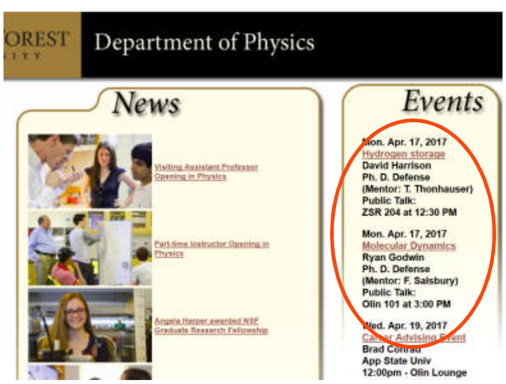
PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 34:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity

	Fri: 03/17/2017		*APS Meeting - no class*		
23	Mon: 03/20/2017	Chap. 9	Sources of Radiation	#17	03/24/2017
24	Wed: 03/22/2017	Chap. 9 & 10	Radiation and Scattering		
25	Fri: 03/24/2017	Chap. 9 & 10	Radiation and Scattering	#18	03/27/2017
26	Mon: 03/27/2017	Chap. 11	Special relativity	#19	03/31/2017
27	Wed: 03/29/2017	Chap. 11	Special relativity		
28	Fri: 03/31/2017	Chap. 11	Special relativity	#20	04/3/2017
29	Mon: 04/03/2017	Chap. 14	Radiation from moving charges	#21	04/5/2017
30	Wed: 04/05/2017	Chap. 14	Radiation from moving charges	#22	04/7/2017
31	Fri: 04/07/2017	Chap. 14	Radiation from moving charges	#23	04/10/2017
32	Mon: 04/10/2017	Chap. 15	Radiation from collisions		
33	Wed: 04/12/2017	Chap. 13	Cherenkov radiation		
	Fri: 04/14/2017		*Good Friday Holiday -- no class*		
[illegible]	Mon: 04/17/2017		Superconductivity		
35	Wed: 04/19/2017		Superconductivity		
36	Fri: 04/21/2017		Review		
	Mon: 04/24/2017		Presentations I		
	Wed: 04/26/2017		Presentations II		



Special topic: Electromagnetic properties of superconductors

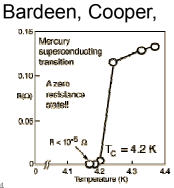
Ref:D. Teplitz, editor, Electromagnetism – paths to research, Plenum Press (1982); Chapter 1 written by Brian Schwartz and Sonia Frota-Pessoa

History:

1908 H. Kamerlingh Onnes successfully liquified He

1911 H. Kamerlingh Onnes discovered that Hg at 4.2 K has vanishing resistance

1957 Theory of superconductivity by Bardeen, Cooper, and Schrieffer



Some phenomenological theories < 1957

Drude model of conductivity in "normal" materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - m\frac{\mathbf{v}}{\tau}$$

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-t/\tau} - \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{J} = -ne\mathbf{v} \qquad \text{for } t >> \tau \qquad \mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} \equiv \sigma\mathbf{E}$$

London model of conductivity in superconducting materials; $\tau \rightarrow 0$

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{e\mathbf{E}}{m} \qquad \frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Fritz London
Poland

Born in 1900 in Breslau (today Wroclaw in Poland), Fritz London studied philosophy before choosing science. After getting a PhD in Munich in 1921, he understood what bonds two hydrogen atoms in a H2 molecule. This work he did with Walter Heitler in Zürich was the starting point for the understanding of chemical bonding. Then he joined Erwin Schrödinger in Berlin but had to leave in 1933 because of the rise of anti-Semitism in Nazi Germany. After a stay in Oxford where he worked on superconductivity with his brother Heinz, he sought refuge at the Institut Henri Poincaré (Paris) in 1936, thanks to a group of intellectuals linked to the Popular Front (Jacques Hadamard, Paul Langevin, Jean Perrin, Frédéric Joliot and Edmond Bauer).

It is at that time, in 1938, that he explained that the superfluidity in liquid helium was a manifestation of Bose-Einstein condensation, a purely quantum phenomenon that could be seen for the first time on a macroscopic scale. This work followed a series of articles about superconductivity that could finally be understood as a superfluidity of charged particles (electron pairs in the case of superconducting metals).

At the beginning of World War II (September 1939), he left France and joined Duke University (USA) where Paul Gross had offered him a professorship in the Chemistry Department and where he felt more comfortable with his wife, the painter Edith London. Einstein wanted the Nobel Prize to be awarded to Fritz London, but London died prematurely in 1954.

Some phenomenological theories < 1957

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times(\nabla\times\mathbf{B}) = -\nabla^2\mathbf{B} = \frac{4\pi}{c}\nabla\times\mathbf{J} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi}{c}\nabla\times\frac{\partial\mathbf{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi ne^2}{mc}\nabla\times\mathbf{E} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = -\frac{4\pi ne^2}{mc^2}\frac{\partial\mathbf{B}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

London model – continued

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

For slowly varying solution:

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2}\right)\mathbf{B} = 0 \qquad \text{for } \frac{\partial\mathbf{B}}{\partial t} = \hat{\mathbf{z}}\frac{\partial B_z(x,t)}{\partial t}:$$

$$\Rightarrow \frac{\partial B_z(x,t)}{\partial t} = \frac{\partial B_z(0,t)}{\partial t}e^{-x/\lambda_L}$$

London leap: $B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Consistent results for current density:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \mathbf{J} = \hat{\mathbf{y}}J_y(x) \quad \Rightarrow \quad J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\mathrm{e}^{-x/\lambda_L}$$

London model – continued

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

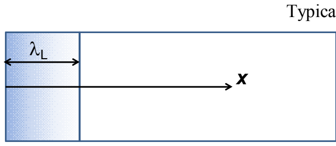
Vector potential for $\nabla\cdot\mathbf{A} = 0$:

$\mathbf{A} = \hat{\mathbf{y}}A_y(x) \qquad A_y(x) = -\lambda_L B_z(0)e^{-x/\lambda_L}$

Recall form for current density: $J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\mathrm{e}^{-x/\lambda_L}$

$$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$$

Typically, $\lambda_L \approx 10^{-7}m$



Magnetization field

Treating London current in terms of corresponding magnetization field **M**:

$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$

$\Rightarrow$ For $x >> \lambda_L$, $\mathbf{H}=-4\pi\mathbf{M}$

Gibbs free energy associated with magnetization for superconductor:

$$G_S(H_a)=G_S(H=0)-\int_0^{H_a} dH M(H)=G_S(0)+\frac{1}{8\pi}H_a^2$$

This relation is true for an applied field $H_a \le H_C$ when the superconducting and normal Gibbs free energies are equal:

$$G_S(H_C)=G_N(H_C)\approx G_N(H=0)$$

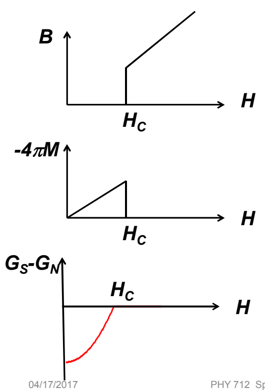
Condition at phase boundary between normal and superconducting states:

$$G_N(H_C)\approx G_N(0)=G_S(H_C)=G_S(0)+\frac{1}{8\pi}H_C^2$$ At *T=0K*

$$\Rightarrow G_S(0)-G_N(0)=-\frac{1}{8\pi}H_C^2$$

$$G_S(H_a)-G_N(H_a)=\begin{cases}-\frac{1}{8\pi}\left(H_C^2-H_a^2\right) & \text{for } H_a<H_C\\ 0 & \text{for } H_a>H_C\end{cases}$$

Magnetization field (for "type I" superconductor)



Inside superconductor

$\mathbf{B}=0=\mathbf{H}+4\pi\mathbf{M}$ for $H<H_C$

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. Bardeen, L. N. Cooper,† and J. R. Schrieffer‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

$$G_S(0)-G_N(0)=-\frac{H_C^2}{8\pi}\approx -2N(E_F)(\hbar\omega)^2 e^{-2/(N(E_F)V)}$$

characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Temperature dependence of critical field

$$H_c(T) \approx H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

From PR **108**, 1175 (1957) Bardeen, Cooper, and Schrieffer, "Theory of Superconductivity"

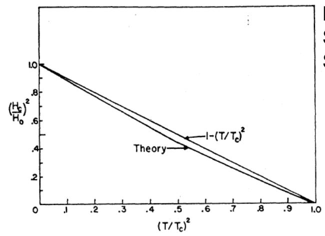


FIG. 2. Ratio of the critical field to its value at $T=0°K$ vs $(T/T_c)^2$. The upper curve is the $1-(T/T_c)^2$ law of the Gorter-Casimir theory and the lower curve is the law predicted by the theory in the weak-coupling limit. Experimental values generally lie between the two curves.

$$T_c \approx \frac{\hbar\omega}{k}e^{-2/(N(E_F)V)}$$

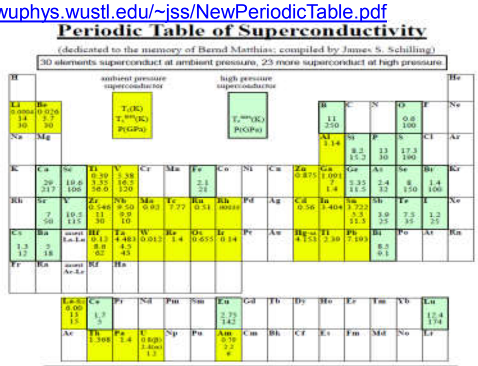
characteristic phonon energy

density of electron states at E_F

attraction potential between electron pairs

Type I elemental superconductors

http://wuphys.wustl.edu/~jss/NewPeriodicTable.pdf



M. Debessai, T. Matsuoka, J.J. Hamlin, W. Bi, Y. Meng, K. Shimizu, and J.S. Schilling, J. Phys.: Conf. Series **215**, 012034 (2010). High pressure data for Ca and Be: K. Shimizu email from 9 Dec 2013.

Behavior of superconducting material – exclusion of magnetic field according to the London model

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

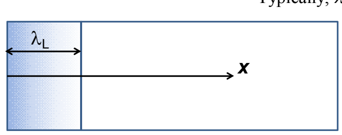
Vector potential for $\nabla\cdot\mathbf{A}=0$:

$\mathbf{A} = \hat{\mathbf{y}}A_y(x) \qquad A_y(x) = -\lambda_L B_z(0)e^{-x/\lambda_L}$

Current density: $J_y(x) = \lambda_L \frac{ne^2}{mc}\mathrm{B}_z(0)\mathrm{e}^{-x/\lambda_L}$

$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$

Typically, $\lambda_L \approx 10^{-7} m$



Behavior of magnetic field lines near superconductor

normal state: superconducting state:

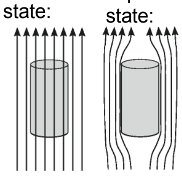
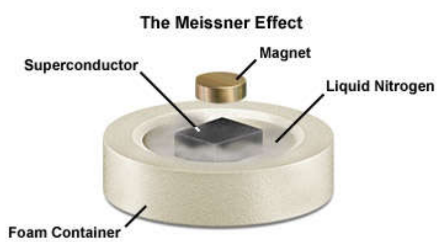


Figure 18.2 Exclusion of a weak external magnetic field from the interior of a superconductor.



Type I superconductors:

$$H_c(T) = H_c(0)\left(1 - \frac{T^2}{T_c^2}\right)$$

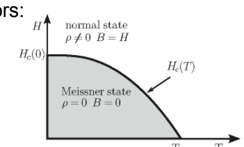


Figure 18.3 Schematic phase diagram illustrating normal and superconducting regions of a type-I superconductor.

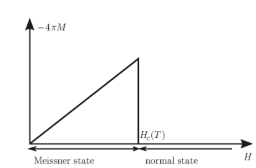


Figure 18.4 Magnetization versus applied field for type-I superconductors.

Type II superconductors

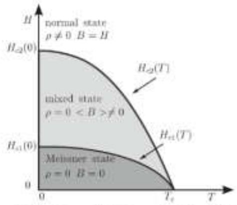


Figure 18.5 Schematic phase diagram illustrating normal, mixed and Meissner regions of a type-II superconductor (the vanishingly small resistivity of the mixed state occurs if flux lines are "pinned" by appropriate material defects); in the mixed state, $\langle B \rangle$ denotes the average magnetic field in the superconductor.

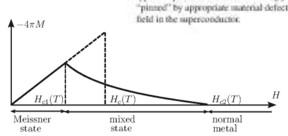


Figure 18.6 Magnetization versus applied field H for a type-II superconductor. The equivalent area construction of the thermodynamic field $H_c(T)$ is also illustrated.

Quantization of current flux associated with the superconducting state (Ref: Ashcroft and Mermin, ***Solid State Physics***)

From the London equations for the interior of the superconductor:

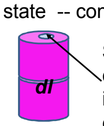
$$\left(m\mathbf{v} + \frac{e}{c}\mathbf{A} \right) = 0$$

Now suppose that the current carrier is a pair of electrons characterized by a wavefunction of the form $\psi = |\psi| e^{i\phi}$

The quantum mechanical current associated with the electron pair is

$$\mathbf{j} = -\frac{e\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) - \frac{2e^2}{mc}\mathbf{A}|\psi|^2$$
$$= -\left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A}\right)|\psi|^2$$

Quantization of current flux associated with the superconducting state -- continued



Suppose a superconducting material has a cylindrical void. Evaluate the integral of the current in a closed path within the superconductor containing the void.

$$\oint \mathbf{j} \cdot d\mathbf{l} = 0 = -\oint \left(\frac{e\hbar}{m}\nabla\phi + \frac{2e^2}{mc}\mathbf{A}\right)|\psi|^2 \cdot d\mathbf{l}$$

$$\oint \mathbf{A} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \int \mathbf{B} \cdot d\mathbf{a} = \Phi \quad \text{magnetic flux}$$

$$\oint \nabla\phi \cdot d\mathbf{l} = 2\pi n \quad \text{for some integer } n$$

$$\Rightarrow \text{Quantization of flux in the void: } |\Phi| = n\frac{hc}{2e} \equiv n\Phi_0$$

Such "vortex" fields can exist within type II superconductors.

Table 18.1 Critical temperature of some selected superconductors, and zero-temperature critical field. For elemental materials, the thermodynamic critical field $H_c(0)$ is given in gauss. For the compounds, which are type-II superconductors, the upper critical field $H_{c2}(0)$ is given in Tesla ($1\ \text{T} = 10^4$ G). The data for metallic elements and binary compounds of V and Nb are taken from G. Burns (1992). The data for MgB_2 and iron pnictide are taken from the references cited in the text, and refer to the two principal crystallographic axes. The data for the other compounds are taken from D. R. Harshman and A. P. Mills, Phys. Rev. B *45*, 10684 (1992)]. A more extensive list of data can be found in the mentioned references.

Metallic elements	$T_c(K)$	$H_c(0)$ (gauss)
Al	1.17	105
Sn	3.72	305
Pb	7.19	803
Hg	4.15	411
Nb	9.25	2060
V	5.40	1410
Binary compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
V_3Ga	16.5	27
V_3Si	17.1	25
Nb_3Al	20.3	34
Nb_3Ge	23.3	38
MgB_2	40	$\approx 5; \approx 20$
Other compounds	$T_c(K)$	$H_{c2}(0)$ (Tesla)
UPt_3 (heavy fermion)	0.53	2.1
$PbMo_6S_8$ (Chevrel phase)	12	55
$\kappa-[BEDT-TTF]_2Cu[NCS]_2$ (organic phase)	10.5	≈ 10
Rb_2CsC_{60} (fullerene)	31.3	≈ 30
$NdFeAsO_{0.7}F_{0.3}$ (iron pnictide)	47	$\approx 30; \approx 50$
Cuprate oxides	$T_c(K)$	$H_{c2}(0)$ (Tesla)
$La_{2-x}Sr_xCuO_4$ ($x \approx 0.15$)	38	≈ 45
$YBa_2Cu_3O_7$	92	≈ 140
$Bi_2Sr_2CaCu_2O_8$	89	≈ 107
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125	≈ 75

Crystal structure of one of the high temperature superconductors

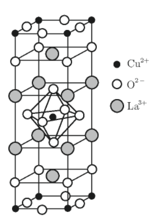


Figure 18.1 Crystal structure of the ceramic material La_2CuO_4. Appropriately doped, lanthanum-based cuprates opened the path to high-T_c superconductivity in 1986.

Some details of single vortex in type II superconductor

London equation without vortices:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \text{where} \quad \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

Equation for field with single quantum of vortex along z-axis:

$$\nabla^2\mathbf{B} - \frac{1}{\lambda_L^2}\mathbf{B} = -\frac{\Phi_0}{\lambda_L^2}\hat{\mathbf{z}}\delta(\mathbf{r}) \qquad \Phi_0 = \frac{hc}{2e} \qquad \mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$$

Solution: $\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}}\frac{\Phi_0}{2\pi\lambda_L^2}K_0\left(\frac{r}{\lambda_L}\right)$

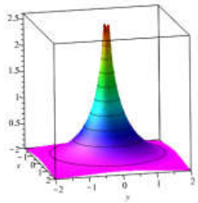
Check:

For $r > 0$ $\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = 0$

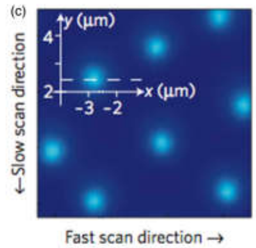
For $r \to 0$ $2\pi\int_0^r dr'\, r'\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{\lambda_L^2}\right)K_0\left(\frac{r}{\lambda_L}\right) = -2\pi$

Since $K_0(u) \underset{u\to 0}{\approx} -\ln u$

$$\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}} \frac{\Phi_0}{2\pi\lambda_L^2} K_0\left(\frac{r}{\lambda_L}\right)$$



Scanning probe images of vortices in YBCO at 22 K



Fundamental studies of superconductors using scanning magnetic imaging

J R Kirtley