PHY 712 Electrodynamics
9-9:50 AM MWF Olin 105

Plan for Lecture 30:

Finish reading Chap. 14 –

Radiation from charged particles

1. **Review of synchrotron radiation**
2. **Free electron laser**
3. **Thompson and Compton scattering**

23	Fri: 03/16/2018	Chap. 9	Harmonic radiation	#15	03/21/2018
24	Mon: 03/19/2018	Chap. 9 & 10	Interference and Scattering	#16	03/23/2018
25	Wed: 03/21/2018	Chap. 11	Special relativity	#17	03/26/2018
26	Fri: 03/23/2018	Chap. 11	Special relativity	#18	03/28/2018
27	Mon: 03/26/2018	Chap. 11	Special relativity		
28	Wed: 03/28/2018	Chap. 14	Radiation from accelerated particles		
	Fri: 03/30/2018	No class	*Good Friday*		
29	Mon: 04/02/2018	Chap. 14	Synchrotron radiation	#19	04/06/2018
30	Wed: 04/04/2018	Chap. 14	Synchrotron radiation	#20	04/09/2018
31	Fri: 04/06/2018				
32	Mon: 04/09/2018				
33	Wed: 04/11/2018				
34	Fri: 04/13/2018				
35	Mon: 04/16/2018				
36	Wed: 04/18/2018				
37	Fri: 04/20/2018				
38	Mon: 04/23/2018				
39	Wed: 04/25/2018				
	Fri: 04/27/2018		Presentations I		
	Mon: 04/30/2018		Presentations II		
	Wed: 05/02/2018		Presentations III		

Presentations I -- Friday April 27, 2018

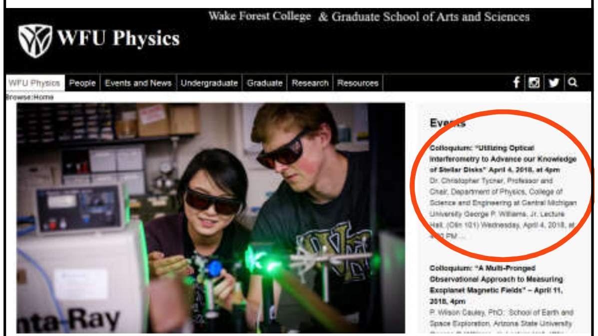
	Presenter name	Presentation title
9:00-9:23 AM	Matthew Waldrip	Magnetrons
9:25-9:47 AM	Yan Li	

Presentations II -- Monday April 30, 2018

	Presenter name	Presentation title
9:00-9:23 AM	Nouf Alharbi	
9:25-9:47 AM	Ellie Alipour	

Presentations III -- Wednesday May 02, 2018

	Presenter name	Presentation title
9:00-9:23 AM	Haardik Pandey	
9:25-9:47 AM	Kevin Roebuck	

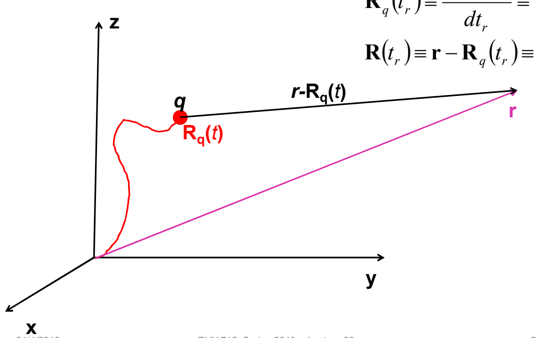


Radiation from a moving charged particle

Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$



Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\left(1 - \frac{v^2}{c^2}\right) + \left(\mathbf{R} \times \left\{\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \times \frac{\dot{\mathbf{v}}}{c^2}\right\}\right)\right]. \quad (19)$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}}\cdot\mathbf{R}}{c^2}\right) - \frac{\mathbf{R}\times\dot{\mathbf{v}}/c}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^2}\right]. \quad (20)$$

In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}. \quad (21)$$

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v} \qquad \mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R} \qquad \dot{\mathbf{v}} \equiv \frac{d^2\mathbf{R}_q(t_r)}{dt_r^{\,2}}$$

Electric and magnetic fields far from source:

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{\left(R-\frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left\{\mathbf{R}\times\left[\left(\mathbf{R}-\frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t)=\frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}$$

$$\text{Let}\quad \hat{\mathbf{R}}\equiv\frac{\mathbf{R}}{R}\qquad \boldsymbol{\beta}\equiv\frac{\mathbf{v}}{c}\qquad \dot{\boldsymbol{\beta}}\equiv\frac{\dot{\mathbf{v}}}{c}$$

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{cR\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\left\{\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t)=\hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

Poynting vector:

$$\mathbf{S}(\mathbf{r},t)=\frac{c}{4\pi}(\mathbf{E}\times\mathbf{B})$$

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{cR\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\left\{\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t)=\hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

$$\mathbf{S}(\mathbf{r},t)=\frac{c}{4\pi}\hat{\mathbf{R}}\left|\mathbf{E}(\mathbf{r},t)\right|^2=\frac{q^2}{4\pi cR^2}\hat{\mathbf{R}}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

$$\frac{dP}{d\Omega}=\mathbf{S}\cdot\hat{\mathbf{R}}R^2=\frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

Spectral composition of electromagnetic radiation

Time integrated power per solid angle :

$$\frac{dW}{d\Omega}=\int_{-\infty}^{\infty}dt\,\frac{dP(t)}{d\Omega}=\int_{-\infty}^{\infty}dt\left|\boldsymbol{a}(t)\right|^2=\int_{-\infty}^{\infty}d\omega\left|\tilde{\boldsymbol{a}}(\omega)\right|^2$$

Fourier amplitude :

$$\tilde{\boldsymbol{a}}(\omega)\equiv\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dt\,\boldsymbol{a}(t)e^{i\omega t}\qquad \boldsymbol{a}(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega\,\tilde{\boldsymbol{a}}(\omega)e^{-i\omega t}$$

Note that: $\tilde{\boldsymbol{a}}(\omega)=\tilde{\boldsymbol{a}}^*(-\omega)$

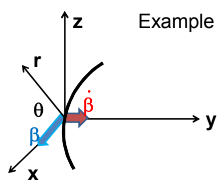
$$\Rightarrow\frac{dW}{d\Omega}=\int_0^{\infty}d\omega\left(\left|\tilde{\boldsymbol{a}}(\omega)\right|^2+\left|\tilde{\boldsymbol{a}}(-\omega)\right|^2\right)\equiv\int_0^{\infty}d\omega\frac{\partial^2 I}{\partial\Omega\partial\omega}$$

$$\frac{\partial^2 I}{\partial\Omega\partial\omega}\equiv 2\left|\tilde{\boldsymbol{a}}(\omega)\right|^2$$

Spectral composition of electromagnetic radiation -- continued

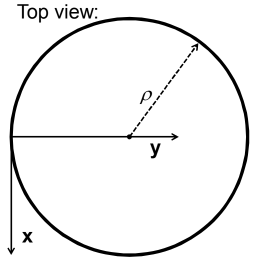
The spectral intensity therefore depends on the following integral:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right] e^{i\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$



Example for circular motion:

Top view:

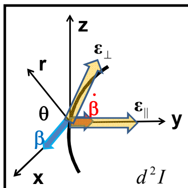


$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho) + \rho\hat{\mathbf{y}}(1-\cos(vt_r/\rho))$$

$$\boldsymbol{\beta}(t_r) = \beta(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho))$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}) = \beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt_r/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt_r/\rho)\right)$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\beta)e^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)}dt\right|^2$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\}$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt\,\sin(vt/\rho)e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt\,\sin\theta\cos(vt/\rho)e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

Synchrotron radiation geometry –
using modified Bessel functions

$$K_{1/3}(\xi)=\sqrt{3}\int_0^\infty dx\cos\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right] \qquad K_{2/3}(\xi)=\sqrt{3}\int_0^\infty dx\, x\sin\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right]$$

Exponential factor

$$\omega\left(t_r-\frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right)=\omega\left(t_r-\frac{\rho}{c}\cos\theta\sin(vt_r/\rho)\right)$$

$$\text{In the limit of}\quad t_r\approx 0,\quad \theta\approx 0,\quad v\approx c\left(1-\frac{1}{2\gamma^2}\right)$$

$$\omega\left(t_r-\frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right)\approx\frac{\omega t_r}{2\gamma^2}\left(1+\gamma^2\theta^2\right)+\frac{\omega c^2 t_r^{\,3}}{6\rho^2}=\frac{3}{2}\xi\left(x+\frac{1}{3}x^3\right)$$

$$\text{where}\quad \xi=\frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2}\quad\text{and}\quad x=\frac{c\gamma t_r}{\rho\left(1+\gamma^2\theta^2\right)^{1/2}}$$

Spectral form of synchrotron radiation in this case:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega}=\frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2\left(1+\gamma^2\theta^2\right)^2\left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\theta^2\right)^{3/2}\right)\right]^2+\frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\theta^2\right)^{3/2}\right)\right]^2\right\}$$

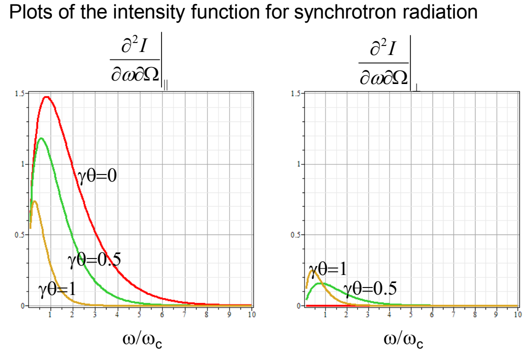
$$\omega_c\equiv\frac{3c\gamma^3}{2\rho}$$

At $\theta=0$:

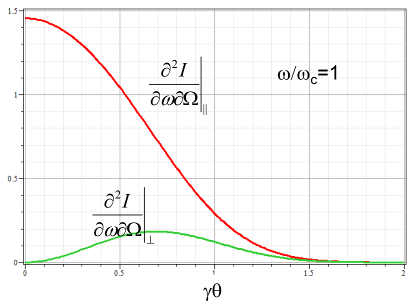
$$\text{note that for}\quad \omega<<\omega_c\quad\Rightarrow\frac{\partial^2 I}{\partial\omega\partial\Omega}\approx\frac{q^2}{\pi^2 c}\left(\Gamma\left(\tfrac{2}{3}\right)\right)^2\left(\frac{3\omega^2\rho^2}{4c^2}\right)^{1/3}$$

$$\text{and for}\quad \omega>>\omega_c\quad\Rightarrow\frac{\partial^2 I}{\partial\omega\partial\Omega}\approx\frac{3q^2}{4\pi c}\gamma^2\left(\frac{\omega}{\omega_c}\right)^2 e^{-\omega/\omega_c}$$

Plots of the intensity function for synchrotron radiation



Plots of the intensity function for synchrotron radiation



Synchrotron facilities worldwide
https://lightsources.org/

There are more than 50 light sources in the world (operational, or under construction). This page lists all the members of the lightsources.org collaboration.

Free electron laser
Reference:

Classical Theory of Free-Electron Lasers

A text for students and researchers

Eric B Szarmes

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1.1 The free-electron laser

A free-electron laser (FEL) is a laser source that produces spatially and temporally coherent optical radiation by stimulated emission, where in place of an atomic or molecular medium to provide amplification the gain medium is comprised of a beam of relativistic electrons traveling in a vacuum through a periodic magnetic field. The basic components common to all FELs are a relativistic electron beam, a periodic magnetic structure (an undulator or wiggler magnet of spatial period λ_w), and an optical resonator providing feedback and amplification. (X-ray FELs such as the Linac Coherent Light Source at Stanford omit the optical resonator by necessity and achieve the required gain on a single pass.) The features that make FELs particularly useful as research devices are the unique combination of continuous and broadband tunability, high peak and average power, and spatial and temporal coherence.

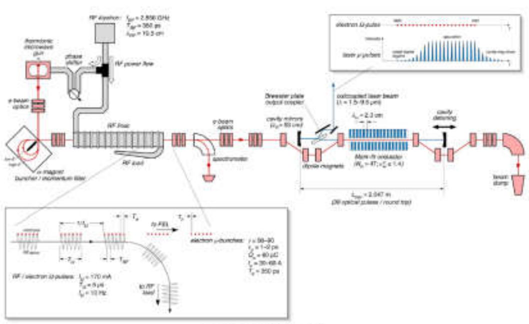
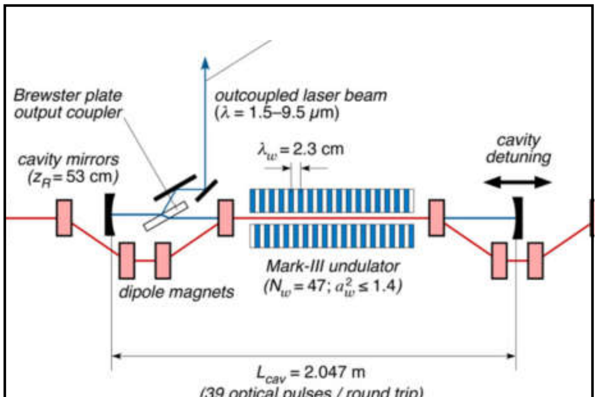


Figure 1.1. The MkIII RF-linac FEL.



Electron emission in periodic magnet

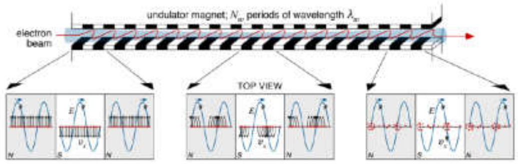


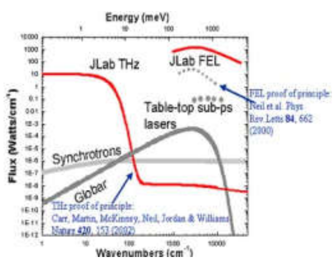
Figure 1.3. Conceptual illustration of the bunching mechanism in an FEL.

Because of Doppler shift and Thompson scattering, effective

wavelength is: $\lambda = \dfrac{\lambda_w}{2\gamma_z^{\,2}}$

frequency is: $\omega = \dfrac{4\pi}{\lambda_w}\gamma_z^{\,2}c$

Spectral output for FEL at Jefferson Lab (in Virginia)



Free-Electron Lasers: Status and Applications

SCIENCE VOL 292 8 JUNE 2001

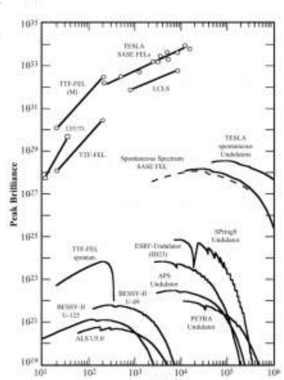
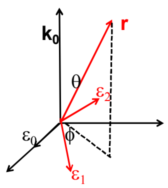


Fig. 3. Peak brilliance of x-ray FELs and undulators for spontaneous radiation at the TESLA Test Facility, in comparison with synchrotron radiation sources. Brilliance is expressed as photons s^{-1} mrad^{-2} mm^{-2} per 0.1% bandwidth. For comparison, the spontaneous spectrum of x-ray FEL undulators is also shown. The label TTF-FEL indicates design values for the FEL at the TESLA Test Facility, with (M) for the planned seeded version (28).

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



$$\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\dot{\mathbf{v}}\right)\right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q) is caused by an electric field : $\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$

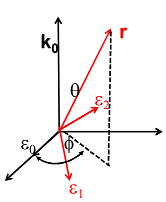
$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Time averaged power : $\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2\left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2\left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\varepsilon_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\varepsilon_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\varepsilon_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization ε^*:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2$$

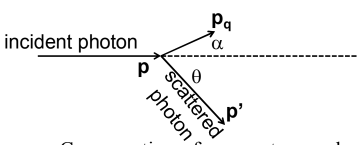
Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

$$\left\langle |\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \left\langle |\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi + \left\langle |\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0|^2 \right\rangle_\phi = \frac{1}{2}\cos^2\theta + \frac{1}{2}$$

Averaged cross section: $\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy:

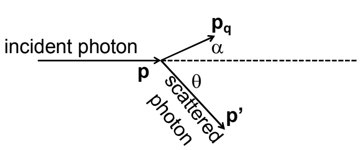
$$p = p'\cos\theta + p_q\cos\alpha \qquad pc = \hbar\omega$$

$$0 = p'\sin\theta - p_q\sin\alpha \qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p_q{}^2 c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$

Thompson scattering – relativistic and quantum modifications



Relativistic and quantum modifications to averaged cross section :

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \left(\frac{p'}{p}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1-\cos\theta\right)}$$