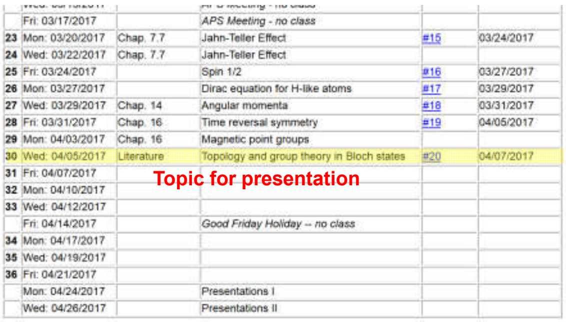
PHY 745 Group Theory
11-11:50 AM MWF Olin 102

Plan for Lecture 30:

Topological analysis of band structures and their relationships to group theory

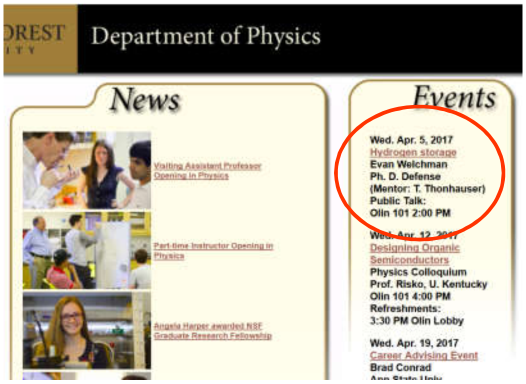
Hasan and Kane, RMP 82, 3045-3067 (2010)

1. What is the notion of topological order
2. Examples



	Fri: 03/17/2017		*APS Meeting - no class*		
23	Mon: 03/20/2017	Chap. 7.7	Jahn-Teller Effect	#15	03/24/2017
24	Wed: 03/22/2017	Chap. 7.7	Jahn-Teller Effect		
25	Fri: 03/24/2017		Spin 1/2	#16	03/27/2017
26	Mon: 03/27/2017		Dirac equation for H-like atoms	#17	03/29/2017
27	Wed: 03/29/2017	Chap. 14	Angular momenta	#18	03/31/2017
28	Fri: 03/31/2017	Chap. 16	Time reversal symmetry	#19	04/05/2017
29	Mon: 04/03/2017	Chap. 16	Magnetic point groups		
30	Wed: 04/05/2017	Literature	Topology and group theory in Bloch states	#20	04/07/2017
31	Fri: 04/07/2017				
32	Mon: 04/10/2017				
33	Wed: 04/12/2017				
	Fri: 04/14/2017		*Good Friday Holiday -- no class*		
34	Mon: 04/17/2017				
35	Wed: 04/19/2017				
36	Fri: 04/21/2017				
	Mon: 04/24/2017		Presentations I		
	Wed: 04/26/2017		Presentations II		

Topic for presentation



Department of Physics

News

Visiting Assistant Professor Opening in Physics

Part-time Instructor Opening in Physics

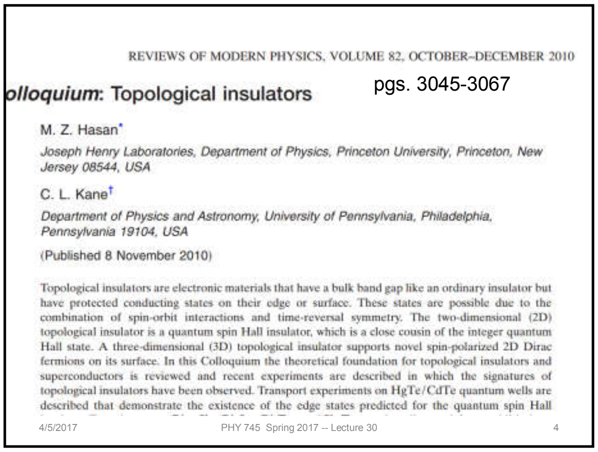
Angela Harper awarded NSF Graduate Research Fellowship

Events

Wed. Apr. 5, 2017
Hydrogen storage
Evan Welchman
Ph. D. Defense
(Mentor: T. Thonhauser)
Public Talk:
Olin 101 2:00 PM

Wed. Apr. 12, 2017
Designing Organic Semiconductors
Physics Colloquium
Prof. Risko, U. Kentucky
Olin 101 4:00 PM
Refreshments:
3:30 PM Olin Lobby

Wed. Apr. 19, 2017
Career Advising Event
Brad Conrad



REVIEWS OF MODERN PHYSICS, VOLUME 82, OCTOBER–DECEMBER 2010

olloquium: Topological insulators

pgs. 3045-3067

M. Z. Hasan*

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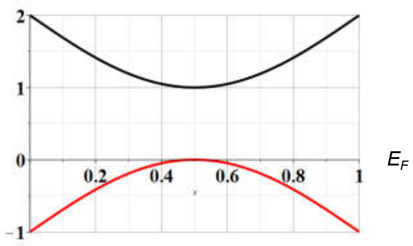
(Published 8 November 2010)

Topological insulators are electronic materials that have a bulk band gap like an ordinary insulator but have protected conducting states on their edge or surface. These states are possible due to the combination of spin-orbit interactions and time-reversal symmetry. The two-dimensional (2D) topological insulator is a quantum spin Hall insulator, which is a close cousin of the integer quantum Hall state. A three-dimensional (3D) topological insulator supports novel spin-polarized 2D Dirac fermions on its surface. In this Colloquium the theoretical foundation for topological insulators and superconductors is reviewed and recent experiments are described in which the signatures of topological insulators have been observed. Transport experiments on HgTe/CdTe quantum wells are described that demonstrate the existence of the edge states predicted for the quantum spin Hall

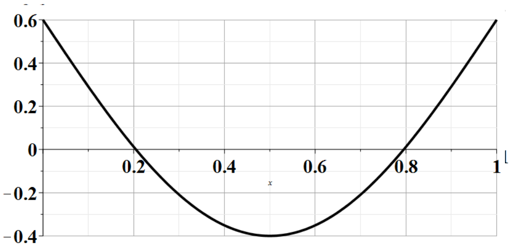
Topology is a mathematical term concerned with the properties of space that are preserved under continuous deformations.

In condensed matter physics, the "space" is typically reciprocal space and the interest is in the behavior of the energy bands in that space, especially near their Fermi levels.

Typical band structure of an insulator



Typical band structure of a metal



Much more exciting band structure -- graphene --

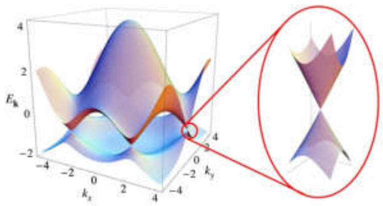


FIG. 3. (Color online) Electronic dispersion in the honeycomb lattice. Left: energy spectrum (in units of t) for finite values of t and t', with t=2.7 eV and t'=−0.2t. Right: zoom in of the energy bands close to one of the Dirac points.

From: Castro Neto *et al.* Rev. Mod. Phys. **81** 109-162 (2009)

Digression – what happens when electrons confined to two dimensions are placed in a magnetic field?

Hamiltonian of an electron in a magnetic field:

$$H=\frac{1}{2m}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2$$

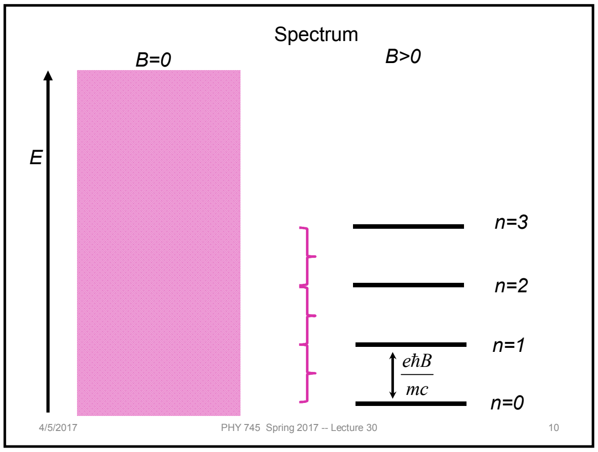
For constant field along z-axis: $\mathbf{B}=B\hat{\mathbf{z}}$

$\mathbf{A}=Bx\hat{\mathbf{y}}$

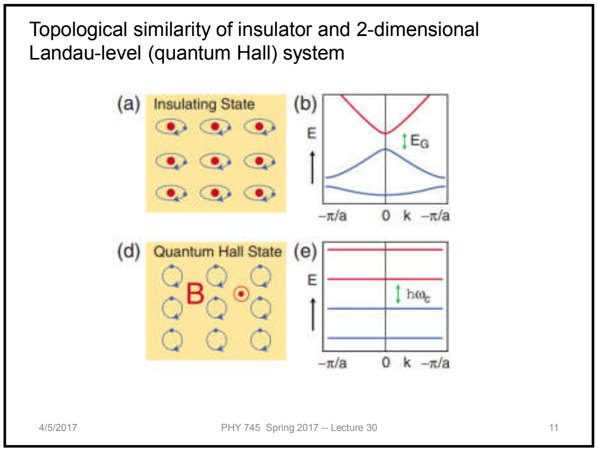
$$H=\frac{p_x^2}{2m}+\frac{1}{2m}\left(p_y+\frac{eB}{c}x\right)^2$$

Eigenstates of Schroedinger equation: $H\Psi_n(x,y)=E_n\Psi_n(x,y)$

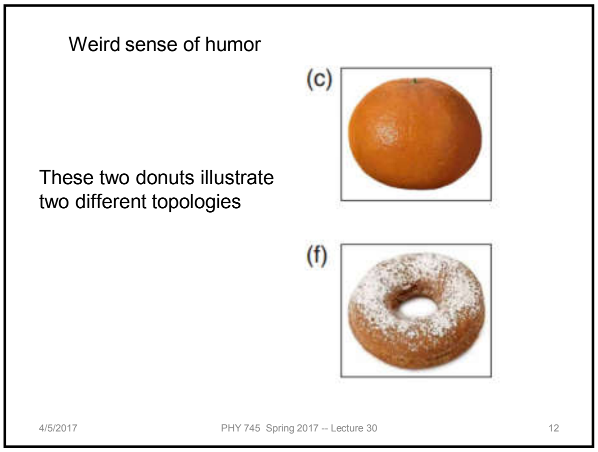
$$E_n=\frac{e\hbar B}{mc}\left(n+\tfrac{1}{2}\right)$$



Spectrum



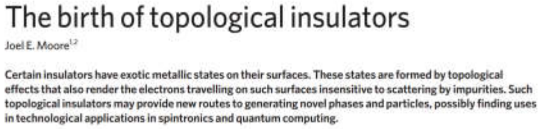
Topological similarity of insulator and 2-dimensional Landau-level (quantum Hall) system



Weird sense of humor

These two donuts illustrate two different topologies

From Nature **464,** 194-198   (2010)



The birth of topological insulators

Joel E. Moore[1,2]

Certain insulators have exotic metallic states on their surfaces. These states are formed by topological effects that also render the electrons travelling on such surfaces insensitive to scattering by impurities. Such topological insulators may provide new routes to generating novel phases and particles, possibly finding uses in technological applications in spintronics and quantum computing.

Examples of these surface states

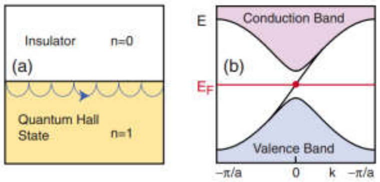
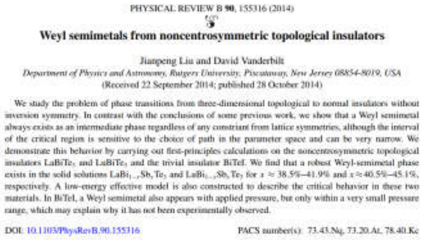


FIG. 2. (Color online) The interface between a quantum Hall state and an insulator has chiral edge mode. (a) The skipping cyclotron orbits. (b) The electronic structure of a semi-infinite strip described by the Haldane model. A single edge state connects the valence band to the conduction band.

Productive strategy for studying these effects

Model Hamiltonian ←→ First principles calculations



PHYSICAL REVIEW B **90**, 155316 (2014)

Weyl semimetals from noncentrosymmetric topological insulators

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(Received 22 September 2014; published 28 October 2014)

We study the problem of phase transitions from three-dimensional topological to normal insulators without inversion symmetry. In contrast with the conclusions of some previous work, we show that a Weyl semimetal always exists as an intermediate phase regardless of any constriant from lattice symmetries, although the interval of the critical region is sensitive to the choice of path in the parameter space and can be very narrow. We demonstrate this behavior by carrying out first-principles calculations on the noncentrosymmetric topological insulators LaBiTe$_3$ and LuBiTe$_3$ and the trivial insulator BiTeI. We find that a robust Weyl-semimetal phase exists in the solid solutions LaBi$_{1-x}$Sb$_x$Te$_3$ and LuBi$_{1-x}$Sb$_x$Te$_3$ for $x \approx 38.5\%$–41.9% and $x \approx 40.5\%$–45.1%, respectively. A low-energy effective model is also constructed to describe the critical behavior in these two materials. In BiTeI, a Weyl semimetal also appears with applied pressure, but only within a very small pressure range, which may explain why it has not been experimentally observed.

DOI: 10.1103/PhysRevB.90.155316 PACS number(s): 73.43.Nq, 73.20.At, 78.40.Kc

A Weyl semimetal is characterized by a Fermi energy that intersects the bulk bands only at one or more pairs of band-touching points (BTPs) between nondegenerate valence and conduction bands. This can occur in the presence of spin-orbit coupling (SOC), typically in a crystal with broken time-reversal or inversion symmetry but not both, so that the pairs are of the form $(\mathbf{k}_0, -\mathbf{k}_0)$ in the Brillouin zone (BZ).

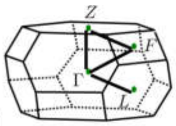
Model Hamiltonian

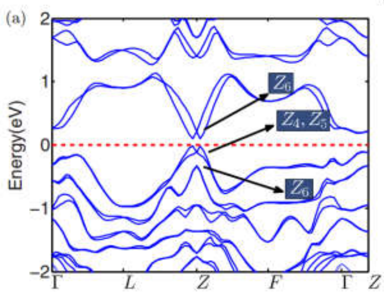
$$H(\mathbf{k},\lambda) = f_x(\mathbf{k},\lambda)\sigma_x + f_y(\mathbf{k},\lambda)\sigma_y + f_z(\mathbf{k},\lambda)\sigma_z, \qquad (1)$$

Many assumptions → quadratic model in region of interest

$$f_1 = p_3 u_{31} + \delta\lambda\,\Lambda_1 + p_1^2 w_1^{11} + p_2^2 w_1^{22} + 2p_1 p_2 w_1^{12},$$
$$f_2 = p_3 u_{32} + \delta\lambda\,\Lambda_2 + p_1^2 w_2^{11} + p_2^2 w_2^{22} + 2p_1 p_2 w_2^{12},$$
$$f_3 = p_3 u_{33} + \delta\lambda\,\Lambda_3 + p_1^2 w_3^{11} + p_2^2 w_3^{22} + 2p_1 p_2 w_3^{12},$$

Band structure of $LaBiTe_3$





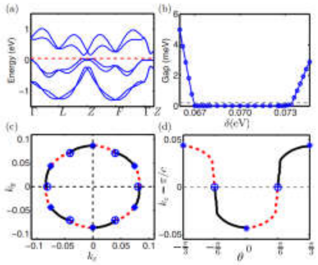
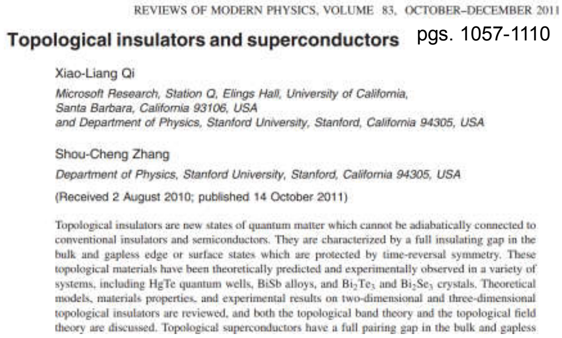


FIG. 9. (Color online) (a) Bulk bandstructure of the six-band model at $\delta = 0.09$ eV. (b) Smallest direct band gap in the BZ vs. δ. (c) Trajectory of Weyl nodes projected onto the (k_x, k_y) plane. Dashed red (solid black) line refers to the Weyl node with positive (negative) chirality. The "*" and "⊕" denote the creation and annihilation point of the Weyl nodes respectively. θ is the azimuthal angle in the (k_x, k_y) plane. (d) Trajectory of Weyl nodes along k_z. Units of k_x, k_y and k_z are Å^{-1}.

Additional review articles --



REVIEWS OF MODERN PHYSICS, VOLUME 83, OCTOBER–DECEMBER 2011

Topological insulators and superconductors

pgs. 1057-1110

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(Received 2 August 2010; published 14 October 2011)

Topological insulators are new states of quantum matter which cannot be adiabatically connected to conventional insulators and semiconductors. They are characterized by a full insulating gap in the bulk and gapless edge or surface states which are protected by time-reversal symmetry. These topological materials have been theoretically predicted and experimentally observed in a variety of systems, including HgTe quantum wells, BiSb alloys, and Bi_2Te_3 and Bi_2Se_3 crystals. Theoretical models, materials properties, and experimental results on two-dimensional and three-dimensional topological insulators are reviewed, and both the topological band theory and the topological field theory are discussed. Topological superconductors have a full pairing gap in the bulk and gapless