PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 12:

Continue reading Chapter 5

A. Examples of magnetostatic fields

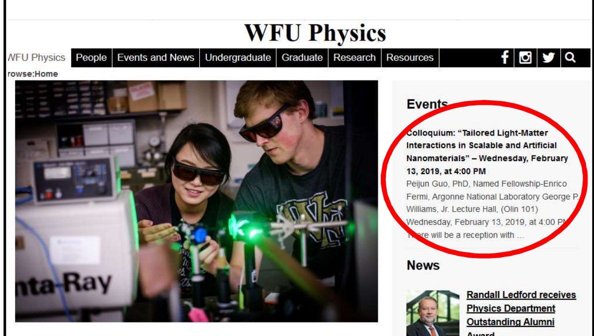
B. Magnetic dipoles

C. Hyperfine interaction

Course schedule for Spring 2019

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	HW	Due date
1	Mon: 01/14/2019	Chap. 1 & Appen.	Introduction, units and Poisson equation	#1	01/23/2019
2	Wed: 01/16/2019	Chap. 1	Electrostatic energy calculations	#2	01/23/2019
3	Fri: 01/18/2019	Chap. 1	Electrostatic potentials and fields	#3	01/23/2019
	Mon: 01/21/2019	No class	Martin Luther King Holiday		
4	Wed: 01/23/2019	Chap. 1 - 3	Poisson's equation in 2 and 3 dimensions		
5	Fri: 01/25/2019	Chap. 1 - 3	Brief introduction to numerical methods	#4	01/28/2019
6	Mon: 01/28/2019	Chap. 2 & 3	Image charge constructions	#5	01/30/2019
7	Wed: 01/30/2019	Chap. 2 & 3	Cylindrical and spherical geometries		
8	Fri: 02/01/2019	Chap. 3 & 4	Spherical geometry and multipole moments	#6	02/04/2019
9	Mon: 02/04/2019	Chap. 4	Dipoles and Dielectrics	#7	02/06/2019
10	Wed: 02/06/2019	Chap. 4	Polarization and Dielectrics		
11	Fri: 02/08/2019	Chap. 5	Magnetostatics	#8	02/11/2019
12	Mon: 02/11/2019	Chap. 5	Magnetic dipoles and hyperfine interaction	#9	02/13/2019
13	Wed: 02/13/2019				
14	Fri: 02/15/2019				
15	Mon: 02/18/2019				



Various forms of Ampere's law :

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

Vector potential : $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$

For Coulomb gauge : $\nabla \cdot \mathbf{A}(\mathbf{r}) = 0$

$$\Rightarrow \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r})$$

For confined current density :

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Other examples of current density sources:

Quantum mechanical expression for current density for a particle of mass M and charge e and of probability amplitude $\Psi(\mathbf{r})$:

$$\mathbf{J}(\mathbf{r}) = -\frac{e\hbar}{2Mi}\left(\Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r}) - \Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r})\right)$$

For an electron in a spherical potential (such as in an atom):

$$\Psi(\mathbf{r}) \equiv \Psi_{nlm_l}(\mathbf{r}) = R_{nl}(r)Y_{lm_l}(\hat{\mathbf{r}})$$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{2Mi}|R_{nl}(r)|^2 \frac{1}{r\sin\theta}\left(Y_{lm_l}^*(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}(\hat{\mathbf{r}})}{\partial\varphi} - Y_{lm_l}(\hat{\mathbf{r}})\frac{\partial Y_{lm_l}^*(\hat{\mathbf{r}})}{\partial\varphi}\right)\hat{\boldsymbol{\varphi}}$$

$$= \frac{e\hbar}{M}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2\hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}} = \frac{\hat{\mathbf{z}}\times\mathbf{r}}{r\sin\theta}$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{M}\frac{m_l}{r^2\sin^2\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2(\hat{\mathbf{z}}\times\mathbf{r})$$

Details of the electron orbital magnetic dipole moment

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{m_e}\frac{m_l}{r\sin\theta}\left|\Psi_{nlm_l}(\mathbf{r})\right|^2\hat{\boldsymbol{\varphi}}$$

Note that: $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}}$

Magnetic dipole moment:

$$\mathbf{m} = \frac{1}{2}\int d^3r'\mathbf{r}'\times\mathbf{J}(\mathbf{r}') = -\frac{e\hbar m_l}{2m_e}\int d^3r' \frac{\mathbf{r}'\times\hat{\boldsymbol{\varphi}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})^2\right|$$

$$= -\frac{e\hbar m_l}{2m_e}\int d^3r' \frac{-r'\hat{\boldsymbol{\theta}}'}{r'\sin\theta'}\left|\Psi_{nlm_l}(\mathbf{r})^2\right|$$

Note that: $\hat{\boldsymbol{\theta}} = \cos\theta\cos\varphi\,\hat{\mathbf{x}} + \cos\theta\sin\varphi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$

$$\mathbf{m} = -\frac{e\hbar m_l\hat{\mathbf{z}}}{2m_e}\int d^3r'\left|\Psi_{nlm_l}(\mathbf{r})^2\right|$$

$$= -\frac{e\hbar m_l}{2m_e}\hat{\mathbf{z}}$$

Summary of magnetic field generated by point magnetic dipole moment:

$$\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\hat{\mathbf{r}}(\boldsymbol{\mu}_e\cdot\hat{\mathbf{r}})-\boldsymbol{\mu}_e}{r^3}+\frac{8\pi}{3}\boldsymbol{\mu}_e\delta(\mathbf{r})\right)$$

Magnetic field near nucleus due to orbiting electron:

$$\mathbf{B}_O(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{e}{m_e}L_z\hat{\mathbf{z}}\left\langle\frac{1}{r^3}\right\rangle$$

"Hyperfine" interaction energy:

$$\mathcal{H}_{HF} = -\boldsymbol{\mu}_N\cdot\left(\mathbf{B}_{\boldsymbol{\mu}_e}(\mathbf{r})+\mathbf{B}_O(\mathbf{r})\right)$$

$$= \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{\mu}_N\cdot\hat{\mathbf{r}})(\boldsymbol{\mu}_e\cdot\hat{\mathbf{r}})-\boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e}{r^3}+\frac{8\pi}{3}\boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e\delta(\mathbf{r})+\frac{e}{m_e}\left\langle\frac{\mathbf{L}\cdot\boldsymbol{\mu}_N}{r^3}\right\rangle\right)$$

$$\mathcal{H}_{HF} = \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{\mu}_N\cdot\hat{\mathbf{r}})(\boldsymbol{\mu}_e\cdot\hat{\mathbf{r}})-\boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e}{r^3}+\frac{8\pi}{3}\boldsymbol{\mu}_N\cdot\boldsymbol{\mu}_e\delta(\mathbf{r})+\frac{e}{m_e}\left\langle\frac{\mathbf{L}\cdot\boldsymbol{\mu}_N}{r^3}\right\rangle\right)$$