PHY 752 Solid State Physics
11-11:50 AM MWF Olin 107

Plan for Lecture 3:

Reading: Chapter 1 & 2 in MPM;

Continued brief introduction to group theory

1. Group multiplication tables
2. Representations of groups
3. The "great" orthogonality theorem

PHY 752 Solid State Physics

MWF 11-11:50 AM	OPL 107	http://www.wfu.edu/~natalie/s15phy752/

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Course schedule for Spring 2015

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	MPM Reading	Topic	Assign.	Due date
1	Mon: 01/12/2015	Chap. 1 & 2	Crystal structures	#1	01/23/2015
2	Wed: 01/14/2015	Chap. 1 & 2	Some group theory	#2	01/23/2015
	Fri: 01/16/2015	No class	NAWH out of town		
	Mon: 01/19/2015	No class	MLK Holiday		
3	Wed: 01/21/2015	Chap. 1 & 2	Some group theory	#3	01/23/2015

WFU Physics Colloquium

TITLE: Quantum Poetics: The Word and Its Earthwork

SPEAKER: Dr. Amy Catanzano,

Department of English
Wake Forest University

TIME: Wednesday January 21, 2015 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

Poetry and science are ordinarily considered to be different disciplines with distinct goals, methods, and questions. I am part of a contemporary and historical lineage of poets who explore the intersections of poetry and science. My work focuses on poetry in relation to relativity, quantum mechanics, and string theory. My methodology follows in the tradition of

Short digression on abstract group theory

What is group theory ?

A group is a collection of "elements" – $A, B, C, \ldots$ and a "multiplication" process. The abstract multiplication $(\cdot)$ pairs two group elements, and associates the "result" with a third element. (For example $(A \cdot B = C)$.) The elements and the multiplication process must have the following properties.

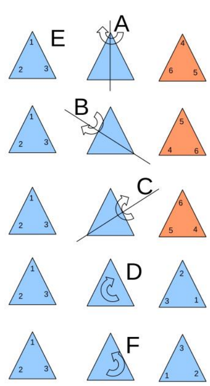
1. The collection of elements is closed under multiplication. That is, if elements A and B are in the group and $A \cdot B = C$, element C must be in the group.
2. One of the members of the group is a "unit element" (E). That is, for any element A of the group, $A \cdot E = E \cdot A = A$.
3. For each element A of the group, there is another element A^{-1} which is its "inverse". That is $A \cdot A^{-1} = A^{-1} \cdot A = E$.
4. The multiplication process is "associative". That is for sequential mulplication of group elements A, B, and C, $(A \cdot B) \cdot C = A \cdot (B \cdot C)$.

Example of a 6-member group *E,A,B,C,D,F,G*

Group multiplication table

Group of order 6

	E	A	B	C	D	F
E	E	A	B	C	D	F
A	A	E	D	F	B	C
B	B	F	E	D	C	A
C	C	D	F	E	A	B
D	D	C	A	B	F	E
F	F	B	C	A	E	D



	E	A	B	C	D	F
E	E	A	B	C	D	F
A	A	E	D	F	B	C
B	B	F	E	D	C	A
C	C	D	F	E	A	B
D	D	C	A	B	F	E
F	F	B	C	A	E	D

Check on group properties:
1. Closed; multiplication table uniquely generates group members.
2. Unit element included.
3. Each element has inverse.
4. Multiplication process is associative.

Definitions

Subgroup: members of larger group which have the property of a group

Class: members of a group which are generated by the construction

$$\mathcal{C} = X_i^{-1} Y X_i \quad \text{where } X_i \text{ and } Y \text{ are group elements}$$

Group theory – some comments

- The elements of the group may be abstract; in general, we will use them to describe symmetry properties of our system

Representations of a group

A representation of a group is a set of matrices (one for each group element) -- $\Gamma(A), \Gamma(B)$... that satisfies the multiplication table of the group. The dimension of the matrices is called the dimension of the representation.

Example:

	E	A	B	C	D	F
E	E	A	B	C	D	F
A	A	E	D	F	B	C
B	B	F	E	D	C	A
C	C	D	F	E	A	B
D	D	C	A	B	F	E
F	F	B	C	A	E	D

Note that the one-dimensional "identical representation"

$\Gamma^1(A)=\Gamma^1(B)=\Gamma^1(C)=\Gamma^1(D)=\Gamma^1(E)=\Gamma^1(F)=1$ is always possible

Another one-dimensional representation is

$$\Gamma^2(A)=\Gamma^2(B)=\Gamma^2(C)=-1$$

$$\Gamma^2(E)=\Gamma^2(D)=\Gamma^2(F)=1$$

Example:

	E	A	B	C	D	F
E	E	A	B	C	D	F
A	A	E	D	F	B	C
B	B	F	E	D	C	A
C	C	D	F	E	A	B
D	D	C	A	B	F	E
F	F	B	C	A	E	D

A two-dimensional representation is

$$\Gamma^3(E)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad \Gamma^3(A)=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\Gamma^3(B)=\begin{pmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix} \quad \Gamma^3(C)=\begin{pmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$$

$$\Gamma^3(D)=\begin{pmatrix}-\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix} \quad \Gamma^3(F)=\begin{pmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix}$$

What about 3 or 4 dimensional representations for this group?

$$\Gamma(E)=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\quad \Gamma(A)=\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}\quad \Gamma(B)=\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&\frac{\sqrt{3}}{2}\\0&\frac{\sqrt{3}}{2}&\frac{1}{2}\end{pmatrix}$$

$$\Gamma(C)=\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\0&-\frac{\sqrt{3}}{2}&\frac{1}{2}\end{pmatrix}\quad \Gamma(D)=\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&\frac{\sqrt{3}}{2}\\0&-\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}\quad \Gamma(F)=\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&-\frac{\sqrt{3}}{2}\\0&\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$$

The only “irreducible” representations for this group are 2 one-dimensional and 1 two-dimensional

Comment about representation matrices

A representation is not fundamentally altered by a similarity transformation

$$\Gamma'(A)=S^{-1}\Gamma(A)S$$

Check:

$$\Gamma'(AB)=S^{-1}\Gamma(AB)S=S^{-1}\Gamma(A)\Gamma(B)S$$
$$=S^{-1}\Gamma(A)SS^{-1}\Gamma(B)S$$
$$=\Gamma'(A)\Gamma'(B)$$

- Typically, unitary matrices are chosen for representations
- Typically representations are reduced to block diagonal form and the irreducible blocks are considered in the representation theory

The great orthogonality theorem

Notation: $h\equiv$ order of the group

$R\equiv$ element of the group

$\Gamma^i(R)_{\alpha\beta}\equiv$ ith representation of R

${}_{\alpha\beta}$ denote matrix indices

$l_i\equiv$ dimension of the representation

$$\sum_R\left(\Gamma^i(R)_{\mu\nu}\right)^*\Gamma^j(R)_{\alpha\beta}=\frac{h}{l_i}\delta_{ij}\delta_{\mu\alpha}\delta_{\nu\beta}$$

Great orthogonality theorem continued

$$\sum_R \left(\Gamma^i(R)_{\mu\nu}\right)^* \Gamma^j(R)_{\alpha\beta} = \frac{h}{l_i}\delta_{ij}\delta_{\mu\alpha}\delta_{\nu\beta}$$

Analysis shows that

$$\sum_i l_i^2 = h$$

Simplified analysis in terms of the "characters" of the representations

$$\chi^j(R) \equiv \sum_{\mu=1}^{l_j} \Gamma^j(R)_{\mu\mu}$$

Character orthogonality theorem

$$\sum_R \left(\chi^i(R)\right)^* \chi^j(R) = h\delta_{ij}$$

Note that all members of a class have the same character for any given representation *i.*