PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 33:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity

20	Mon: 03/16/2015	Chap. 8	Review Exam; Wave guides	#19	03/18/2015
21	Wed: 03/18/2015	Chap. 8	Wave guides	#20	03/20/2015
22	Fri: 03/20/2015	Chap. 9	Radiation sources	#21	03/23/2015
23	Mon: 03/23/2015	Chap. 9 & 10	Radiation and scattering	#22	03/25/2015
24	Wed: 03/25/2015	Chap. 9 & 10	Radiation and scattering		
25	Fri: 03/27/2015	Chap. 11	Special relativity	#23	03/30/2015
26	Mon: 03/30/2015	Chap. 11	Special relativity	#24	04/01/2015
27	Wed: 04/01/2015	Chap. 11	Special relativity	#25	04/06/2015
	Fri: 04/03/2015	Good Friday	No class		
28	Mon: 04/06/2015	Chap. 14	Radiation from moving charges	#26	04/08/2015
29	Wed: 04/08/2015	Chap. 14	Radiation from moving charges	#27	04/10/2015
30	Fri: 04/10/2015	Chap. 14	Radiation from moving charges	#28	04/13/2015
31	Mon: 04/13/2015	Chap. 15	Radiation due to scattering	#29	04/15/2015
32	Wed: 04/16/2015	Chap. 13	Cherenkov radiation	#30	04/17/2015
33	Fri: 04/17/2015		Special topics -- superconductivity		
34	Mon: 04/20/2015		Special topics -- superconductivity		
35	Wed: 04/22/2015		Review		
36	Fri: 04/24/2015		Review		
	Mon: 04/27/2015		Presentations I		
	Wed: 04/29/2015		Presentations II		
	Fri: 05/01/2015		Presentations III & Take home exam		

Special topic: Electromagnetic properties of superconductors

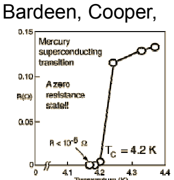
Ref:D. Teplitz, editor, Electromagnetism – paths to research, Plenum Press (1982); Chapter 1 written by Brian Schwartz and Sonia Frota-Pessoa

History:

1908 H. Kamerlingh Onnes successfully liquified He

1911 H. Kamerlingh Onnes discovered that Hg at 4.2 K has vanishing resistance

1957 Theory of superconductivity by Bardeen, Cooper, and Schrieffer



Some phenomenological theories < 1957

Drude model of conductivity in "normal" materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - m\frac{\mathbf{v}}{\tau}$$

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-t/\tau} - \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{J} = -ne\mathbf{v} \qquad \text{for } t >> \tau \qquad \mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} \equiv \sigma\mathbf{E}$$

London model of conductivity in superconducting materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E}$$

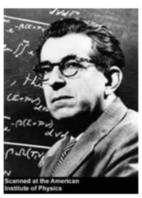
$$\frac{d\mathbf{v}}{dt} = -\frac{e\mathbf{E}}{m} \qquad \frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

Fritz London
Poland



Born in 1900 in Breslau (today Wrocław in Poland), Fritz London studied philosophy before choosing science. After getting a PhD in Munich in 1921, he understood what bonds two hydrogen atoms in a H2 molecule. This work he did with Walter Heitler in Zürich was the starting point for the understanding of chemical bonding. Then he joined Erwin Schrödinger in Berlin but had to leave in 1933 because of the rise of anti-Semitism in Nazi Germany. After a stay in Oxford where he worked on superconductivity with his brother Heinz, he sought refuge at the Institut Henri Poincaré (Paris) in 1936, thanks to a group of intellectuals linked to the Popular Front (Jacques Hadamard, Paul Langevin, Jean Perrin, Frédéric Joliot and Edmond Bauer).

It is at that time, in 1938, that he explained that the superfluidity in liquid helium was a manifestation of Bose-Einstein condensation, a purely quantum phenomenon that could be seen for the first time on a macroscopic scale. This work followed a series of articles about superconductivity that could finally be understood as a superfluidity of charged particles (electron pairs in the case of superconducting metals).

At the beginning of World War II (September 1939), he left France and joined Duke University (USA) where Paul Gross had offered him a professorship in the Chemistry Department and where he felt more comfortable with his wife, the painter Edith London. Einstein wanted the Nobel Prize to be awarded to Fritz London, but London died prematurely in 1954.

Some phenomenological theories < 1957

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\left(\nabla\times\mathbf{B}\right) = -\nabla^2\mathbf{B} = \frac{4\pi}{c}\nabla\times\mathbf{J} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi}{c}\nabla\times\frac{\partial\mathbf{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi ne^2}{mc}\nabla\times\mathbf{E} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = -\frac{4\pi ne^2}{mc^2}\frac{\partial\mathbf{B}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

London model – continued

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

For slowly varying solution:

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2}\right)\mathbf{B} = 0 \qquad \text{for } \frac{\partial \mathbf{B}}{\partial t} = \hat{\mathbf{z}}\frac{\partial B_z(x,t)}{\partial t}:$$

$$\Rightarrow \frac{\partial B_z(x,t)}{\partial t} = \frac{\partial B_z(0,t)}{\partial t}e^{-x/\lambda_L}$$

London leap: $B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Consistent results for current density:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \mathbf{J} = \hat{\mathbf{y}}J_y(x) \quad \Rightarrow \quad J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$$

London model – continued

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

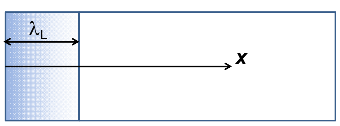
$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Vector potential for $\nabla\cdot\mathbf{A} = 0$:

$\mathbf{A} = \hat{\mathbf{y}}A_y(x) \qquad A_y(x) = -\lambda_L B_z(0)e^{-x/\lambda_L}$

Recall form for current density: $J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$

$$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$$



Magnetization field

Treating London current in terms of corresponding magnetization field **M**:

$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

$\Rightarrow$ For $x >> \lambda_L$, $\mathbf{H} = -4\pi\mathbf{M}$

Gibbs free energy associated with magnetization for superconductor:

$$G_S(H_a) = G_S(H=0) - \int_0^{H_a} dH M(H) = G_S(0) + \frac{1}{8\pi}H_a^2$$

Gibbs free energy associated with magnetization for normal conductor:

$G_N(H_a) \approx G_N(H=0)$

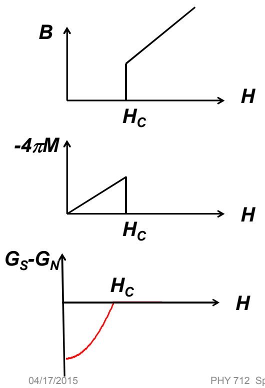
Condition at phase boundary between normal and superconducting states:

$$G_N(H_C) \approx G_N(0) = G_S(H_C) = G_S(0) + \frac{1}{8\pi}H_C^2$$

$$\Rightarrow G_S(0) - G_N(0) = -\frac{1}{8\pi}H_C^2$$

$$G_S(H_a) - G_N(H_a) = \begin{cases} -\frac{1}{8\pi}\left(H_C^2 - H_a^2\right) & \text{for } H_a < H_C \\ 0 & \text{for } H_a > H_C \end{cases}$$

Magnetization field (for "type I" superconductor)



Inside superconductor

$\mathbf{B}=0=\mathbf{H}+4\pi\mathbf{M}$ for $H<H_C$

Behavior of superconducting material – exclusion of magnetic field according to the London model

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

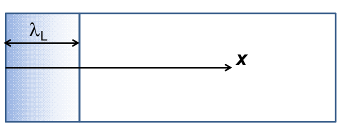
Vector potential for $\nabla\cdot\mathbf{A}=0$:

$\mathbf{A}=\hat{\mathbf{y}}A_y(x) \qquad A_y(x) = -\lambda_L B_z(0)e^{-x/\lambda_L}$

Current density: $J_y(x) = \lambda_L \frac{ne^2}{mc} \mathrm{B}_z(0)\mathrm{e}^{-x/\lambda_L}$

$\Rightarrow \mathbf{J}+\frac{ne^2}{mc}\mathbf{A}=0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v}+\frac{e}{c}\mathbf{A}\right)=0$

Typically, $\lambda_L \approx 10^{-7} m$



Examples of type I superconductors

http://hyperphysics.phy-astr.gsu.edu/hbase/tables/supcon.html#c1

Superconductivity Transition Temperatures and Critical Fields

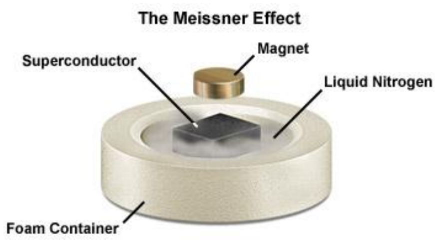
Li	Be 0.026 ...	Superconductivity parameters for elements Transition temperature in Kelvin Critical magnetic field in gauss (10^{-4} tesla)										B	C	N	O	F	Ne ... ...
Na	Mg	...										Al 1.140 105	Si* 7 ...	P* 5 ...	S*	Cl	Ar ... ...
K	Ca	Sc	Ti 0.39 100	V 5.38 1420	Cr*	Mn	Fe	Co	Ni	Cu	Zn 0.875 53	Ga 1.091 51	Ge* 5 ...	As* 0.5 ...	Se* 7 ...	Br	Kr ... ...
Rb	Sr	Y*	Zr 0.546 47	Nb 9.50 1980	Mo 0.90 95	Tc 7.77 1410	Ru 0.51 70	Rh 0.0003 0.049	Pd	Ag	Cd 0.56 30	In 3.4035 293	Sn(w) 3.722 309	Sb* 3.5 ...	Te* 4 ...	I	Xe ... ...
Cs* 1.5 ...	Ba* 5 ...	La(fcc) 6.00 1100	Hf 0.12 ...	Ta 4.483 830	W 0.012 1.07	Re 1.4 198	Os 0.655 65	Ir 0.14 19	Pt	Au	Hg 4.153 412	Tl 2.39 171	Pb 7.193 803	Bi* 8 ...	Po	At	Rn ... ...

Index

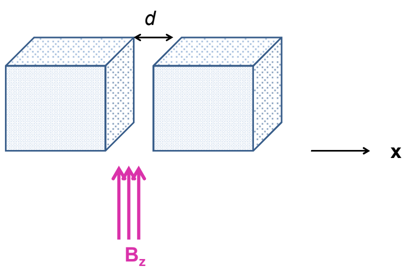
Data from Kittel, Introduction to Solid State Physics, 7th Ed., Ch 12

*Superconducting only in thin films or under high pressure in a crystal modification not normally stable. Critical temperatures for those elements from Myers, Ch 13.

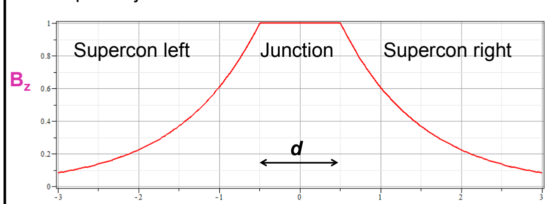
It is notable that in the range of data covered by this table, the best conductors like Cu do not become superconducting at all. Neither the noble metals or the magnetic materials become superconducting. That is not to be taken as a statement that they cannot be made superconducting, it is just that the transitions to superconductivity must be at such low temperatures and require such great purity of material that they have not been demonstrated conclusively.



Josephson junction -- tunneling current between two superconductors

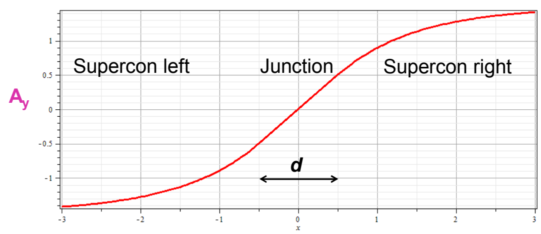


Josephson junction -- continued



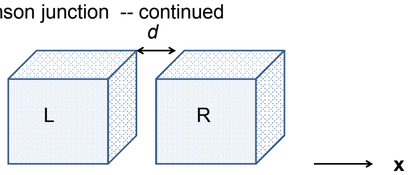
$$B_z(x) = \begin{cases} B_0 e^{(x+d/2)/\lambda_L} & x < -d/2 \\ B_0 & -d/2 < x < d/2 \\ B_0 e^{(-x+d/2)/\lambda_L} & x > d/2 \end{cases}$$

Josephson junction -- continued



$$A_y(x) = \begin{cases} B_0\left(\lambda_L e^{(x+d/2)/\lambda_L} - \left(\lambda_L + d/2\right)\right) & x < -d/2 \\ B_0 x & -d/2 < x < d/2 \\ B_0\left(-\lambda_L e^{(-x+d/2)/\lambda_L} + \left(\lambda_L + d/2\right)\right) & x > d/2 \end{cases}$$

Josephson junction -- continued



Quantum mechanical model of tunnelling current

Let $\Psi_L = \Psi_L{}^0 e^{i\phi_L}$ denote a wavefunction for a Cooper pair on left

Let $\Psi_R = \Psi_R{}^0 e^{i\phi_R}$ denote a wavefunction for a Cooper pair on right

$$-i\hbar \frac{\partial \Psi_L}{\partial t} = E_L \Psi_L + \varepsilon \Psi_R$$

$$-i\hbar \frac{\partial \Psi_R}{\partial t} = E_R \Psi_R + \varepsilon \Psi_L$$

Josephson junction -- continued

Solving for wavefunctions

$$\frac{1}{2}\frac{\partial \left|\Psi_L{}^0\right|^2}{\partial t} + i\left|\Psi_L{}^0\right|^2 \frac{\partial \phi_L}{\partial t} = -\frac{i}{\hbar}\left(E_L \left|\Psi_L{}^0\right|^2 + \varepsilon \Psi_L{}^0 \Psi_R{}^0 e^{i(\phi_R - \phi_L)}\right)$$

$$\frac{1}{2}\frac{\partial \left|\Psi_R{}^0\right|^2}{\partial t} + i\left|\Psi_R{}^0\right|^2 \frac{\partial \phi_R}{\partial t} = -\frac{i}{\hbar}\left(E_R \left|\Psi_R{}^0\right|^2 + \varepsilon \Psi_L{}^0 \Psi_R{}^0 e^{-i(\phi_R - \phi_L)}\right)$$

$$\left|\Psi_L{}^0\right|^2 \equiv n_L \qquad \left|\Psi_R{}^0\right|^2 \equiv n_R \qquad \phi_{LR} \equiv \phi_L - \phi_R$$

$$\frac{\partial n_L}{\partial t} = -\frac{\partial n_R}{\partial t} = -\frac{2\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$$

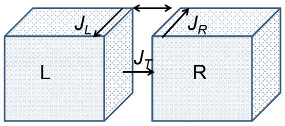
$$\frac{\partial \phi_L}{\partial t} = -\frac{E_L}{\hbar} - \varepsilon\sqrt{\frac{n_R}{n_L}}\cos\phi_{LR}$$

$$\frac{\partial \phi_R}{\partial t} = -\frac{E_R}{\hbar} - \varepsilon\sqrt{\frac{n_L}{n_R}}\cos\phi_{LR}$$

Josephson junction -- continued

Tunneling current: $J_T = 2e\frac{\partial n_L}{\partial t} = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

If $n_L = n_R$ and in absense of magnetic field, $\phi_{LR}(t) = \phi_{LR}(0) + \frac{E_R - E_L}{\hbar}t$



$$\Rightarrow J_L = \frac{2e}{2m}\left|\Psi_L^0\right|^2\left(\hbar\nabla\phi_L - \frac{2e}{c}\mathbf{A}\right)$$

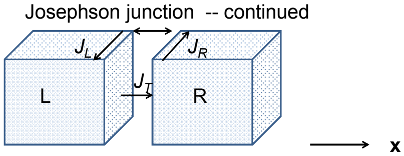
$$\Rightarrow J_R = \frac{2e}{2m}\left|\Psi_R^0\right|^2\left(\hbar\nabla\phi_R - \frac{2e}{c}\mathbf{A}\right)$$

x

Relationship between superconductor currents J_L and J_R and tunneling current. Within the superconductor, denote the generalize current operator acting on pair wavefunction $\Psi = \Psi^0 e^{i\phi}$

$$\hat{\mathbf{v}} \equiv \frac{1}{2m}\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right) \quad \text{with current } J = \frac{2e}{2}\left(\Psi^*(\hat{\mathbf{v}}\Psi) + \Psi(\hat{\mathbf{v}}\Psi)^*\right)$$

Josephson junction -- continued



$$\Rightarrow J_L = \frac{2e}{2m}\left|\Psi_L^0\right|^2\left(\hbar\nabla\phi_L - \frac{2e}{c}\mathbf{A}\right) \equiv 2en_L\mathbf{v}_L$$

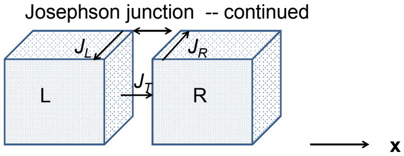
$$\Rightarrow J_R = \frac{2e}{2m}\left|\Psi_R^0\right|^2\left(\hbar\nabla\phi_R - \frac{2e}{c}\mathbf{A}\right) \equiv 2en_R\mathbf{v}_R$$

$$\nabla\phi_L = \frac{2m\mathbf{v}_L}{\hbar} + \frac{2e}{\hbar c}\mathbf{A} \qquad \nabla\phi_R = \frac{2m\mathbf{v}_R}{\hbar} + \frac{2e}{\hbar c}\mathbf{A}$$

Tunneling current: $J_T = 2e\frac{\partial n_L}{\partial t} = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

Need to evaluate ϕ_{LR} in presence of magnetic field

Josephson junction -- continued



$$\nabla\phi_L = \frac{2m\mathbf{v}_L}{\hbar} + \frac{2e}{\hbar c}\mathbf{A} \qquad \nabla\phi_R = \frac{2m\mathbf{v}_R}{\hbar} + \frac{2e}{\hbar c}\mathbf{A}$$

Recall that for $x \to -\infty$ $\mathbf{v}_L \to 0$ and $\mathbf{A} \to -\left(\lambda_L + d/2\right)B_0\hat{\mathbf{y}}$

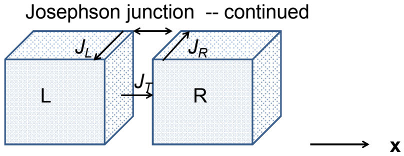
for $x \to \infty$ $\mathbf{v}_R \to 0$ and $\mathbf{A} \to \left(\lambda_L + d/2\right)B_0\hat{\mathbf{y}}$

Integrating the difference of the phase angles along y:

$$\phi_{LR} = \phi_{LR}^0 + B_0(2\lambda_L + d)y$$

Tunneling current: $J_T = -\frac{4e\varepsilon}{\hbar}\sqrt{(n_L n_R)}\sin\phi_{LR}$

Josephson junction -- continued



Integrating the difference of the phase angles along y:

$$\phi_{LR} = \phi_{LR}^0 + \frac{2e}{\hbar c} B_0 (2\lambda_L + d) y$$

Tunneling current density: $J_T = \frac{4e\varepsilon}{\hbar} n_L \sin\phi_{LR}$

Integrating current density throughout width w of superconductors

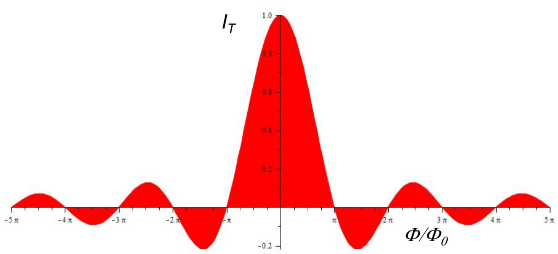
$$I_T = h \int_{-w/2}^{w/2} J_T dy = hwJ_{T0} \sin(\phi_{LR}^0) \frac{\sin(\pi\Phi/\Phi^0)}{\pi\Phi/\Phi^0}$$

where $\Phi = B_0 w (2\lambda_L + d)$ and $\Phi^0 = \frac{2\pi\hbar c}{2e}$

04/17/2015 PHY 712 Spring 2015 -- Lecture 33 22

Josephson junction -- continued

SQUID = *superconducting quantum interference device*



Note: This very sensitive "SQUID" technology has been used in scanning probe techniques. See for example, J. R. Kirtley, Rep. Prog. Physics 73, 126501 (2010).

04/17/2015 PHY 712 Spring 2015 -- Lecture 33 23

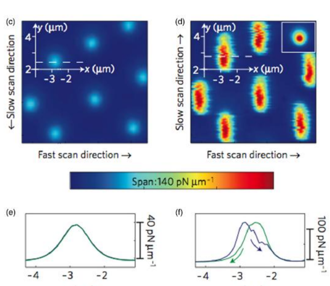


Figure 31. MFM imaging and manipulation of individual vortices in YBCO at $T = 22.3$ K. (*a*), (*b*) Schematic drawings of an MFM tip (triangle) that attracts a vortex (thick lines) in a sample with randomly distributed pinning sites (dots): at large heights (*a*) the force between tip and vortex is too weak to move the vortex; at small heights (*b*) the vortex moves right and then left as the tip rasters over it. (*c*) MFM image taken at $z = 420$ nm (maximum applied lateral force $F_{max}^{lat} \approx 6$ nN). (*d*) $z = 170$ nm ($F_{max}^{lat} \approx 12$ nm). Inset: scan taken at a comparable height at $T = 5.2$ K. (*e*) Line cut through the data in (*c*) along the dashed line, showing the signal from a stationary vortex (solid curve with peak to the left, blue online). Overlapping it is a line cut from the reverse scan (solid curve with peak to the right, green online). (*f*) Line cut through the data in (*d*) along the dashed line, showing a typical signal from a dragged vortex. Figure reprinted by permission from [312]. Copyright 2008 Macmillan Publishers Ltd.

04/17/2015 PHY 712 Spring 2015 -- Lecture 33 24