

Various forms of Ampere's law :

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{J}(\mathbf{r})$$

$$\text{Vector potential: } \mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

$$\text{For Coulomb gauge: } \nabla \cdot \mathbf{A}(\mathbf{r}) = 0$$

$$\Rightarrow \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r})$$

For confined current density :

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d^3 r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

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Other examples of current density sources:

Quantum mechanical expression for current density

for a particle of mass M and charge e and of probability amplitude $\Psi(\mathbf{r})$:

$$\mathbf{J}(\mathbf{r}) = -\frac{e\hbar}{2Mi} (\Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r}) - \Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r}))$$

For an electron in a spherical potential (such as in an atom):

$$\Psi(\mathbf{r}) = \Psi_{nlm}(\mathbf{r}) = R_{nl}(r)Y_{lm}(\hat{\mathbf{r}})$$

$$\begin{aligned} \mathbf{J}(\mathbf{r}) &= \frac{e\hbar}{2Mi} |R_{nl}(r)|^2 \frac{1}{r \sin \theta} \left(Y_{lm}^*(\hat{\mathbf{r}}) \frac{\partial Y_{lm}(\hat{\mathbf{r}})}{\partial \phi} - Y_{lm}(\hat{\mathbf{r}}) \frac{\partial Y_{lm}^*(\hat{\mathbf{r}})}{\partial \phi} \right) \hat{\phi} \\ &= \frac{e\hbar}{M} \frac{m}{r \sin \theta} |\Psi_{nlm}(\mathbf{r})|^2 \hat{\phi} \end{aligned}$$

$$\text{Note that: } \hat{\phi} = -\sin \theta \hat{x} + \cos \theta \hat{y} = \frac{\hat{z} \times \mathbf{r}}{r \sin \theta}$$

$$\mathbf{J}(\mathbf{r}) = \frac{e\hbar}{M} \frac{m}{r^2 \sin^2 \theta} |\Psi_{nlm}(\mathbf{r})|^2 (\hat{z} \times \mathbf{r})$$

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