

PHY 712 Electrodynamics
9-9:50 AM Olin 103

Plan for Lecture 18:

Complete reading of Chapter 7

- 1. Comments on reflectivity of plane waves**
- 2. Summary of complex response functions for electromagnetic fields**


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10	Fri: 02/03/2017	Chap. 4	Dipoles and dielectrics	#10	02/06/2017
11	Mon: 02/06/2017	Chap. 5	Magnetostatics	#11	02/08/2017
12	Wed: 02/08/2017	Chap. 5	Magnetostatics and the Hyperfine Interaction	#12	02/10/2017
13	Fri: 02/10/2017	Chap. 5	Magnetic dipoles and dipolar fields	#13	02/13/2017
14	Mon: 02/13/2017	Chap. 6	Maxwell's Equations	#14	02/15/2017
15	Wed: 02/15/2017	Chap. 6	Electromagnetic energy and forces	#15	02/17/2017
16	Fri: 02/17/2017	Chap. 7	Electromagnetic plane waves	#16	02/20/2017
17	Mon: 02/20/2017	Chap. 7	Dielectric media		
18	Wed: 02/22/2017	Chap. 7	Complex dielectrics		
19	Fri: 02/24/2017	Chap. 1-7	Review -- Take home exam distributed		
20	Mon: 02/27/2017				Exam
21	Wed: 03/01/2017				Exam
22	Fri: 03/03/2017				Exam Due
	Mon: 03/06/2017		Spring break - no class		
	Wed: 03/08/2017		Spring break - no class		
	Fri: 03/10/2017		Spring break - no class		
	Mon: 03/13/2017		APS Meeting - no class		
	Wed: 03/15/2017		APS Meeting - no class		
	Fri: 03/17/2017		APS Meeting - no class		
23	Mon: 03/20/2017				
24	Wed: 03/22/2017				
25	Fri: 03/24/2017				


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Department of Physics


News



Diana Jurchescu receives the Rufynus Family Omicron Delta Kappa Award



Angela Harper named Churchill Scholar



Major Mansel Ahmoudou featured in article on gender diversity in STEM

Events

Wed. Feb. 22, 2017
Data Compression Methods
 Professor Ballard, WFU
 4:00pm - Olin 101
 Refreshments served 3:30pm - Olin Lounge

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Comment on HW #16

PHY 712 -- Assignment #16

February 17, 2017

Start reading Chapter 7 in Jackson

1. Consider the reflectivity of a plane polarized electromagnetic wave incident from air ($n=1$) on a material with refractive index $n=1.5$ at an angle of incidence i . Plot the reflectance

$$R(i) = |E_{0R}|^2 / |E_{0i}|^2$$

as a function of i for both cases of polarization (E_0 in the plane of incidence or perpendicular to the plane of incidence). What is the qualitative difference between the two cases?

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Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)

For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n \cos i - \frac{\mu}{\mu'} n' \cos \theta}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta} \quad \frac{E'_{0t}}{E_{0i}} = \frac{2n \cos i}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta}$$

Note that: $n' \cos \theta = \sqrt{n'^2 - n^2} \sin^2 i$

For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos \theta}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta} \quad \frac{E'_{0t}}{E_{0i}} = \frac{2n \cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta}$$

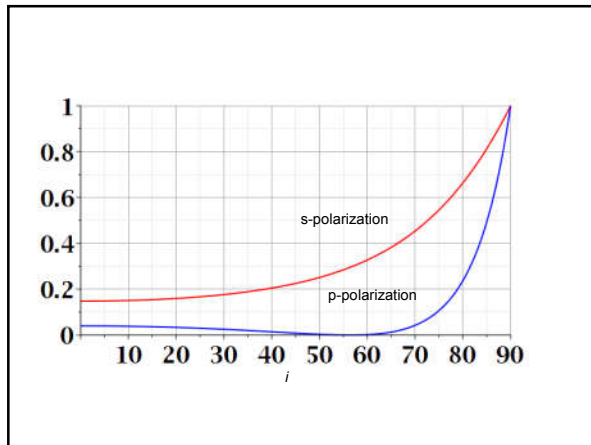
Note that: $n' \cos \theta = \sqrt{n'^2 - n^2} \sin^2 i$

Reflectance for s-polarization

$$R_s = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{n \cos i - \frac{\mu}{\mu'} \sqrt{n'^2 - n^2 \sin^2 i}}{n \cos i + \frac{\mu}{\mu'} \sqrt{n'^2 - n^2 \sin^2 i}} \right|^2$$

Reflectance for p-polarization

$$R_p = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{\frac{\mu}{\mu'} n' \cos i - \frac{n}{n'} \sqrt{n'^2 - n^2 \sin^2 i}}{\frac{\mu}{\mu'} n' \cos i + \frac{n}{n'} \sqrt{n'^2 - n^2 \sin^2 i}} \right|^2$$



Review: Drude model dielectric function:

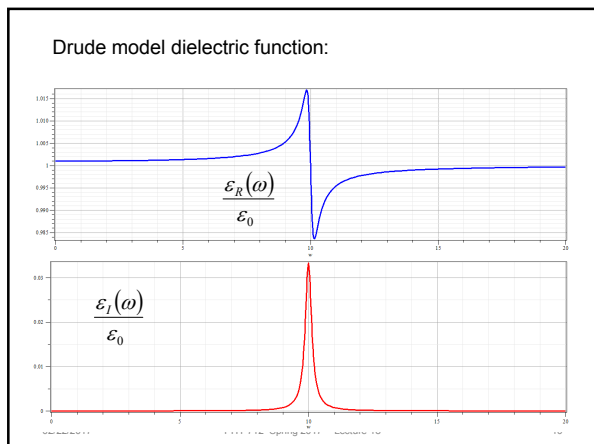
$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + N \sum_i f_i \frac{q_i^2}{\epsilon_0 m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$= \frac{\epsilon_R(\omega)}{\epsilon_0} + i \frac{\epsilon_I(\omega)}{\epsilon_0}$$

$$\frac{\epsilon_R(\omega)}{\epsilon_0} = 1 + N \sum_i f_i \frac{q_i^2}{\epsilon_0 m_i} \frac{\omega_i^2 - \omega^2}{(\omega_i^2 - \omega^2)^2 + \omega^2 \gamma_i^2}$$

$$\frac{\epsilon_I(\omega)}{\epsilon_0} = N \sum_i f_i \frac{q_i^2}{\epsilon_0 m_i} \frac{\omega \gamma_i}{(\omega_i^2 - \omega^2)^2 + \omega^2 \gamma_i^2}$$

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Drude model dielectric function – some analytic properties:

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + N \sum_i f_i \frac{q_i^2}{\epsilon_0 m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

For $\omega \gg \omega_i$, $\frac{\epsilon(\omega)}{\epsilon_0} \approx 1 - \frac{1}{\omega^2} \left(N \sum_i f_i \frac{q_i^2}{\epsilon_0 m_i} \right)$

$$\equiv 1 - \frac{\omega_p^2}{\omega^2}$$

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Analysis for Drude model dielectric function – continued --

Analytic properties:

$$f(z) = \frac{\epsilon(z)}{\epsilon_0} - 1 = N \sum_i f_i \frac{q_i^2}{\epsilon_0 m_i} \frac{1}{\omega_i^2 - z^2 - iz\gamma_i}$$

$f(z)$ has poles z_p at $\omega_i^2 - z_p^2 - iz_p\gamma_i = 0$

$$z_p = -i\frac{\gamma_i}{2} \pm \sqrt{\omega_i^2 - \left(\frac{\gamma_i}{2}\right)^2}$$

Note that $\Im(z_p) \leq 0 \Rightarrow f(z)$ is analytic for $\Im(z_p) > 0$

$f(z)$ analytic

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Because of these analytic properties, Cauchy's integral theorem results in:

Kramers-Kronig transform – for dielectric function:

$$\frac{\epsilon_R(\omega)}{\epsilon_0} - 1 = \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\epsilon_I(\omega')}{\epsilon_0} \frac{1}{\omega' - \omega}$$

$$\frac{\epsilon_I(\omega)}{\epsilon_0} = -\frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \left(\frac{\epsilon_R(\omega')}{\epsilon_0} - 1 \right) \frac{1}{\omega' - \omega}$$

with $\epsilon_R(-\omega) = \epsilon_R(\omega)$; $\epsilon_I(-\omega) = -\epsilon_I(\omega)$

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Further comments on analytic behavior of dielectric function

"Causal" relationship between **E** and **D** fields:

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_0 \left\{ \mathbf{E}(\mathbf{r}, t) + \int_0^{\infty} d\tau G(\tau) \mathbf{E}(\mathbf{r}, t - \tau) \right\}$$

$$G(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{\epsilon(\omega)}{\epsilon_0} - 1 \right) e^{-i\omega\tau} d\omega \quad \frac{\epsilon(\omega)}{\epsilon_0} - 1 = \int_0^{\infty} d\tau G(\tau) e^{i\omega\tau}$$

For $\frac{\epsilon(\omega)}{\epsilon_0} - 1 = \sum_i^N f_i \frac{q_i^2}{m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$

$$G(\tau) = \sum_i^N f_i \frac{q_i^2}{m_i} e^{-\gamma_i \tau / 2} \frac{\sin(\nu_i \tau)}{\nu_i} \Theta(\tau)$$

where $\nu_i \equiv \sqrt{\omega_i^2 - \gamma_i^2 / 4}$

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Review: Electromagnetic plane waves in isotropic medium with real permeability and permittivity: μ, ϵ .

$$\mathbf{E}(\mathbf{r}, t) = \Re \left(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - ct} \right) \quad n^2 = c^2 \mu \epsilon$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{n}{c} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t) = \sqrt{\mu \epsilon} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t)$$

Poynting vector for plane electromagnetic waves:

$$\langle \mathbf{S} \rangle_{\text{avg}} = \frac{n |\mathbf{E}_0|^2}{2 \mu c} \hat{\mathbf{k}} = \frac{1}{2} \sqrt{\frac{\epsilon}{\mu}} |\mathbf{E}_0|^2 \hat{\mathbf{k}}$$

Energy density for plane electromagnetic waves:

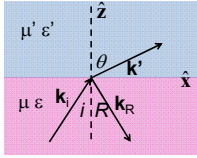
$$\langle u \rangle_{\text{avg}} = \frac{1}{2} \epsilon |\mathbf{E}_0|^2$$

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Review:
Reflection and refraction between two isotropic media



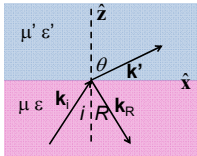
Reflectance, transmittance :

$$R = \frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E_{0R}}{E_{0i}} \right|^2 \quad T = \frac{\mathbf{S}' \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n' \mu \cos \theta}{n \mu' \cos i}$$

Note that $R + T = 1$

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Reflection and refraction between two isotropic media -- continued



For each wave:

$$\mathbf{E}(\mathbf{r}, t) = \Re\left\{ \mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - ct} \right\} \quad n^2 = c^2 \mu \epsilon$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{n}{c} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t) = \sqrt{\mu \epsilon} \hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r}, t)$$

Matching condition at interface:

$$n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$$

If $n > n'$, for $i > i_0 \equiv \sin^{-1}\left(\frac{n'}{n}\right)$,
refracted field no longer propagates in medium $\mu' \epsilon'$

Total internal reflection:

$$n' \cos \theta = i \sqrt{n^2 \sin^2 i - n'^2} = i n \sqrt{\frac{\sin^2 i}{\sin^2 i_0} - 1}$$

$$\mathbf{E}'(\mathbf{r}, t) = e^{-\left(\frac{\mu n}{\mu' \sqrt{\sin^2 i - \sin^2 i_0}}\right) z} \Re\left\{ \mathbf{E}'_0 e^{i\mathbf{k}' \cdot \mathbf{r} - ct} \right\}$$

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For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n \cos i - \frac{\mu}{\mu'} n' \cos \theta}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta} \quad \frac{E'_{0i}}{E_{0i}} = \frac{2n \cos i}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta}$$

Note that : $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos \theta}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta} \quad \frac{E'_{0i}}{E_{0i}} = \frac{2n \cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta}$$

Note that : $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

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Special case: normal incidence ($i=0$, $\theta=0$)

$$\frac{E_{0R}}{E_{0i}} = \frac{\mu}{\mu'} \frac{n'-n}{n'+n} \quad \frac{E'_0}{E_{0i}} = \frac{2n}{\mu' n'+n}$$

Reflectance, transmittance:

$$R = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{\frac{\mu}{\mu'} n' - n}{\frac{\mu}{\mu'} n' + n} \right|^2$$

$$T = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n' \mu}{n \mu'} = \left| \frac{2n}{\frac{\mu}{\mu'} n' + n} \right|^2 \frac{n' \mu}{n \mu'}$$

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Extension to complex refractive index $n = n_R + i n_I$

Suppose $\mu = \mu'$, $n = \text{real}$, $n' = n'_R + i n'_I$

Reflectance at normal incidence:

$$R = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{\frac{\mu}{\mu'} n' - n}{\frac{\mu}{\mu'} n' + n} \right|^2 = \frac{(n'_R - n)^2 + (n'_I)^2}{(n'_R + n)^2 + (n'_I)^2}$$

Note that for $n'_I \gg |n'_R \pm n|$:

$$R \approx 1$$

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