PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 32:

Special Topics in Electrodynamics:

Cherenkov radiation

References: Jackson Chapter 13.4
Zangwill Chapter 23.7
Smith Chapter 6.4

24	Mon: 03/19/2018	Chap. 9 & 10	Interference and Scattering	#16	03/23/2018
25	Wed: 03/21/2018	Chap. 11	Special relativity	#17	03/26/2018
26	Fri: 03/23/2018	Chap. 11	Special relativity	#18	03/28/2018
27	Mon: 03/26/2018	Chap. 11	Special relativity		
28	Wed: 03/28/2018	Chap. 14	Radiation from accelerated particles		
	Fri: 03/30/2018	No class	*Good Friday*		
29	Mon: 04/02/2018	Chap. 14	Synchrotron radiation	#19	04/06/2018
30	Wed: 04/04/2018	Chap. 14	Synchrotron radiation	#20	04/09/2018
31	Fri: 04/06/2018	Chap. 15	Radiation from collisions of charged particles		
32	Mon: 04/09/2018	Chap. 13	Cherenkov radiation		
33	Wed: 04/11/2018		Review		
34	Fri: 04/13/2018		Review		
35	Mon: 04/16/2018				
36	Wed: 04/18/2018				
37	Fri: 04/20/2018				
38	Mon: 04/23/2018				
39	Wed: 04/25/2018				
	Fri: 04/27/2018		Presentations I		
	Mon: 04/30/2018		Presentations II		
	Wed: 05/02/2018		Presentations III		

Wake Forest College & Graduate School of Arts and Sciences

WFU Physics

WFU Physics | People | Events and News | Undergraduate | Graduate | Research | Resources

Browse:Home

Events

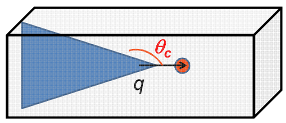
PhD Defense: "Non-Traditional Study of Renewable Generators in Thermoelectric and Piezoelectric Meta Structures." April 9, 2018, at 12 PM
David Montgomery, PhD Candidate Public Presentation in Olin 107 Monday, April 9, 2018, at 12:00 PM David L. Carroll, PhD, Advisor The defense will follow. ABSTRACT This work focuses on ....

Colloquium: "A Multi-Pronged Observational Approach to Measuring Exoplanet Magnetic Fields." April 11, 2018, 4pm
P. Wilson Cauley, PhD, School of Earth and Space Exploration, Arizona State University George P. Williams, Jr. Lecture Hall, (Olin 101) Wednesday, April 11, 2018, at 4:00 PM

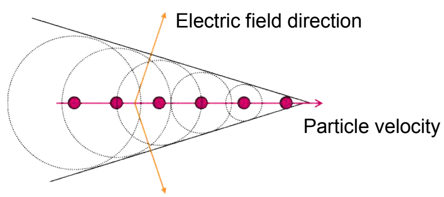
References for notes: Glenn S. Smith, *An Introduction to Electromagnetic Radiation* (Cambridge UP, 1997), Andrew Zangwill, Modern Electrodynamics (Cambridge UP, 2013)

Cherenkov radiation

Discovered ~1930; bluish light emitted by energetic charged particles traveling within dielectric materials



From: http://large.stanford.edu/courses/2014/ph241/alaeian2/



Maxwell's potential equations within a material having permittivity and permeability (Lorentz gauge; cgs Gaussian units)

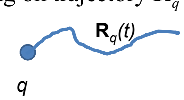
$$\nabla^2\Phi - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}\rho$$

$$\nabla^2\mathbf{A} - \mu\varepsilon\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\mathbf{J}$$

Source: charged particle moving on trajectory $\mathbf{R}_q(t)$:

$$\rho(\mathbf{r},t) = q\delta(\mathbf{r}-\mathbf{R}_q(t))$$

$$\mathbf{J}(\mathbf{r},t) = q\dot{\mathbf{R}}_q(t)\delta(\mathbf{r}-\mathbf{R}_q(t))$$



Liénard-Wiechert potential solutions:

$$\Phi(\mathbf{r},t)=\frac{q}{\varepsilon}\frac{1}{R(t_r)-\boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)}$$

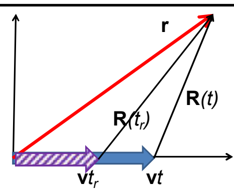
$$\mathbf{A}(\mathbf{r},t)=q\mu\frac{\boldsymbol{\beta}_n}{R(t_r)-\boldsymbol{\beta}_n\cdot\mathbf{R}(t_r)}$$

$$\mathbf{R}(t_r)\equiv\mathbf{r}-\mathbf{R}_q(t_r)$$

$$\boldsymbol{\beta}_n(t_r)\equiv\frac{\dot{\mathbf{R}}_q(t_r)}{c_n}\qquad c_n\equiv\sqrt{\mu\varepsilon}\,c\equiv\frac{c}{n}$$

$$t_r=t-\frac{R(t_r)}{c_n}$$

Consider a particle moving at constant velocity $\mathbf{v}$; $v>c_n$



Some algebra

$$\mathbf{R}(t)=\mathbf{r}-\mathbf{v}t$$

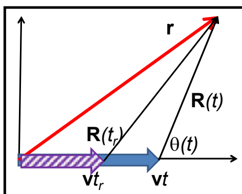
$$\mathbf{R}(t_r)=\mathbf{r}-\mathbf{v}t_r=\mathbf{R}(t)+\mathbf{v}(t-t_r)$$

$$(t-t_r)c_n=R(t_r)=|\mathbf{R}(t)+\mathbf{v}(t-t_r)|$$

Quadratic equation for $(t-t_r)c_n$:

$$((t-t_r)c_n)^2=R^2(t)+2\mathbf{R}(t)\cdot\boldsymbol{\beta}_n(t-t_r)c_n+\beta_n^{\,2}((t-t_r)c_n)^2$$

$$(t-t_r)c_n=\frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\pm\sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2-(\beta_n^{\,2}-1)R^2(t)}}{\beta_n^{\,2}-1}$$



$$\mathbf{R}(t_r)=\mathbf{r}-\mathbf{v}t_r=\mathbf{R}(t)+\mathbf{v}(t-t_r)$$

$$(t-t_r)c_n=R(t_r)$$

$$R(t_r)-\mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n=$$

$$(t-t_r)c_n\left(1-\beta_n^2\right)-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$=R(t_r)\left(1-\beta_n^2\right)-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n$$

$$R(t_r)=\frac{-\mathbf{R}(t)\cdot\boldsymbol{\beta}_n\pm\sqrt{(\mathbf{R}(t)\cdot\boldsymbol{\beta}_n)^2-(\beta_n^{\,2}-1)R^2(t)}}{\beta_n^{\,2}-1}$$

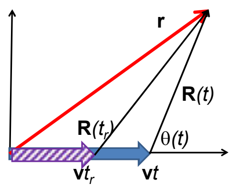
$$R(t_r)=\frac{R(t)}{\beta_n^{\,2}-1}\left(-\beta_n\cos\theta\pm\sqrt{1-\beta_n^{\,2}\sin^2\theta}\right)=(t-t_r)c_n$$

$$R(t_r)-\mathbf{R}(t_r)\cdot\boldsymbol{\beta}_n=\mp R(t)\sqrt{1-\beta_n^{\,2}\sin^2\theta}$$

Liénard-Wiechert potentials for two different retarded times:

$$\Phi(\mathbf{r},t) = \pm\frac{q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}$$

$$\mathbf{A}(\mathbf{r},t) = \pm q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}$$

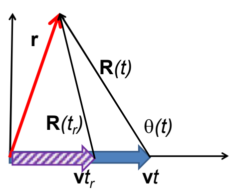


For $\beta_n > 1$, range of θ is limited:

$$R(t_r) = \frac{R(t)}{\beta_n^2 - 1}\left(-\beta_n\cos\theta \pm \sqrt{1-\beta_n^2\sin^2\theta}\right) \geq 0$$

$$\Rightarrow \theta \leq \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Physical fields for $\beta_n > 1$



$$\theta \leq \sin^{-1}\left(\frac{1}{\beta_n}\right)$$

Define $\cos\theta_C \equiv -\sqrt{1-\frac{1}{\beta_n^2}}$

$$\Rightarrow \cos\theta \leq \cos\theta_C$$

Adding two solutions; in terms of Heaviside $\Theta(x)$:

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

Physical fields for $\beta > 1$

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{1}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = 2q\mu\frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n^2\sin^2\theta}}\Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi - \frac{1}{c_n}\frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$

$$\left(\frac{\beta_n^2-1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\;\delta(\cos\theta_C - \cos\theta(t))\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n\sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Intermediate steps:

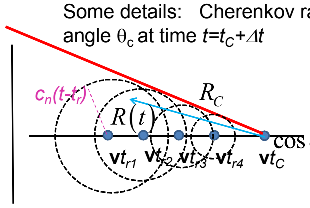
$$\frac{d\theta}{dt} = \frac{v\sin\theta}{R} \qquad \frac{dR}{dt} = -v\cos\theta$$

$$\text{Using instantaneous polar coordinates:} \quad \nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial \theta}$$

$$\nabla\Theta(\cos\theta_C - \cos\theta(t)) = \delta(\cos\theta_C - \cos\theta(t))\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta(\cos\theta_C - \cos\theta(t))}{\partial t} = \delta(\cos\theta_C - \cos\theta(t))\frac{v\sin^2\theta(t)}{R(t)}$$

Some details: Cherenkov radiation observed near the angle θ_c at time $t=t_C+\Delta t$



$$\cos\theta_C - \cos\theta(t) \approx \frac{c_n\Delta t}{\beta_n R_C}$$

$$1-\beta_n\sin^2\theta(t) \approx \frac{2c_n\Delta t\sqrt{\beta_n^2-1}}{R_C}$$

When the dust clears ....

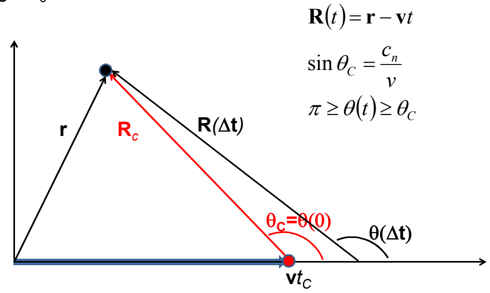
$$\frac{d^2I}{d\omega d\ell} \propto \omega$$

Some details: Cherenkov radiation observed near the angle θ_c

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

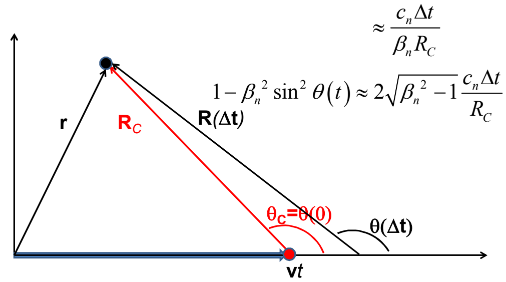
$$\sin\theta_C = \frac{c_n}{v}$$

$$\pi \geq \theta(t) \geq \theta_C$$



Cherenkov radiation observed near the angle θ_c -- continued

$$\cos\theta_C - \cos(\theta_C + \Delta\theta) \approx \sin\theta_C \ \Delta\theta$$
$$\approx \frac{c_n \Delta t}{\beta_n R_C}$$
$$1 - \beta_n^2 \sin^2\theta(t) \approx 2\sqrt{\beta_n^2 - 1}\,\frac{c_n \Delta t}{R_C}$$



Cherenkov radiation observed near the angle θ_c -- continued

$$\mathbf{E}(\mathbf{r},t) = \frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}}{(R(t))^2\sqrt{1-\beta_n^2\sin^2\theta}} \times$$
$$\left(\frac{\beta_n^2 - 1}{1-\beta_n^2\sin^2\theta}\Theta(\cos\theta_C - \cos\theta(t)) + \sqrt{\beta_n^2-1}\ \delta(\cos\theta_C - \cos\theta(t))\right)$$
$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta\left(\hat{\theta}\times\mathbf{E}(\mathbf{r},t)\right)$$

Estimates at $t = t_C + \Delta t$

$$\mathbf{E}(\mathbf{r},t) \approx -\frac{2q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3 R_C)^{1/2}}\left[(\Delta t)^{-1/2}\delta(\Delta t) - \frac{1}{2}(\Delta t)^{-3/2}\Theta(\Delta t)\right]$$
$$\mathbf{B}(\mathbf{r},t) = -\left(\hat{\boldsymbol{\theta}}(t_C)\times\mathbf{E}(\mathbf{r},t)\right)$$

Cherenkov radiation observed near the angle θ_c -- continued

Spectral analysis:

$$\tilde{\mathbf{E}}(\omega) = -\frac{2q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3R_C)^{1/2}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dt\left[t^{-1/2}\delta(t) - \frac{1}{2}t^{-3/2}\Theta(t)\right]e^{i\omega t}$$
$$= -i\omega\frac{2q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3R_C)^{1/2}}\frac{1}{\sqrt{2\pi}}\int_0^{\infty}dt\ t^{-1/2}e^{i\omega t}$$
$$= \frac{q}{\varepsilon}\hat{\mathbf{R}}_C\frac{(\beta_n^2-1)^{1/4}}{(2c_n^3R_C)^{1/2}}(1-i)\sqrt{\omega}$$

Spectral intensity: $$\frac{d^2I}{d\Omega d\omega} \propto \left|\tilde{\mathbf{E}}(\omega)\right|^2 = \frac{q^2}{\varepsilon^2}\frac{(\beta_n^2-1)^{1/2}}{c_n^3R_C}\omega$$

Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013.
http://www.britannica.com/EBchecked/media/174732