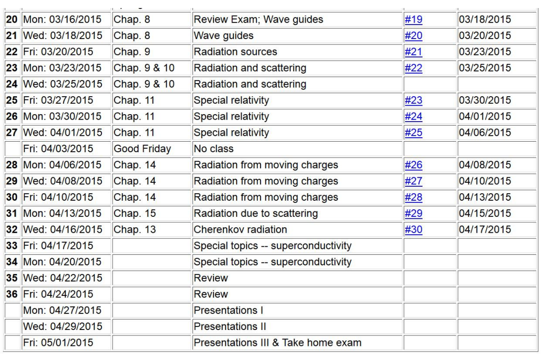
PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 35:

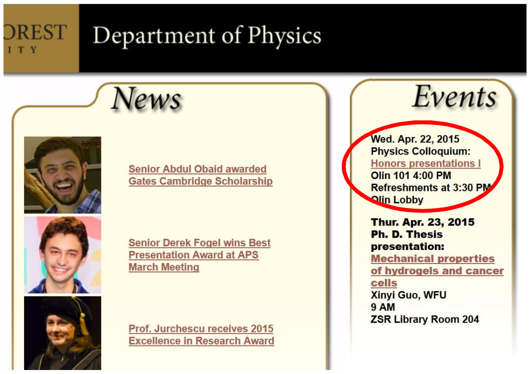
Review Part I:

Problem solving examples

(mainly from Chapters 7 & 9)



20	Mon: 03/16/2015	Chap. 8	Review Exam; Wave guides	#19	03/18/2015
21	Wed: 03/18/2015	Chap. 8	Wave guides	#20	03/20/2015
22	Fri: 03/20/2015	Chap. 9	Radiation sources	#21	03/23/2015
23	Mon: 03/23/2015	Chap. 9 & 10	Radiation and scattering	#22	03/25/2015
24	Wed: 03/25/2015	Chap. 9 & 10	Radiation and scattering		
25	Fri: 03/27/2015	Chap. 11	Special relativity	#23	03/30/2015
26	Mon: 03/30/2015	Chap. 11	Special relativity	#24	04/01/2015
27	Wed: 04/01/2015	Chap. 11	Special relativity	#25	04/06/2015
	Fri: 04/03/2015	Good Friday	No class		
28	Mon: 04/06/2015	Chap. 14	Radiation from moving charges	#26	04/08/2015
29	Wed: 04/08/2015	Chap. 14	Radiation from moving charges	#27	04/10/2015
30	Fri: 04/10/2015	Chap. 14	Radiation from moving charges	#28	04/13/2015
31	Mon: 04/13/2015	Chap. 15	Radiation due to scattering	#29	04/15/2015
32	Wed: 04/16/2015	Chap. 13	Cherenkov radiation	#30	04/17/2015
33	Fri: 04/17/2015		Special topics -- superconductivity		
34	Mon: 04/20/2015		Special topics -- superconductivity		
35	Wed: 04/22/2015		Review		
36	Fri: 04/24/2015		Review		
	Mon: 04/27/2015		Presentations I		
	Wed: 04/29/2015		Presentations II		
	Fri: 05/01/2015		Presentations III & Take home exam		



Department of Physics

News

Senior Abdul Obaid awarded Gates Cambridge Scholarship

Senior Derek Fogel wins Best Presentation Award at APS March Meeting

Prof. Jurchescu receives 2015 Excellence in Research Award

Events

Wed. Apr. 22, 2015
Physics Colloquium:
Honors presentations I
Olin 101 4:00 PM
Refreshments at 3:30 PM
Olin Lobby

Thur. Apr. 23, 2015
Ph. D. Thesis presentation:
Mechanical properties of hydrogels and cancer cells
Xinyi Guo, WFU
9 AM
ZSR Library Room 204

WFU Physics Colloquium

TITLE: Physics Honors Theses Presentations I

SPEAKERS: Five Undergraduate Thesis Students

TIME: Wednesday April 22, 2015 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

PROGRAM

- Jay Einhorn & Andy Lundeen -- “Effects of Conformally invariant Quantum Fields on Future Singularities”
- Erica Freund -- “Long-term Storage Conditions of Nanoparticle Encapsulated Orlistat to Maintain Cytotoxicity"
- Billy Nicholson -- “Quantifying the Stability of Acridness to Putative Ribosomal DNA G-Quadruplexes”
- Kelli Simms --"TBA"

Schedule for PHY 712 Presentations

Monday 4/27/2015

	Presenter	Topic
9:00 -9:25 AM	Larry Rush	"Superconductivity"
9:25-9:50 AM	Junwei Xu	"Electrodynamics in alternating current electroluminescent device"

Wednesday 4/29/2015

	Presenter	Topic
9:00 -9:25 AM	Jason Howard	"Ewald summations with anisotropic dielectric screening"
9:25-9:50 AM	Eric Chapman	The Physics of MRI

Friday 5/1/2015

	Presenter	Topic
9:00 -9:25 AM	Lauren Nelson	Solar Cells
9:25-9:50 AM	Hysun Lee	Surface Plasmon and It's application

Review topic – analytic properties of dielectric function
Material from Chapter 7 in Jackson

The displacement field **D** is related to the electric field **E**

$$\mathbf{D} = \varepsilon\mathbf{E} \quad = \varepsilon_0\mathbf{E} + \mathbf{P}$$

Dielectric function $\varepsilon(\omega) = \varepsilon_R(\omega) + i\varepsilon_I(\omega)$

can be shown to be analyic for $\omega \to z$ for $\Im(z) > 0$

Kramers-Kronig transform – for dielectric function:

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} - 1 = \frac{1}{\pi} P\int_{-\infty}^{\infty} d\omega' \frac{\varepsilon_I(\omega')}{\varepsilon_0}\frac{1}{\omega'-\omega}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = -\frac{1}{\pi} P\int_{-\infty}^{\infty} d\omega' \left(\frac{\varepsilon_R(\omega')}{\varepsilon_0} - 1\right)\frac{1}{\omega'-\omega}$$

with $\varepsilon_R(-\omega) = \varepsilon_R(\omega); \quad \varepsilon_I(-\omega) = -\varepsilon_I(\omega)$

Drude model dielectric function:

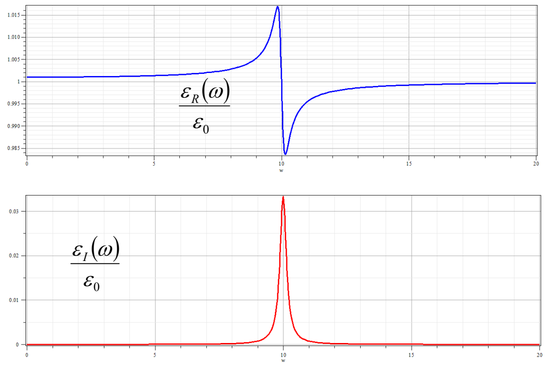
$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1+N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2-\omega^2-i\omega\gamma_i}$$

$$= \frac{\varepsilon_R(\omega)}{\varepsilon_0}+i\frac{\varepsilon_I(\omega)}{\varepsilon_0}$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} = 1+N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{\omega_i^2-\omega^2}{\left(\omega_i^2-\omega^2\right)^2+\omega^2\gamma_i^2}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{\omega\gamma_i}{\left(\omega_i^2-\omega^2\right)^2+\omega^2\gamma_i^2}$$

Drude model dielectric function:



Practical evaluation of Kramers-Kronig relation

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0}-1=\frac{1}{\pi}P\int_{-\infty}^{\infty}d\omega'\frac{\varepsilon_I(\omega')}{\varepsilon_0}\frac{1}{\omega'-\omega}$$

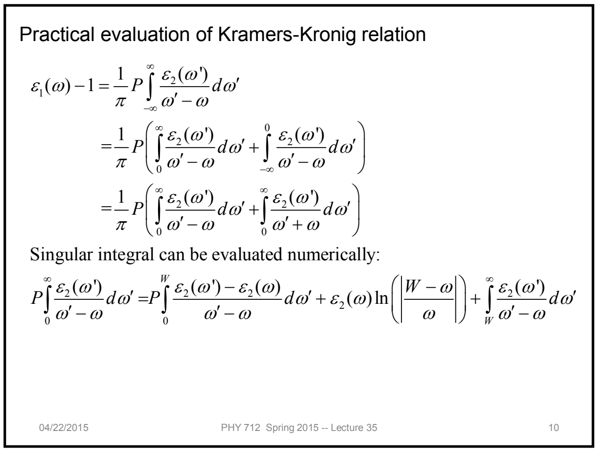
$$\frac{\varepsilon_I(\omega)}{\varepsilon_0}=-\frac{1}{\pi}P\int_{-\infty}^{\infty}d\omega'\left(\frac{\varepsilon_R(\omega')}{\varepsilon_0}-1\right)\frac{1}{\omega'-\omega}$$

with $\varepsilon_R(-\omega)=\varepsilon_R(\omega);\quad \varepsilon_I(-\omega)=-\varepsilon_I(\omega)$

Let $\varepsilon_1(\omega)=\frac{\varepsilon_R(\omega)}{\varepsilon_0}\qquad \varepsilon_2(\omega)=\frac{\varepsilon_I(\omega)}{\varepsilon_0}$

$$\varepsilon_1(\omega)-1=\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega'$$

$$\varepsilon_2(\omega)=-\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\varepsilon_1(\omega')-1}{\omega'-\omega}d\omega'$$



Practical evaluation of Kramers-Kronig relation

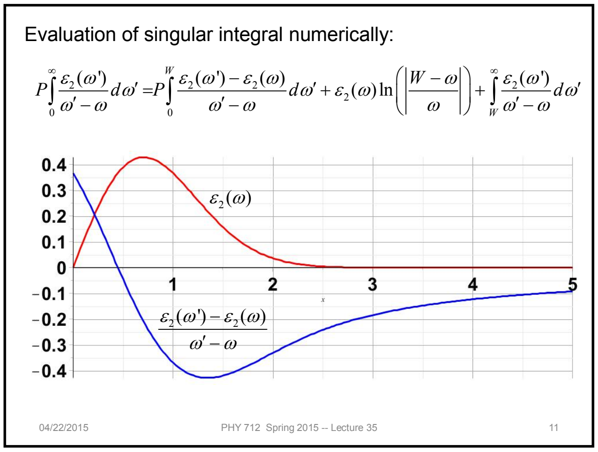
$$\varepsilon_1(\omega)-1=\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega'$$

$$=\frac{1}{\pi}P\left(\int_{0}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega'+\int_{-\infty}^{0}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega'\right)$$

$$=\frac{1}{\pi}P\left(\int_{0}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega'+\int_{0}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'+\omega}d\omega'\right)$$

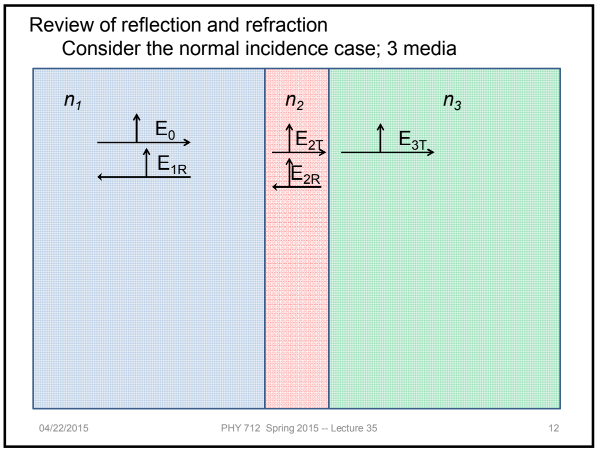
Singular integral can be evaluated numerically:

$$P\int_{0}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega' = P\int_{0}^{W}\frac{\varepsilon_2(\omega')-\varepsilon_2(\omega)}{\omega'-\omega}d\omega'+\varepsilon_2(\omega)\ln\left(\left|\frac{W-\omega}{\omega}\right|\right)+\int_{W}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega'$$



Evaluation of singular integral numerically:

$$P\int_{0}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega' = P\int_{0}^{W}\frac{\varepsilon_2(\omega')-\varepsilon_2(\omega)}{\omega'-\omega}d\omega'+\varepsilon_2(\omega)\ln\left(\left|\frac{W-\omega}{\omega}\right|\right)+\int_{W}^{\infty}\frac{\varepsilon_2(\omega')}{\omega'-\omega}d\omega'$$

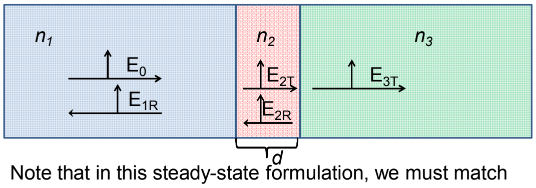


Review of reflection and refraction

Consider the normal incidence case; 3 media

Review of reflection and refraction -- continued

Consider the normal incidence case; 3 media



Note that in this steady-state formulation, we must match the tangential components of the E and H fields at each boundary

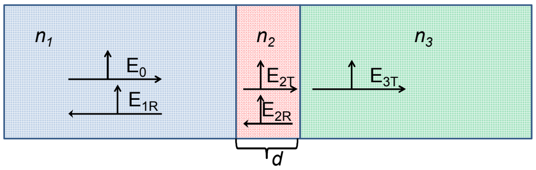
Each plane wave component has the form:

$$\mathbf{E}_j(\mathbf{r},t)=E_j\hat{\mathbf{y}}e^{i(\omega/c)(n_jx-ct)}$$

$$\mathbf{H}_j(\mathbf{r},t)=\frac{n_jE_j}{\mu_jc}\hat{\mathbf{z}}e^{i(\omega/c)(n_jx-ct)}=\frac{n_jE_j}{\mu_0c}\hat{\mathbf{z}}e^{i(\omega/c)(n_jx-ct)} \quad \text{in our case}$$

Review of reflection and refraction -- continued

Consider the normal incidence case; 3 media



Matching equations:

$$E_0+E_{1R}=E_2+E_{2R}$$

$$\frac{n_1}{n_2}\left(E_0-E_{1R}\right)=E_2-E_{2R}$$

$$E_2e^{i\theta}+E_{2R}e^{-i\theta}=E_3$$

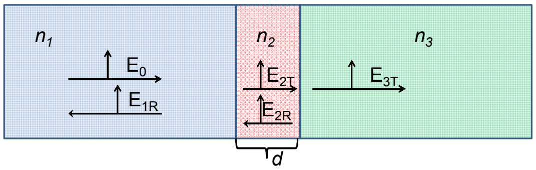
$$\frac{n_2}{n_3}\left(E_2e^{i\theta}-E_{2R}e^{-i\theta}\right)=E_3$$

Here:

$$\theta\equiv\frac{n_2\omega d}{c}$$

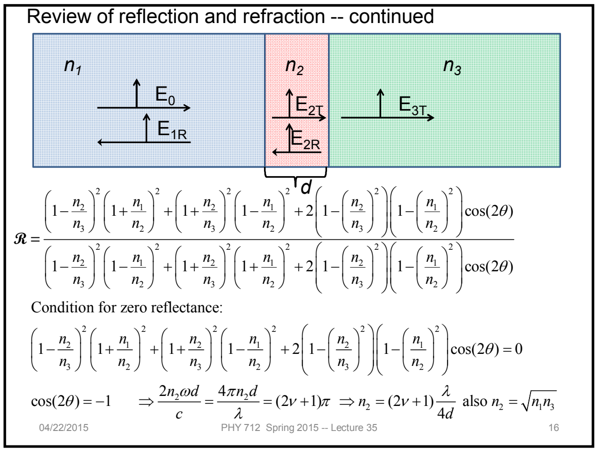
Review of reflection and refraction -- continued

Consider the normal incidence case; 3 media



After some algebra:

$$\mathcal{R}=\frac{\left(1-\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2+\left(1+\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2+2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}{\left(1-\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2+\left(1+\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2+2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}$$



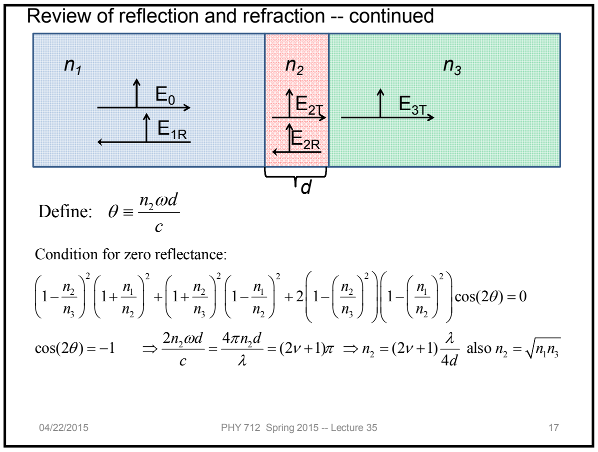
Review of reflection and refraction -- continued

$$\mathcal{R}=\frac{\left(1-\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2+\left(1+\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2+2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}{\left(1-\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2+\left(1+\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2+2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}$$

Condition for zero reflectance:

$$\left(1-\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2+\left(1+\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2+2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)=0$$

$$\cos(2\theta)=-1 \qquad \Rightarrow \frac{2n_2\omega d}{c}=\frac{4\pi n_2 d}{\lambda}=(2\nu+1)\pi \;\Rightarrow n_2=(2\nu+1)\frac{\lambda}{4d} \text{ also } n_2=\sqrt{n_1 n_3}$$



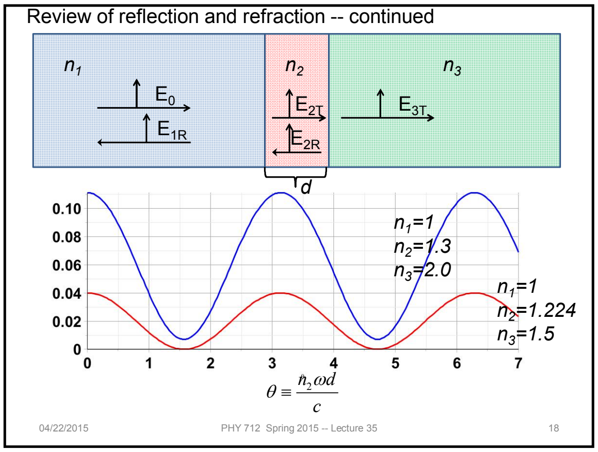
Review of reflection and refraction -- continued

Define: $\theta \equiv \frac{n_2\omega d}{c}$

Condition for zero reflectance:

$$\left(1-\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2+\left(1+\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2+2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)=0$$

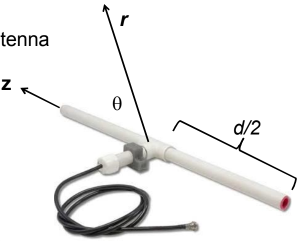
$$\cos(2\theta)=-1 \qquad \Rightarrow \frac{2n_2\omega d}{c}=\frac{4\pi n_2 d}{\lambda}=(2\nu+1)\pi \;\Rightarrow n_2=(2\nu+1)\frac{\lambda}{4d} \text{ also } n_2=\sqrt{n_1 n_3}$$



Review of reflection and refraction -- continued

Review of radiation from antenna

Linear center-fed antenna



$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx$$

$$\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r'\, e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\,\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{J}}(\mathbf{r}',\omega) = I_0 \sin\left(\frac{kd}{2} - k\,|\,z\,|\right)\delta(x)\delta(y)\hat{\mathbf{z}}$$

Linear center-fed antenna continued

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx \hat{\mathbf{z}}\frac{\mu_0 I_0}{4\pi}\frac{e^{ikr}}{r}\int_{-d/2}^{d/1} dz'\, e^{-ik\cos(\theta)z'}\ \sin\left(\frac{kd}{2} - k\,|\,z'\,|\right)$$

$$= \hat{\mathbf{z}}\frac{\mu_0 I_0}{2\pi}\frac{e^{ikr}}{r}\left(\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin^2\theta}\right)$$

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^{\,2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{8\pi^2}\left|\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta}\right|^2$$

Linear center-fed antenna continued

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^{\,2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{8\pi^2}\left|\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta}\right|^2$$

$$\text{for}\quad kd = \pi: \quad \frac{dP}{d\Omega} = I_0^{\,2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{8\pi^2}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$\text{for}\quad kd = 2\pi: \quad \frac{dP}{d\Omega} = I_0^{\,2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{4}{8\pi^2}\frac{\cos^4\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

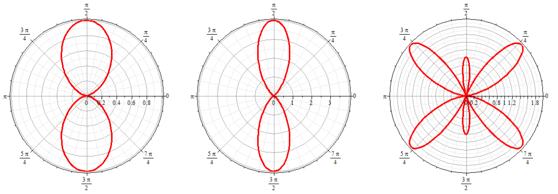
Linear center-fed antenna continued

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$

Radiation patterns $kd=m\pi$:

m=1 *m=2* *m=3*



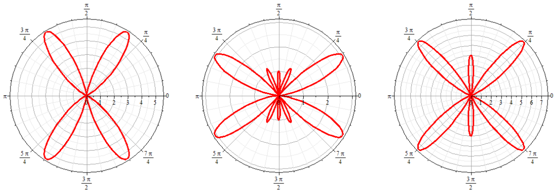
Linear center-fed antenna continued

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$

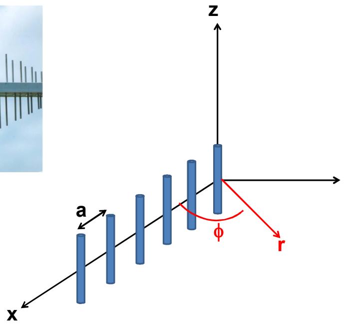
Radiation patterns $kd=m\pi$:

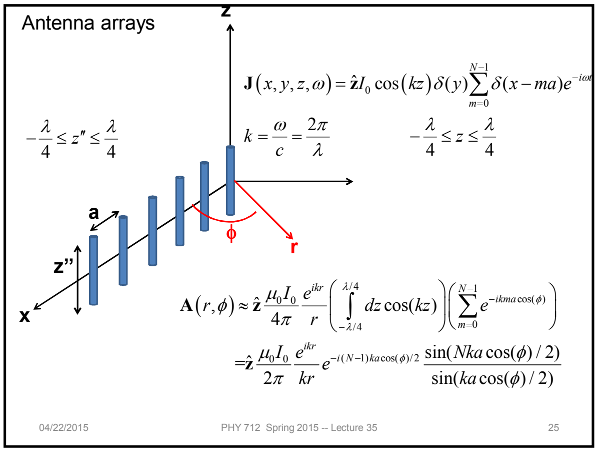
m=4 *m=5* *m=6*



Antenna arrays

http://www.tennadyne.com/company.htm





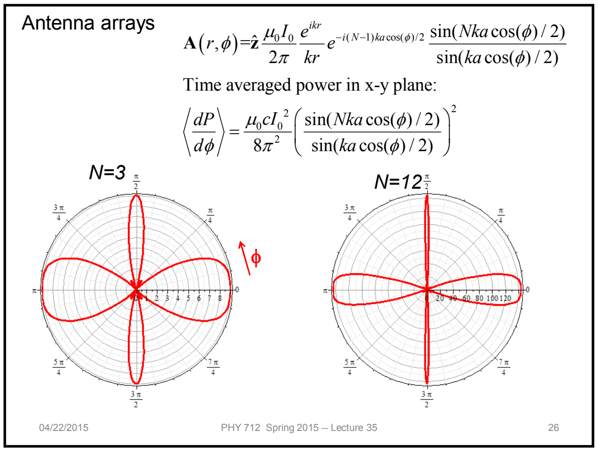
Antenna arrays

$$\mathbf{J}\left(x,y,z,\omega\right)=\hat{\mathbf{z}}I_0\cos\left(kz\right)\delta(y)\sum_{m=0}^{N-1}\delta(x-ma)e^{-i\omega t}$$

$$-\frac{\lambda}{4}\leq z''\leq\frac{\lambda}{4} \qquad k=\frac{\omega}{c}=\frac{2\pi}{\lambda} \qquad -\frac{\lambda}{4}\leq z\leq\frac{\lambda}{4}$$

$$\mathbf{A}\left(r,\phi\right)\approx\hat{\mathbf{z}}\frac{\mu_0 I_0}{4\pi}\frac{e^{ikr}}{r}\left(\int_{-\lambda/4}^{\lambda/4}dz\cos(kz)\right)\left(\sum_{m=0}^{N-1}e^{-ikma\cos(\phi)}\right)$$

$$=\hat{\mathbf{z}}\frac{\mu_0 I_0}{2\pi}\frac{e^{ikr}}{kr}e^{-i(N-1)ka\cos(\phi)/2}\frac{\sin(Nka\cos(\phi)/2)}{\sin(ka\cos(\phi)/2)}$$



Antenna arrays

$$\mathbf{A}\left(r,\phi\right)=\hat{\mathbf{z}}\frac{\mu_0 I_0}{2\pi}\frac{e^{ikr}}{kr}e^{-i(N-1)ka\cos(\phi)/2}\frac{\sin(Nka\cos(\phi)/2)}{\sin(ka\cos(\phi)/2)}$$

Time averaged power in x-y plane:

$$\left\langle\frac{dP}{d\phi}\right\rangle=\frac{\mu_0 c I_0{}^2}{8\pi^2}\left(\frac{\sin(Nka\cos(\phi)/2)}{\sin(ka\cos(\phi)/2)}\right)^2$$

N=3

N=12