PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 36:

Review of Electrodynamics

1. **Units**
2. **Problem solving strategies**
3. **Examples**
4. **Course evaluation forms**

	Fri: 03/17/2017		APS Meeting -- no class		
23	Mon: 03/20/2017	Chap. 9	Sources of Radiation	#17	03/24/2017
24	Wed: 03/22/2017	Chap. 9 & 10	Radiation and Scattering		
25	Fri: 03/24/2017	Chap. 9 & 10	Radiation and Scattering	#18	03/27/2017
26	Mon: 03/27/2017	Chap. 11	Special relativity	#19	03/31/2017
27	Wed: 03/29/2017	Chap. 11	Special relativity		
28	Fri: 03/31/2017	Chap. 11	Special relativity	#20	04/3/2017
29	Mon: 04/03/2017	Chap. 14	Radiation from moving charges	#21	04/5/2017
30	Wed: 04/05/2017	Chap. 14	Radiation from moving charges	#22	04/7/2017
31	Fri: 04/07/2017	Chap. 14	Radiation from moving charges	#23	04/10/2017
32	Mon: 04/10/2017	Chap. 15	Radiation from collisions		
33	Wed: 04/12/2017	Chap. 13	Cherenkov radiation		
	Fri: 04/14/2017		*Good Friday Holiday -- no class*		
34	Mon: 04/17/2017		Superconductivity		
35	Wed: 04/19/2017		Superconductivity		
6	Fri: 04/21/2017		Review		
	Mon: 04/24/2017		Presentations I		
	Wed: 04/26/2017		Presentations II		

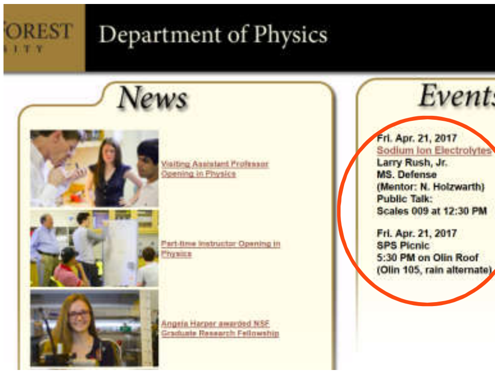
PHY 712 Presentation Schedule

Monday April 24, 2017

Time	Name	Topic/Title
9:00-9:25 AM	Ali Daraei	Light scattering of fibrin fibers
9:25-9:50 AM	Taylor Ordines	Prob. 7.2

Wednesday April 26, 2017

Time	Name	Topic/Title
9:00-9:25 AM	TJ Colvin	Smooth Particle Mesh Ewald
9:25-9:50 AM	Colin Tyznik	



Example problems --

PHY 712 -- Assignment #22

April 5, 2017

Continue reading Chap. 14 in **Jackson** .

1. Consider an electron moving at constant speed βc ≈ c in a circular trajectory of radius ρ. Its total energy is E= γ m c^2. Determine the ratio of the energy lost during one full cycle to the total energy. Evaluate the expression for an electron with total energy 200 GeV in a synchroton of radius ρ=10^3 m.

Radiated power from circular motion (Jackson 14.46):

$$P = \frac{2}{3}\frac{e^2|\dot{\mathbf{v}}|^2}{c^3}\gamma^4 \qquad \text{For circular orbit } |\dot{\mathbf{v}}| \approx \frac{v^2}{\rho} \approx \frac{c^2}{\rho}$$

$$\Delta E \approx P\left(\frac{2\pi\rho}{v}\right) \approx P\left(\frac{2\pi\rho}{c}\right)$$

$$\frac{\Delta E}{E} \approx \frac{2}{3}\frac{e^2}{c^3}\gamma^4\left(\frac{c^2}{\rho}\right)^2\left(\frac{2\pi\rho}{c}\right)\frac{1}{\gamma mc^2}$$

$$\frac{\Delta E}{E} \approx \frac{4\pi}{3}\frac{e^2/\rho}{mc^2}\gamma^3 = \frac{4\pi}{3}\cdot 2.8\times10^{-18}\cdot 6.4\times10^{16}$$

$$\approx 0.75$$

$$\frac{e^2}{\rho} = \frac{\left(4.8\times10^{-10}\,\text{stat C}\right)^2}{10^5\ \text{cm}}$$

$$= 2.3\times10^{-24}\,\text{erg}\,\frac{10^{-7}\,\text{J}}{\text{erg}} = 2.3\times10^{-31}\ \text{J}$$

$$\frac{e^2/\rho}{mc^2} = \frac{2.3\times10^{-31}\ \text{J}}{8.2\times10^{-14}\ \text{J}} = 2.8\times10^{-18}$$

Jackson
pg. 783

Table 4 Conversion Table for Given Amounts of a Physical Quantity

The table is arranged so that a given amount of some physical quantity, expressed as so many SI or Gaussian units of that quantity, can be expressed as an equivalent number of units in the other system. Thus the entries in each row stand for the same amount, expressed in different units. All factors of 3 (apart from exponents) should, for accurate work, be replaced by (2.997 924 58), arising from the numerical value of the velocity of light. For example, in the row for displacement (D), the entry ($12\pi \times 10^5$) is actually ($2.997\,924\,58 \times 4\pi \times 10^5$) and "9" is actually $10^{-16} c^2 = 8.987\,55\ldots$. Where a name for a unit has been agreed on or is in common usage, that name is given. Otherwise, one merely reads so many Gaussian units, or SI units.

Physical Quantity	Symbol	SI		Gaussian
Length	l	1 meter (m)	10^2	centimeters (cm)
Mass	m	1 kilogram (kg)	10^3	grams (g)
Time	t	1 second (s)	1	second (s)
Frequency	ν	1 hertz (Hz)	1	hertz (Hz)
Force	F	1 newton (N)	10^5	dynes
Work / Energy	W / U	1 joule (J)	10^7	ergs
Power	P	1 watt (W)	10^7	ergs s^{-1}
Charge	q	1 coulomb (C)	3×10^9	statcoulombs
Charge density	ρ	1 C m^{-3}	3×10^3	statcoul cm^{-3}
Current	I	1 ampere (A)	3×10^9	statamperes
Current density	J	1 A m^{-2}	3×10^5	statamp cm^{-2}
Electric field	E	1 volt m^{-1} (Vm^{-1})	$\frac{1}{3} \times 10^{-4}$	statvolt cm^{-1}
Potential	Φ, V	1 volt (V)	$\frac{1}{300}$	statvolt
Polarization	P	1 C m^{-2}	3×10^5	dipole moment cm^{-3}
Displacement	D	1 C m^{-2}	$12\pi \times 10^5$	statvolt cm^{-1} (statcoul cm^{-2})
Conductivity	σ	1 mho m^{-1}	9×10^9	s^{-1}
Resistance	R	1 ohm (Ω)	$\frac{1}{9} \times 10^{-11}$	s cm^{-1}
Capacitance	C	1 farad (F)	9×10^{11}	cm
Magnetic flux	ϕ, F	1 weber (Wb)	10^8	gauss cm^2 or maxwells
Magnetic induction	B	1 tesla (T)	10^4	gauss (G)
Magnetic field	H	1 A m^{-1}	$4\pi \times 10^{-3}$	oersted (Oe)
Magnetization	M	1 A m^{-1}	10^{-3}	magnetic moment cm^{-3}
Inductance*	L	1 henry (H)	$\frac{1}{9} \times 10^{-11}$	

*There is some confusion about the unit of inductance in Gaussian units. This stems from the use by some authors of a modified system of Gaussian units in which current is measured in electromagnetic units, so that the connection between charge and current is $I_m = (1/c)(dq/dt)$. Since inductance is defined through the induced voltage $V = L(dI/dt)$ or the energy $U = \frac{1}{2}LI^2$, the choice of current defined in Section 2 means that our Gaussian unit of inductance is equal in magnitude and dimensions (t^2l^{-1}) to the electrostatic unit of inductance. The electromagnetic current I_m is related to our Gaussian current I by the relation $I_m = (1/c)I$. From the energy definition of inductance, we see that the electromagnetic inductance L_m is related to our Gaussian inductance L through $L_m = c^2L$. Thus L_m has the dimensions of length. The modified Gaussian system generally uses the electromagnetic unit of inductance, as well as current. Then the voltage relation reads $V = (L_m/c)(dI_m/dt)$. The numerical connection between units of inductance is

$$1 \text{ henry} = \tfrac{1}{9} \times 10^{-11} \text{ Gaussian (es) unit} = 10^9 \text{ emu}$$

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Length	l	1 meter (m)	10^2	centimeters (cm)
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Inductance*	L	1 henry (H)	$\frac{1}{9} \times 10^{-11}$	

Basic equations of electrodynamics

CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$	$u = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

More SI relationships:

$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ $\quad \mathbf{D} = \varepsilon \mathbf{E}$

$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M})$ $\quad \mathbf{B} = \mu \mathbf{H}$ $\quad \mathbf{B} = F(\mathbf{H})$

for ferromagnet

More Gaussian relationships:

$\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P}$ $\quad \mathbf{D} = \varepsilon \mathbf{E}$

$\mathbf{H} = \mathbf{B} - 4\pi \mathbf{M})$ $\quad \mathbf{B} = \mu \mathbf{H}$ $\quad \mathbf{B} = F(\mathbf{H})$

for ferromagnet

elementary charge: e=1.6021766208 x 10^{-19} C
=4.80320467299766 x 10^{-10} statC

Energy and power (SI units)

Electromagnetic energy density: $u \equiv \frac{1}{2}\left(\mathbf{E}\cdot\mathbf{D}+\mathbf{H}\cdot\mathbf{B}\right)$

Poynting vector: $\mathbf{S} \equiv \mathbf{E}\times\mathbf{H}$

Equations for time harmonic fields:

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)e^{-i\omega t}\right) \equiv \frac{1}{2}\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)e^{-i\omega t} + \tilde{\mathbf{E}}^*(\mathbf{r},\omega)e^{i\omega t}\right)$$

$$\left\langle u(\mathbf{r},t)\right\rangle_{t\text{ avg}} = \frac{1}{4}\Re\left(\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\cdot\tilde{\mathbf{D}}^*(\mathbf{r},\omega) + \tilde{\mathbf{B}}(\mathbf{r},\omega)\cdot\tilde{\mathbf{H}}^*(\mathbf{r},\omega)\right)\right)$$

$$\left\langle \mathbf{S}(\mathbf{r},t)\right\rangle_{t\text{ avg}} = \frac{1}{2}\Re\left(\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{H}}^*(\mathbf{r},\omega)\right)\right)$$

Solution of Maxwell's equations:

$$\nabla\cdot\mathbf{E} = \rho/\varepsilon_0 \qquad \nabla\times\mathbf{B} - \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} = \mu_0\mathbf{J}$$

$$\nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

Introduction of vector and scalar potentials:

$$\nabla\cdot\mathbf{B} = 0 \quad \Rightarrow \quad \mathbf{B} = \nabla\times\mathbf{A}$$

$$\nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0 \quad \Rightarrow \nabla\times\left(\mathbf{E} + \frac{\partial\mathbf{A}}{\partial t}\right) = 0$$

$$\mathbf{E} + \frac{\partial\mathbf{A}}{\partial t} = -\nabla\Phi \quad \text{or} \quad \mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}$$

Scalar and vector potentials continued:

$\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$:

$$-\nabla^2\Phi - \frac{\partial\left(\nabla\cdot\mathbf{A}\right)}{\partial t} = \rho/\varepsilon_0$$

$$\nabla\times\mathbf{B} - \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} = \mu_0\mathbf{J}$$

$$\nabla\times\left(\nabla\times\mathbf{A}\right) + \frac{1}{c^2}\left(\frac{\partial\left(\nabla\Phi\right)}{\partial t} + \frac{\partial^2\mathbf{A}}{\partial t^2}\right) = \mu_0\mathbf{J}$$

Analysis of the scalar and vector potential equations :

$$-\nabla^2\Phi - \frac{\partial(\nabla\cdot\mathbf{A})}{\partial t} = \rho/\varepsilon_0$$

$$\nabla\times(\nabla\times\mathbf{A}) + \frac{1}{c^2}\left(\frac{\partial(\nabla\Phi)}{\partial t} + \frac{\partial^2\mathbf{A}}{\partial t^2}\right) = \mu_0\mathbf{J}$$

Lorentz gauge form - - require $\nabla\cdot\mathbf{A}_L + \frac{1}{c^2}\frac{\partial\Phi_L}{\partial t} = 0$

$$-\nabla^2\Phi_L + \frac{1}{c^2}\frac{\partial^2\Phi_L}{\partial t^2} = \rho/\varepsilon_0$$

$$-\nabla^2\mathbf{A}_L + \frac{1}{c^2}\frac{\partial^2\mathbf{A}_L}{\partial t^2} = \mu_0\mathbf{J}$$

Solution methods for scalar and vector potentials
and their electrostatic and magnetostatic analogs:

$$-\nabla^2\Phi_L + \frac{1}{c^2}\frac{\partial^2\Phi_L}{\partial t^2} = \rho/\varepsilon_0$$

$$-\nabla^2\mathbf{A}_L + \frac{1}{c^2}\frac{\partial^2\mathbf{A}_L}{\partial t^2} = \mu_0\mathbf{J}$$

In your "bag" of tricks:
- ❑ **Direct (analytic or numerical) solution of differential equations**
- ❑ **Solution by expanding in appropriate orthogonal functions**
- ❑ **Green's function techniques**

How to choose most effective solution method --
- ❑ In general, Green's functions methods work well when source is contained in a finite region of space

Consider the electrostatic problem:

$$-\nabla^2\Phi_L = \rho/\varepsilon_0$$

Define: $\nabla'^2 G(\mathbf{r},\mathbf{r}') = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$

$$\Phi_L(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_V d^3r'\,\rho(\mathbf{r}')G(\mathbf{r},\mathbf{r}') +$$

$$\frac{1}{4\pi}\int_S d^2r'\left[G(\mathbf{r},\mathbf{r}')\nabla'\Phi(\mathbf{r}') - \Phi(\mathbf{r}')\nabla'G(\mathbf{r},\mathbf{r}')\right]\cdot\hat{\mathbf{r}}'.$$

For electrostatic problems where $\rho(\mathbf{r})$ is contained in a small region of space and $S \to \infty$, $\quad G(\mathbf{r},\mathbf{r}') = \frac{1}{|\mathbf{r}-\mathbf{r}'|}$

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{lm} \frac{4\pi}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y_{lm}(\theta,\varphi) Y_{lm}^*(\theta',\varphi')$$

Electromagnetic waves from time harmonic sources

Charge density : $\rho(\mathbf{r},t) = \Re\left(\tilde{\rho}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Current density : $\mathbf{J}(\mathbf{r},t) = \Re\left(\tilde{\mathbf{J}}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Note that the continuity condition :

$$\frac{\partial \rho(\mathbf{r},t)}{\partial t} + \nabla\cdot\mathbf{J}(\mathbf{r},t) = 0 \quad \Rightarrow -i\omega\tilde{\rho}(\mathbf{r},\omega) + \nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega) = 0$$

For dynamic problems where $\tilde{\rho}(\mathbf{r},\omega)$ and $\tilde{\mathbf{J}}(\mathbf{r},\omega)$ are contained in a small region of space and $S \to \infty$,

$$\tilde{G}(\mathbf{r},\mathbf{r}',\omega) = \frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$

Electromagnetic waves from time harmonic sources – continued:

For scalar potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

For vector potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \frac{\mu_0}{4\pi}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

Electromagnetic waves from time harmonic sources – continued:

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr) = j_l(kr) + in_l(kr)$

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \sum_{lm} \tilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\phi}_{lm}(r,\omega) = \frac{ik}{\varepsilon_0}\int d^3r'\,\tilde{\rho}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

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Electromagnetic waves from time harmonic sources – continued:

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik\sum_{lm} j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr) = j_l(kr) + in_l(kr)$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \sum_{lm} \tilde{\mathbf{a}}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\mathbf{a}}_{lm}(r,\omega) = ik\mu_0\int d^3r'\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

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Electromagnetic waves from time harmonic sources – continued:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) = -\nabla\tilde{\Phi}(\mathbf{r},\omega) + i\omega\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$\tilde{\mathbf{B}}(\mathbf{r},\omega) = \nabla\times\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

Power radiated :

$$\frac{dP}{d\Omega} = r^2\hat{\mathbf{r}}\cdot\langle\mathbf{S}\rangle_{avg} = \frac{r^2\hat{\mathbf{r}}}{2\mu_0}\hat{\mathbf{r}}\cdot\Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)\right)$$

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Example of dipole radiation source

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\,(ik\mu_0)\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0 k}{\varepsilon_0\omega R}\cos\theta\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

Evaluation for $r >> R$:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\;\frac{2R^3}{(1+k^2R^2)^2}$$

$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{J_0 k}{\varepsilon_0\omega}\cos\theta\,\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right)\frac{2R^3}{(1+k^2R^2)^2}$$

Example of dipole radiation source -- continued

Evaluation for $r >> R$:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\;\frac{2R^3}{(1+k^2R^2)^2}$$

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Relationship to pure dipole approximation (exact when kR→0)

$$\mathbf{p}(\omega)\equiv\int d^3r\,\mathbf{r}\tilde{\rho}(\mathbf{r},\omega)=-\frac{1}{i\omega}\int d^3r\,\tilde{\mathbf{J}}(\mathbf{r},\omega)=-\frac{8\pi R^3 J_0}{i\omega}\hat{\mathbf{z}}$$

Corresponding dipole fields: $\tilde{\mathbf{A}}(\mathbf{r},\omega)=-\frac{i\mu_0\omega}{4\pi}\mathbf{p}(\omega)\frac{e^{ikr}}{r}$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{i}{4\pi\omega\varepsilon_0}\mathbf{p}(\omega)\cdot\hat{\mathbf{r}}\left(1+\frac{i}{kr}\right)\frac{e^{ikr}}{r}$$

Electromagnetic waves from time harmonic sources – for dipole radiation --:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega)=-\nabla\tilde{\Phi}(\mathbf{r},\omega)+i\omega\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\left(k^2\left((\hat{\mathbf{r}}\times\mathbf{p}(\omega))\times\hat{\mathbf{r}}\right)+\left(\frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{p}(\omega))-\mathbf{p}(\omega)}{r^2}\right)(1-ikr)\right)$$

$$\tilde{\mathbf{B}}(\mathbf{r},\omega)=\nabla\times\tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$=\frac{1}{4\pi\varepsilon_0 c^2}\frac{e^{ikr}}{r}k^2(\hat{\mathbf{r}}\times\mathbf{p}(\omega))\left(1-\frac{1}{ikr}\right)$$

Power radiated for $kr >> 1$:

$$\frac{dP}{d\Omega}=r^2\hat{\mathbf{r}}\cdot\langle\mathbf{S}\rangle_{avg}=\frac{r^2\hat{\mathbf{r}}}{2\mu_0}\hat{\mathbf{r}}\cdot\Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{B}}^*(\mathbf{r},\omega)\right)$$

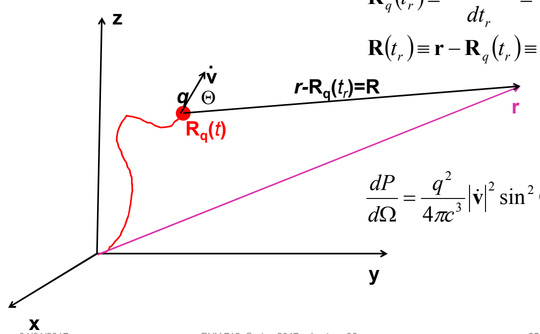
$$=\frac{c^2k^4}{32\pi^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left|(\hat{\mathbf{r}}\times\mathbf{p}(\omega))\times\hat{\mathbf{r}}\right|^2$$

Radiation from a moving charged particle

Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$



$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}|\dot{\mathbf{v}}|^2 \sin^2\Theta$$

Liènard-Wiechert potentials –(Gaussian units)

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\left(1 - \frac{v^2}{c^2}\right) + \left(\mathbf{R}\times\left\{\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right\}\right)\right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}}\cdot\mathbf{R}}{c^2}\right) - \frac{\mathbf{R}\times\dot{\mathbf{v}}/c}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^2}\right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}.$$

Electric and magnetic fields far from source:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left\{\mathbf{R}\times\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}$$

Let $\hat{\mathbf{R}} \equiv \frac{\mathbf{R}}{R} \qquad \boldsymbol{\beta} \equiv \frac{\mathbf{v}}{c} \qquad \dot{\boldsymbol{\beta}} \equiv \frac{\dot{\mathbf{v}}}{c}$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}})^3}\left\{\hat{\mathbf{R}}\times\left[(\hat{\mathbf{R}} - \boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

Poynting vector:

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{B})$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\left\{\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\hat{\mathbf{R}}\left|\mathbf{E}(\mathbf{r},t)\right|^2 = \frac{q^2}{4\pi cR^2}\hat{\mathbf{R}}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

$$\frac{dP}{d\Omega} = \mathbf{S}\cdot\hat{\mathbf{R}}R^2 = \frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

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