

**PHY 712 Electrodynamics**  
**9-9:50 AM Olin 103**

**Plan for Lecture 16:**

**Read Chapter 7**

1. Plane polarized electromagnetic waves
2. Reflectance and transmittance of electromagnetic waves – extension to anisotropy and complexity

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8	Mon: 01/30/2017	Chap. 3 & 4	Multipole analysis	<a href="#">#8</a>	02/01/2017
9	Wed: 02/01/2017	Chap. 4	Dipoles and dielectrics	<a href="#">#9</a>	02/03/2017
10	Fri: 02/03/2017	Chap. 4	Dipoles and dielectrics	<a href="#">#10</a>	02/06/2017
11	Mon: 02/06/2017	Chap. 5	Magnetostatics	<a href="#">#11</a>	02/08/2017
12	Wed: 02/08/2017	Chap. 5	Magnetostatics and the Hyperfine Interaction	<a href="#">#12</a>	02/10/2017
13	Fri: 02/10/2017	Chap. 5	Magnetic dipoles and dipolar fields	<a href="#">#13</a>	02/13/2017
14	Mon: 02/13/2017	Chap. 6	Maxwell's Equations	<a href="#">#14</a>	02/15/2017
15	Wed: 02/15/2017	Chap. 6	Electromagnetic energy and forces	<a href="#">#15</a>	02/17/2017
16	Fri: 02/17/2017	Chap. 7	Electromagnetic plane waves	<a href="#">#16</a>	02/20/2017
17	Mon: 02/20/2017				
18	Wed: 02/22/2017				
19	Fri: 02/24/2017				
20	Mon: 02/27/2017				Exam
21	Wed: 03/01/2017				Exam
22	Fri: 03/03/2017				Exam Due
	Mon: 03/06/2017		Spring break - no class		
	Wed: 03/08/2017		Spring break - no class		
	Fri: 03/10/2017		Spring break - no class		
	Mon: 03/13/2017		APS Meeting - no class		
	Wed: 03/15/2017		APS Meeting - no class		
	Fri: 03/17/2017		APS Meeting - no class		
23	Mon: 03/20/2017				
24	Wed: 03/22/2017				

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## Maxwell's equations

For linear isotropic media and no sources:  $\mathbf{D} = \epsilon \mathbf{E}$ ;  $\mathbf{B} = \mu \mathbf{H}$

Coulomb's law:  $\nabla \cdot \mathbf{E} = 0$

Ampere-Maxwell's law:  $\nabla \times \mathbf{B} - \mu \epsilon \frac{\partial \mathbf{E}}{\partial t} = 0$

Faraday's law:  $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles:  $\nabla \cdot \mathbf{B} = 0$

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Analysis of Maxwell's equations without sources -- continued:

Coulomb's law :  $\nabla \cdot \mathbf{E} = 0$

Ampere- Maxwell's law :  $\nabla \times \mathbf{B} - \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} = 0$

Faraday's law :  $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles:  $\nabla \cdot \mathbf{B} = 0$

$$\nabla \times \left( \nabla \times \mathbf{B} - \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} \right) = -\nabla^2 \mathbf{B} - \mu\epsilon \frac{\partial (\nabla \times \mathbf{E})}{\partial t}$$

$$= -\nabla^2 \mathbf{B} + \mu\epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$\nabla \times \left( \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} \right) = -\nabla^2 \mathbf{E} + \frac{\partial (\nabla \times \mathbf{B})}{\partial t}$$

$$= -\nabla^2 \mathbf{E} + \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

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Analysis of Maxwell's equations without sources -- continued:

Both E and B fields are solutions to a wave equation:

$$\nabla^2 \mathbf{B} - \frac{1}{v^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$\nabla^2 \mathbf{E} - \frac{1}{v^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

where  $v^2 \equiv c^2 \frac{\mu_0 \epsilon_0}{\mu\epsilon} \equiv \frac{c^2}{n^2}$

Plane wave solutions to wave equation :

$$\mathbf{B}(\mathbf{r}, t) = \Re(\mathbf{B}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}) \quad \mathbf{E}(\mathbf{r}, t) = \Re(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t})$$

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Analysis of Maxwell's equations without sources -- continued:

Plane wave solutions to wave equation :

$$\mathbf{B}(\mathbf{r}, t) = \Re(\mathbf{B}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}) \quad \mathbf{E}(\mathbf{r}, t) = \Re(\mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t})$$

$$|\mathbf{k}|^2 = \left( \frac{\omega}{v} \right)^2 = \left( \frac{n\omega}{c} \right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

Note:  $\epsilon, \mu, n, k$  can all be complex; for the moment we will assume that they are all real (no dissipation).

Note that  $\mathbf{E}_0$  and  $\mathbf{B}_0$  are not independent;

from Faraday's law :  $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

$$\Rightarrow \mathbf{B}_0 = \frac{\mathbf{k} \times \mathbf{E}_0}{\omega} = \frac{n\hat{\mathbf{k}} \times \mathbf{E}_0}{c}$$

also note:  $\hat{\mathbf{k}} \cdot \mathbf{E}_0 = 0$  and  $\hat{\mathbf{k}} \cdot \mathbf{B}_0 = 0$

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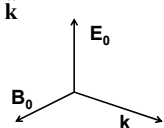
Analysis of Maxwell's equations without sources -- continued:  
 Summary of plane electromagnetic waves:

$$\mathbf{B}(\mathbf{r}, t) = \Re\left(\frac{n\hat{\mathbf{k}} \times \mathbf{E}_0}{c} e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}\right) \quad \mathbf{E}(\mathbf{r}, t) = \Re(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t})$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} \quad \text{and } \hat{\mathbf{k}} \cdot \mathbf{E}_0 = 0$$

Poynting vector and energy density:

$$\langle \mathbf{S} \rangle_{avg} = \frac{n|\mathbf{E}_0|^2}{2\mu c} \hat{\mathbf{k}} = \frac{1}{2} \sqrt{\frac{\epsilon}{\mu}} |\mathbf{E}_0|^2 \hat{\mathbf{k}}$$

$$\langle u \rangle_{avg} = \frac{1}{2} \epsilon |\mathbf{E}_0|^2$$


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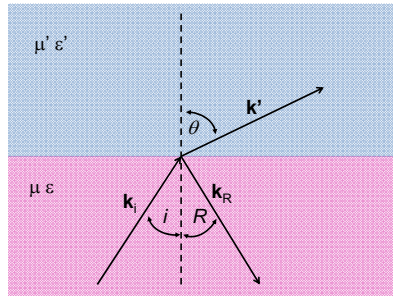
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Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



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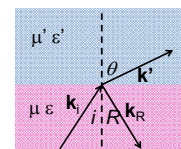
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Reflection and refraction -- continued



In medium  $\mu' \epsilon'$ :

$$\mathbf{E}'(\mathbf{r}, t) = \Re(\mathbf{E}'_0 e^{i\mathbf{k}'\cdot\mathbf{r} - ct})$$

$$\mathbf{B}'(\mathbf{r}, t) = \frac{n'}{c} \hat{\mathbf{k}}' \times \mathbf{E}'(\mathbf{r}, t) = \sqrt{\mu' \epsilon'} \hat{\mathbf{k}}' \times \mathbf{E}'(\mathbf{r}, t)$$

In medium  $\mu \epsilon$ :

$$\mathbf{E}_i(\mathbf{r}, t) = \Re(\mathbf{E}_{0i} e^{i\mathbf{k}_i\cdot\mathbf{r} - ct})$$

$$\mathbf{B}_i(\mathbf{r}, t) = \frac{n}{c} \hat{\mathbf{k}}_i \times \mathbf{E}_i(\mathbf{r}, t) = \sqrt{\mu \epsilon} \hat{\mathbf{k}}_i \times \mathbf{E}_i(\mathbf{r}, t)$$

$$\mathbf{E}_R(\mathbf{r}, t) = \Re(\mathbf{E}_{0R} e^{i\mathbf{k}_R\cdot\mathbf{r} - ct})$$

$$\mathbf{B}_R(\mathbf{r}, t) = \frac{n}{c} \hat{\mathbf{k}}_R \times \mathbf{E}_R(\mathbf{r}, t) = \sqrt{\mu \epsilon} \hat{\mathbf{k}}_R \times \mathbf{E}_R(\mathbf{r}, t)$$

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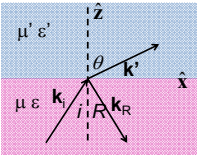
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Reflection and refraction -- continued



Snell's law -- matching phase factors at boundary plane:

$$\mathbf{r} = x\hat{x} + y\hat{y} + 0\hat{z}$$

$$\hat{\mathbf{k}}' \cdot \mathbf{r} = x \sin \theta$$

$$\hat{\mathbf{k}}_i \cdot \mathbf{r} = x \sin i = \hat{\mathbf{k}}_R \cdot \mathbf{r} \Rightarrow i = R$$

$$n' \hat{\mathbf{k}}' \cdot \mathbf{r} = n \hat{\mathbf{k}}_i \cdot \mathbf{r} \Rightarrow n' x \sin \theta = n x \sin i$$

Snell's law:  $n' \sin \theta = n \sin i$

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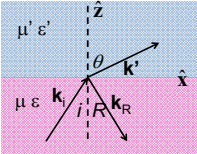
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Reflection and refraction -- continued



Continuity equations at boundary with no sources:

$$\nabla \cdot \mathbf{D} = 0 \quad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0 \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

Matching field amplitudes at boundary plane:

$\mathbf{D} \cdot \hat{\mathbf{z}}, \mathbf{B} \cdot \hat{\mathbf{z}}$  continuous

$\mathbf{H} \times \hat{\mathbf{z}}, \mathbf{E} \times \hat{\mathbf{z}}$  continuous

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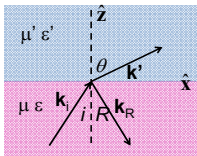
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Reflection and refraction -- continued



Matching field amplitudes at boundary plane:

$\mathbf{D} \cdot \hat{\mathbf{z}}$  continuous:

$$\epsilon (\mathbf{E}_{0i} + \mathbf{E}_{0R}) \cdot \hat{\mathbf{z}} = \epsilon' \mathbf{E}'_0 \cdot \hat{\mathbf{z}}$$

$\mathbf{B} \cdot \hat{\mathbf{z}}$  continuous:

$$n (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \cdot \hat{\mathbf{z}} = n' \hat{\mathbf{k}}' \times \mathbf{E}'_0 \cdot \hat{\mathbf{z}}$$

$\mathbf{E} \times \hat{\mathbf{z}}$  continuous:

$$(\mathbf{E}_{0i} + \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \mathbf{E}'_0 \times \hat{\mathbf{z}}$$

$\mathbf{H} \times \hat{\mathbf{z}}$  continuous:

$$\frac{n}{\mu} (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \frac{n'}{\mu'} \hat{\mathbf{k}}' \times \mathbf{E}'_0 \times \hat{\mathbf{z}}$$

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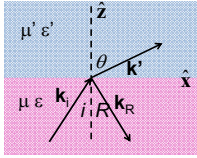
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Reflection and refraction -- continued



s-polarization – **E** field “polarized” perpendicular to plane of incidence

$\mathbf{E} \times \hat{\mathbf{z}}$  continuous:  
 $(\mathbf{E}_{0i} + \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \mathbf{E}'_0 \times \hat{\mathbf{z}}$

$\mathbf{H} \times \hat{\mathbf{z}}$  continuous:  
 $\frac{n}{\mu} (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \frac{n'}{\mu'} \hat{\mathbf{k}}' \times \mathbf{E}'_0 \times \hat{\mathbf{z}}$

$$\frac{E_{0R}}{E_{0i}} = \frac{n \cos i - \frac{\mu}{\mu'} n' \cos \theta}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta} \quad \frac{E'_0}{E_{0i}} = \frac{2n \cos i}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta}$$

Note that:  $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

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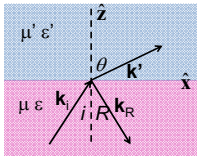
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Reflection and refraction -- continued



p-polarization – **E** field “polarized” parallel to plane of incidence

$\mathbf{D} \cdot \hat{\mathbf{z}}$  continuous:  
 $\varepsilon (\mathbf{E}_{0i} + \mathbf{E}_{0R}) \cdot \hat{\mathbf{z}} = \varepsilon' \mathbf{E}'_0 \cdot \hat{\mathbf{z}}$

$\mathbf{H} \times \hat{\mathbf{z}}$  continuous:  
 $\frac{n}{\mu} (\hat{\mathbf{k}}_i \times \mathbf{E}_{0i} + \hat{\mathbf{k}}_R \times \mathbf{E}_{0R}) \times \hat{\mathbf{z}} = \frac{n'}{\mu'} \hat{\mathbf{k}}' \times \mathbf{E}'_0 \times \hat{\mathbf{z}}$

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos \theta}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta} \quad \frac{E'_0}{E_{0i}} = \frac{2n \cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta}$$

Note that:  $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

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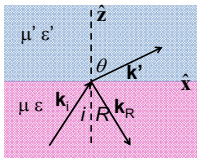
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Reflection and refraction -- continued



Reflectance, transmittance:

$$R = \frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E_{0R}}{E_{0i}} \right|^2 \quad T = \frac{\mathbf{S}' \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n' \mu \cos \theta}{n \mu' \cos i}$$

Note that  $R + T = 1$

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For s-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{n \cos i - \frac{\mu}{\mu'} n' \cos \theta}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta} \quad \frac{E'_{0t}}{E_{0i}} = \frac{2n \cos i}{n \cos i + \frac{\mu}{\mu'} n' \cos \theta}$$

Note that:  $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

For p-polarization

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos \theta}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta} \quad \frac{E'_{0t}}{E_{0i}} = \frac{2n \cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos \theta}$$

Note that:  $n' \cos \theta = \sqrt{n'^2 - n^2 \sin^2 i}$

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Special case: normal incidence ( $i=0, \theta=0$ )

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' - n}{\frac{\mu}{\mu'} n' + n} \quad \frac{E'_{0t}}{E_{0i}} = \frac{2n}{\frac{\mu}{\mu'} n' + n}$$

Reflectance, transmittance:

$$R = \left| \frac{E_{0R}}{E_{0i}} \right|^2 = \left| \frac{\frac{\mu}{\mu'} n' - n}{\frac{\mu}{\mu'} n' + n} \right|^2$$

$$T = \left| \frac{E'_{0t}}{E_{0i}} \right|^2 \frac{n' \mu'}{n \mu} = \left| \frac{2n}{\frac{\mu}{\mu'} n' + n} \right|^2 \frac{n' \mu'}{n \mu}$$

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Multilayer dielectrics (Problem #7.2)

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Extension of analysis to anisotropic media --

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Consider the problem of determining the reflectance from an anisotropic medium with isotropic permeability  $\mu_0$  and anisotropic permittivity  $\epsilon_0 \boldsymbol{\kappa}$  where:

$$\boldsymbol{\kappa} \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}$$

By assumption, the wave vector in the medium is confined to the x-y plane and will be denoted by  $\mathbf{k}_t \equiv \frac{\omega}{c} (n_x \hat{\mathbf{x}} + n_y \hat{\mathbf{y}})$ , where  $n_x$  and  $n_y$  are to be determined.

The electric field inside the medium is given by:

$$\mathbf{E} = (E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{z}}) e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}$$

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Inside the anisotropic medium, Maxwell's equations are:

$$\nabla \cdot \mathbf{H} = 0 \quad \nabla \cdot \boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} - i\omega \mu_0 \mathbf{H} = 0 \quad \nabla \times \mathbf{H} + i\omega \epsilon_0 \boldsymbol{\kappa} \cdot \mathbf{E} = 0$$

After some algebra, the equation for  $\mathbf{E}$  is:

$$\begin{pmatrix} \kappa_{xx} - n_y^2 & n_x n_y & 0 \\ n_x n_y & \kappa_{yy} - n_x^2 & 0 \\ 0 & 0 & \kappa_{zz} - (n_x^2 + n_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0.$$

From  $\mathbf{E}$ ,  $\mathbf{H}$  can be determined from

$$\mathbf{H} = \frac{1}{\mu_0 c} \{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) + (E_y n_x - E_x n_y) \hat{\mathbf{z}} \} e^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}$$

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The fields for the incident and reflected waves are the same as for the isotropic case.

$$\mathbf{k}_i = \frac{\omega}{c} (\sin i \hat{\mathbf{x}} + \cos i \hat{\mathbf{y}}),$$

$$\mathbf{k}_R = \frac{\omega}{c} (\sin i \hat{\mathbf{x}} - \cos i \hat{\mathbf{y}}).$$

Note that, consistent with Snell's law:  $n_x = \sin i$   
Continuity conditions at the  $y=0$  plane must be applied for the following fields:

$\mathbf{H}(x, 0, z, t)$ ,  $E_x(x, 0, z, t)$ ,  $E_z(x, 0, z, t)$ , and  $D_y(x, 0, z, t)$ .

There will be two different solutions, depending of the polarization of the incident field.

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Solution for s-polarization

$$E_x = E_y = 0 \quad \Rightarrow \quad n_y^2 = \kappa_{zz} - n_x^2$$

$$\mathbf{E} = E_z \hat{\mathbf{z}} e^{i\omega(n_x x + n_y y) - i\omega t} \quad \mathbf{H} = \frac{1}{\mu_0 c} \{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) \} e^{i\omega(n_x x + n_y y) - i\omega t}$$

$E_z$  must be determined from the continuity conditions:

$$E_0 + E_0'' = E_z \quad (E_0 - E_0'') \cos i = E_z n_y \quad (E_0 + E_0'') \sin i = E_z n_x$$

$$\frac{E_0''}{E_0} = \frac{\cos i - n_y}{\cos i + n_y}$$

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Solution for p-polarization

$$E_z = 0 \quad \Rightarrow \quad n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}} (\kappa_{yy} - n_x^2).$$

$$\mathbf{E} = E_x \left( \hat{\mathbf{x}} - \frac{\kappa_{xx} n_x}{\kappa_{yy} n_y} \hat{\mathbf{y}} \right) e^{i\omega(n_x x + n_y y) - i\omega t}$$

$$\mathbf{H} = -\frac{E_x}{\mu_0 c} \frac{\kappa_{xx}}{n_y} \hat{\mathbf{z}} e^{i\omega(n_x x + n_y y) - i\omega t}$$

$E_x$  must be determined from the continuity conditions:

$$(E_0 - E_0'') \cos i = E_x \quad (E_0 + E_0'') = \frac{\kappa_{xx}}{n_y} E_x \quad (E_0 + E_0'') \sin i = \frac{\kappa_{xx} n_x}{n_y} E_x$$

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx} \cos i - n_y}{\kappa_{xx} \cos i + n_y}$$

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Extension of analysis to complex dielectric functions

For simplicity assume that  $\mu = \mu_0$

Suppose the dielectric function is complex :

$$\epsilon = \epsilon_R + i\epsilon_I \quad \frac{\epsilon}{\epsilon_0} = (n_R + in_I)^2 \equiv \alpha + i\beta$$

$$n_R = \left( \frac{\sqrt{\alpha^2 + \beta^2} + \alpha}{2} \right)^{1/2} \quad n_I = \left( \frac{\sqrt{\alpha^2 + \beta^2} - \alpha}{2} \right)^{1/2}$$

$$\mathbf{E}(\mathbf{r}, t) = \Re(\mathbf{E}_0 e^{i\omega(\mathbf{\hat{k}}\cdot\mathbf{r} - ct)}) = \Re(\mathbf{E}_0 e^{i\omega(n_R \mathbf{\hat{k}}\cdot\mathbf{r} - ct)}) e^{-\frac{\omega}{c} n_I \mathbf{\hat{k}}\cdot\mathbf{r}}$$

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