dx}\right|_{x_i}\right)\right)dx$$

$$= \sum_i \frac{F(x_i)}{\left|\frac{dp}{dx}\right|_{x_i}}$$

In three-dimensions:

$$\delta^3(\mathbf{r}-\mathbf{r}_0) \equiv \delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \qquad \text{Cartesian}$$

$$= \frac{1}{r^2}\delta(r-r_0)\delta(\phi-\phi_0)\delta(\cos\theta-\cos\theta_0) \qquad \text{Spherical}$$

$$= \frac{1}{r}\delta(r-r_0)\delta(\phi-\phi_0)\delta(z-z_0) \qquad \text{Cylindrical}$$

Orthogonal functions useful for angular representations

Legendre polynomials for $-1 \le x \le 1$:

$P_0(x) = 1$ $\quad P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$ $\quad P_3(x) = \frac{1}{2}(5x^3 - 3x)$

Spherical harmonic functions

$l = 0: \quad Y_{00}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{4\pi}}$

$l = 1: \quad Y_{1(\pm1)}(\hat{\mathbf{r}}) = \mp\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{\pm i\varphi} \quad Y_{10}(\hat{\mathbf{r}}) = \sqrt{\frac{3}{4\pi}} \cos\theta$

$l = 2: \quad Y_{2(\pm2)}(\hat{\mathbf{r}}) = \sqrt{\frac{15}{32\pi}} \sin^2\theta \, e^{\pm 2i\varphi} \quad Y_{2(\pm1)}(\hat{\mathbf{r}}) = \mp\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta \, e^{\pm i\varphi}$

$Y_{20}(\hat{\mathbf{r}}) = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$

Note that $Y_{l0}(\hat{\mathbf{r}}) = \sqrt{\frac{2l+1}{4\pi}} P_l(\cos\theta)$

Useful identities related to Coulomb kernel:

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{\sqrt{r^2 + r'^2 - 2\mathbf{r} \cdot \mathbf{r}'}} = \sum_{l=0}^{\infty} \frac{r_<^l}{r_>^{l+1}} P_l(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}')$$

$$P_l(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}') = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\hat{\mathbf{r}}) Y_{lm}^*(\hat{\mathbf{r}}')$$

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \sum_{lm} \frac{4\pi}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{lm}(\theta, \varphi) Y_{lm}^*(\theta', \varphi')$$

Also note that: $\nabla^2 \frac{1}{|\mathbf{r} - \mathbf{r}'|} = -4\pi\delta^3(\mathbf{r} - \mathbf{r}')$

Green's theorem for electrostatics

Poisson equation: $\nabla^2 \Phi(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\varepsilon_0}$

Green's relation: $\nabla'^2 G(\mathbf{r}, \mathbf{r}') = -4\pi\delta^3(\mathbf{r} - \mathbf{r}')$.

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_V d^3r' \rho(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') +$$

volume integral over source

$$\frac{1}{4\pi} \int_S d^2r' \left[G(\mathbf{r}, \mathbf{r}') \nabla' \Phi(\mathbf{r}') - \Phi(\mathbf{r}') \nabla' G(\mathbf{r}, \mathbf{r}') \right] \cdot \hat{\mathbf{r}}'.$$

surface integrals incorporating boundary values

Review

Maxwell's equations

Coulomb's law: $\nabla \cdot \mathbf{D} = \rho_{free}$

Ampere-Maxwell's law: $\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_{free}$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

For linear isotropic media and no sources: $\mathbf{D} = \varepsilon \mathbf{E}$; $\mathbf{B} = \mu \mathbf{H}$

Review -- continued

Maxwell's equations

Microscopic or vacuum form $(\mathbf{P} = 0;\ \mathbf{M} = 0)$:

Coulomb's law: $\nabla \cdot \mathbf{E} = \rho / \varepsilon_0$

Ampere-Maxwell's law: $\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

$$\Rightarrow c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

Review -- continued

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad \Rightarrow \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \Phi$$

$$\text{or} \quad \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

Review -- continued

Analysis of the scalar and vector potential equations :

$$-\nabla^2\Phi - \frac{\partial(\nabla\cdot\mathbf{A})}{\partial t} = \rho/\varepsilon_0$$

$$\nabla\times(\nabla\times\mathbf{A}) + \frac{1}{c^2}\left(\frac{\partial(\nabla\Phi)}{\partial t} + \frac{\partial^2\mathbf{A}}{\partial t^2}\right) = \mu_0\mathbf{J}$$

Lorentz gauge form -- require $\nabla\cdot\mathbf{A}_L + \frac{1}{c^2}\frac{\partial\Phi_L}{\partial t} = 0$

$$-\nabla^2\Phi_L + \frac{1}{c^2}\frac{\partial^2\Phi_L}{\partial t^2} = \rho/\varepsilon_0$$

$$-\nabla^2\mathbf{A}_L + \frac{1}{c^2}\frac{\partial^2\mathbf{A}_L}{\partial t^2} = \mu_0\mathbf{J}$$

Review – continued – focusing on statics --

When to solve equations using integral form versus differential form?

Examples from electrostatic and magnetostatic cases:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r'\,\frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Useful for spatially confined sources.

Review -- continued

General form of electrostatic potential with boundary value $r\to\infty$, for isolated charge density $\rho(\mathbf{r})$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{1}{4\pi\varepsilon_0}\int d^3r'\,\rho(\mathbf{r}')\left(\sum_{lm}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}(\theta,\varphi)Y_{lm}^*(\theta',\varphi')\right)$$

Suppose that $\rho(\mathbf{r}) = \sum_{lm}\rho_{lm}(r)Y_{lm}(\theta,\varphi)$

$$\Rightarrow \Phi(\mathbf{r}) = \frac{1}{\varepsilon_0}\sum_{lm}\frac{1}{2l+1}Y_{lm}(\theta,\varphi)\left(\frac{1}{r^{l+1}}\int_0^r r'^{2+l}\,dr'\,\rho_{lm}(r') + r^l\int_r^\infty r'^{1-l}\,dr'\rho_{lm}(r')\right)$$

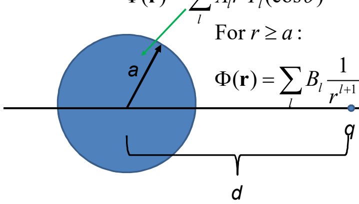
Example of use of spherical harmonic expansion: Problem #4.9 in Jackson. A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a ($a<d$) and dielectric constant $\varepsilon/\varepsilon_0$. Find the electrostatic potential.

For $r \le a$:

$$\Phi(\mathbf{r}) = \sum_l A_l r^l P_l(\cos\theta)$$

For $r \ge a$:

$$\Phi(\mathbf{r}) = \sum_l B_l \frac{1}{r^{l+1}} P_l(\cos\theta) + \frac{q}{4\pi\epsilon_0} \frac{1}{|\mathbf{r} - d\hat{\mathbf{z}}|}$$



Review of HW -- continued

For $r \le a$:

$$\Phi(\mathbf{r}) = \sum_l A_l r^l P_l(\cos\theta)$$

For $r \ge a$:

$$\Phi(\mathbf{r}) = \sum_l B_l \frac{1}{r^{l+1}} P_l(\cos\theta) + \frac{q}{4\pi\epsilon_0} \frac{1}{|\mathbf{r} - d\hat{\mathbf{z}}|}$$

In order to match BC's at $r = a$:

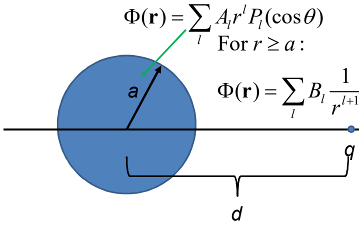
$$\frac{1}{|\mathbf{r} - d\hat{\mathbf{z}}|} = \sum_{l=0}^{\infty} \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta) = \sum_{l=0}^{\infty} \frac{a^l}{d^{l+1}} P_l(\cos\theta)$$

For $r \le a$:

$$\Phi(\mathbf{r}) = \sum_l A_l r^l P_l(\cos\theta)$$

For $r \ge a$:

$$\Phi(\mathbf{r}) = \sum_l B_l \frac{1}{r^{l+1}} P_l(\cos\theta) + \frac{q}{4\pi\epsilon_0} \frac{1}{|\mathbf{r} - d\hat{\mathbf{z}}|}$$



Boundary conditions:

$$\mathbf{D} \cdot \hat{\mathbf{r}}\Big|_{r=a} = \text{continuous} \qquad \mathbf{E} \cdot \hat{\boldsymbol{\theta}}\Big|_{r=a} = \text{continuous}$$

$$\varepsilon \frac{\partial \Phi_{in}(r)}{\partial r}\bigg|_{r=a} = \varepsilon_0 \frac{\partial \Phi_{out}(r)}{\partial r}\bigg|_{r=a} \qquad \frac{\partial \Phi_{in}(r)}{\partial \theta}\bigg|_{r=a} = \frac{\partial \Phi_{out}(r)}{\partial \theta}\bigg|_{r=a}$$

Boundary conditions:

$\mathbf{D}\cdot\hat{\mathbf{r}}\big|_{r=a} = \text{continuous}$ $\qquad$ $\mathbf{E}\cdot\hat{\boldsymbol{\theta}}\big|_{r=a} = \text{continuous}$

$$\varepsilon \frac{\partial \Phi_{in}(r)}{\partial r}\bigg|_{r=a} = \varepsilon_0 \frac{\partial \Phi_{out}(r)}{\partial r}\bigg|_{r=a} \qquad \frac{\partial \Phi_{in}(r)}{\partial \theta}\bigg|_{r=a} = \frac{\partial \Phi_{out}(r)}{\partial \theta}\bigg|_{r=a}$$

Equality for each l:

$$\varepsilon l a^{l-1} A_l = -\frac{\varepsilon_0 (l+1)}{a^{l+2}} B_l + \frac{q}{4\pi}\frac{l a^{l-1}}{d^{l+1}}$$

$$a^l A_l = +\frac{1}{a^{l+1}} B_l + \frac{q}{4\pi}\frac{a^l}{d^{l+1}}$$

2 equations and 2 unknowns for each *l*

Review -- continued

Hyperfine interaction energy:

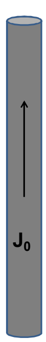
$$E_{int} \equiv H_{HF} = -\mu_e \cdot \mathbf{B}_{\mu_N} - \mu_N \cdot \mathbf{B}_{\mathbf{J}_e}(0)$$

Putting all of the terms together:

$$H_{\text{HF}} = -\frac{\mu_0}{4\pi}\left(\left\langle \frac{3(\boldsymbol{\mu}_\text{N}\cdot\hat{\mathbf{r}})(\boldsymbol{\mu}_\text{e}\cdot\hat{\mathbf{r}}) - \boldsymbol{\mu}_\text{N}\cdot\boldsymbol{\mu}_\text{e}}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_\text{N}\cdot\boldsymbol{\mu}_\text{e}\delta^3(\mathbf{r}) \right\rangle + \frac{e}{m_e}\left\langle \frac{\mathbf{L}\cdot\boldsymbol{\mu}_\text{N}}{r^3} \right\rangle \right).$$

In this expression the brackets $\langle\ \rangle$ indicate evaluating the expectation value relative to the electronic state.

Comment on HW #8

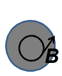


1. Consider an infinitely long wire with radius a, oriented along the **z** axis. There is a steady uniform current inside the wire. Specifically the current is along the z-axis with the magnitude of J_0 for ρ ≤ a and zero for ρ > a, where ρ denotes the radial parameter of the natural cylindrical coordinates of the system.
   a. Find the vector potential (**A**) for all ρ.
   b. Find the magnetic flux field (**B**) for all ρ.

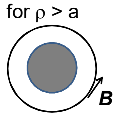
Solution to problem using PHY 114 ideas

In this case, it is convenient to solve part b first.

Top view for ρ < a

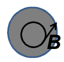


Top view for ρ > a



Comment on HW #11 -- continued

Top view for ρ < a



$$\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 \int \mathbf{J}\cdot d\mathbf{A}$$

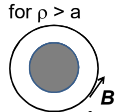
$$2\pi\rho B = \mu_0 J_0 \pi \rho^2$$

$$B = \frac{\mu_0 J_0 \rho}{2}$$

$$\mathbf{B} = \frac{\mu_0 J_0 \rho}{2}\hat{\boldsymbol{\varphi}} = \nabla\times\mathbf{A}$$

$$\mathbf{A} = -\frac{\mu_0 J_0\left(\rho^2 - a^2\right)}{4}\hat{\mathbf{z}}$$

Top view for ρ > a



$$\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 \int \mathbf{J}\cdot d\mathbf{A}$$

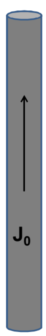
$$2\pi\rho B = \mu_0 J_0 \pi a^2$$

$$B = \frac{\mu_0 J_0 a^2}{2\rho}$$

$$\mathbf{B} = \frac{\mu_0 J_0 a^2}{2\rho}\hat{\boldsymbol{\varphi}} = \nabla\times\mathbf{A}$$

$$\mathbf{A} = -\frac{\mu_0 J_0 a^2 \ln\left(\rho/a\right)}{2}\hat{\mathbf{z}}$$

Comment on HW #8 -- continued



Alternative treatment using differential equations:

$$-\nabla^2\mathbf{A} = \begin{cases} \mu_0 J_0 \hat{\mathbf{z}} & \text{for } \rho \le a \\ 0 & \text{for } \rho > a \end{cases}$$

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial A_z(\rho)}{\partial\rho} = \begin{cases} \mu_0 J_0 & \text{for } \rho \le a \\ 0 & \text{for } \rho > a \end{cases}$$

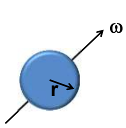
$$A_z(\rho) = \begin{cases} -\frac{\mu_0 J_0 \rho^2}{4} + C_1 & \text{for } \rho \le a \\ C_2 + C_3\ln(\rho) & \text{for } \rho > a \end{cases}$$

Choosing constants from continuity requirements:

$$A_z(\rho) = \begin{cases} -\frac{\mu_0 J_0 \rho^2}{4} + \frac{\mu_0 J_0 a^2}{4} & \text{for } \rho \le a \\ -\frac{\mu_0 J_0 a^2}{2}\ln(\rho/a) & \text{for } \rho > a \end{cases}$$

$$\mathbf{B} = -\frac{\partial A_z(\rho)}{\partial\rho}\hat{\boldsymbol{\varphi}}$$

Comment on HW #9



A sphere of radius a carries a uniform surface charge distribution σ. The sphere is rotated about a diameter with constant angular velocity ω. Find the vector potential **A** and magnetic field **B** both inside and outside the sphere.

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}.$$

$$\mathbf{J}(\mathbf{r}') = \begin{cases} \sigma\ \delta(r'-a)\ \boldsymbol{\omega}\times\mathbf{r}' & \text{for } r' \le a \\ 0 & \text{otherwise} \end{cases}$$

Note that: $\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{lm}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}(\hat{\mathbf{r}})Y_{lm}^*(\hat{\mathbf{r}}')$

and: $\int d\Omega' \sum_m Y_{lm}(\hat{\mathbf{r}})Y_{lm}^*(\hat{\mathbf{r}}')\,\mathbf{r}' = \frac{r'}{r}\mathbf{r}\,\delta_{l1}.$

Comment on HW #9 -- continued

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} = \frac{\mu_0\sigma}{4\pi}\frac{\boldsymbol{\omega}\times\mathbf{r}}{r}\frac{4\pi}{3}\int_0^a r'^3 dr'\,\delta(r'-a)\frac{r_<}{r_>^2}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0\sigma}{3}\boldsymbol{\omega}\times\mathbf{r}\begin{cases} a & \text{for } r \le a \\ \dfrac{a^4}{r^3} & \text{for } r > a \end{cases}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0\sigma}{3}\begin{cases} 2\boldsymbol{\omega}a & \text{for } r \le a \\ \dfrac{a^4}{r^3}\left(3\left(\hat{\mathbf{r}}\cdot\boldsymbol{\omega}\right)\hat{\mathbf{r}}-\boldsymbol{\omega}\right) & \text{for } r > a \end{cases}$$

Correction to Lecture 18

Reflectance for s-polarization

$$R_s = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Reflectance for p-polarization

$$R_p = \left|\frac{E_{0R}}{E_{0i}}\right|^2 = \left|\frac{\frac{\mu}{\mu'}n'\cos i - \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'\cos i + \frac{n}{n'}\sqrt{n'^2 - n^2\sin^2 i}}\right|^2$$

Example for $\mu = \mu'$; $n = 1$ and $n' = 1.5$