PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 31:

Start reading Chap. 15 –

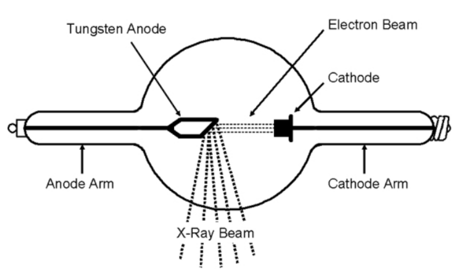
Radiation from collisions of charged particles

1. Overview
2. X-ray tube
3. Radiation from Rutherford scattering
4. Continuum models (Chap. 13)

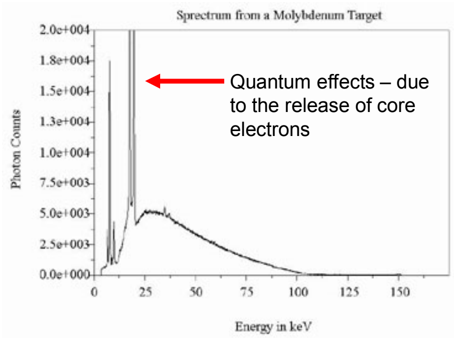
20	Mon: 03/16/2015	Chap. 8	Review Exam; Wave guides	#19	03/18/2015
21	Wed: 03/18/2015	Chap. 8	Wave guides	#20	03/20/2015
22	Fri: 03/20/2015	Chap. 9	Radiation sources	#21	03/23/2015
23	Mon: 03/23/2015	Chap. 9 & 10	Radiation and scattering	#22	03/25/2015
24	Wed: 03/25/2015	Chap. 9 & 10	Radiation and scattering		
25	Fri: 03/27/2015	Chap. 11	Special relativity	#23	03/30/2015
26	Mon: 03/30/2015	Chap. 11	Special relativity	#24	04/01/2015
27	Wed: 04/01/2015	Chap. 11	Special relativity	#25	04/06/2015
	Fri: 04/03/2015	Good Friday	No class		
28	Mon: 04/06/2015	Chap. 14	Radiation from moving charges	#26	04/08/2015
29	Wed: 04/08/2015	Chap. 14	Radiation from moving charges	#27	04/10/2015
30	Fri: 04/10/2015	Chap. 14	Radiation from moving charges	#28	04/13/2015
31	Mon: 04/13/2015	Chap. 15	Radiation due to scattering	#29	04/15/2015
32	Wed: 04/16/2015	Chap. 13	Cherenkov radiation	#30	04/17/2015
33	Fri: 04/17/2015		Special topics -- superconductivity		04/20/2015
34	Mon: 04/20/2015		Special topics -- superconductivity		
35	Wed: 04/22/2015		Review		
36	Fri: 04/24/2015		Review		
	Mon: 04/27/2015		Presentations I		
	Wed: 04/29/2015		Presentations II		
	Fri: 05/01/2015		Presentations III & Take home exam		

Generation of X-rays in a Coolidge tube

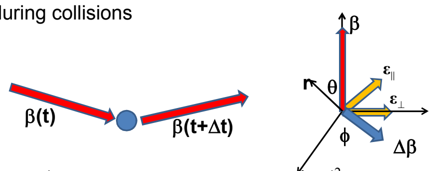
https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm



http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Radiation during collisions



Intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\ e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\ \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

Note that $\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right) = \left(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_1 + \left(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_2$

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Radiation during collisions -- continued

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Non-relativistic limit:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\Delta\boldsymbol{\beta}\right)\right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$$

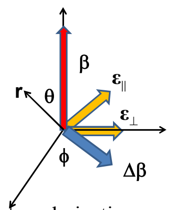
Relativistic collision with small $\left|\Delta\boldsymbol{\beta}\right|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times\left(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta}\right)}{\left(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}\right)^2}\right)\right|^2$$

Radiation during collisions -- continued

Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$



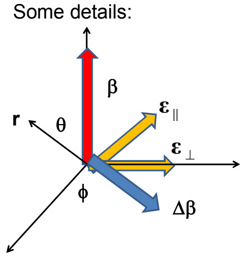
Also assume $\Delta\boldsymbol{\beta}$ is perpendicular to $\mathbf{r}$ – $\boldsymbol{\beta}$ plane

Expressions (averaging over φ) for || or ⊥ polarization:

$$\frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{(\beta-\cos\theta)^2}{(1-\beta\cos\theta)^4}$$ polarization in $\boldsymbol{r}$ and β plane

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$ polarization perpendicular to $\boldsymbol{r}$ and β plane

Some details:



$$\boldsymbol{\beta} = \beta\hat{\mathbf{z}} \qquad \hat{\mathbf{r}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\varepsilon}_{\parallel} = -\cos\theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{z}} \qquad \boldsymbol{\varepsilon}_{\perp} = \hat{\mathbf{y}}$$

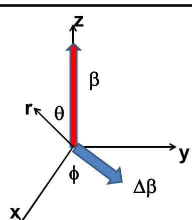
$$\Delta\boldsymbol{\beta} = \Delta\beta(\cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}})$$

Radiation during collisions -- continued

Intensity expressions:

$$\frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{(\beta-\cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$

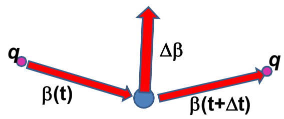


Relativistic collision at low ω and with small $|\Delta\boldsymbol{\beta}|$ and $\Delta\boldsymbol{\beta}$ perpendicular to plane of $\hat{\mathbf{r}}$ and $\boldsymbol{\beta}$, as a function of θ where $\hat{\mathbf{r}}\cdot\boldsymbol{\beta} = \beta\cos\theta$;

Integrating over solid angle:

$$\frac{dI}{d\omega} = \int d\Omega\left(\frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega}\right) = \frac{2}{3\pi}\frac{q^2}{c}\gamma^2\ |\Delta\boldsymbol{\beta}|^2$$

Estimation of Δβ



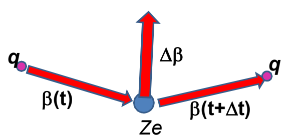
Momentum transfer:

$$Q \equiv |\mathbf{p}(t+\tau)-\mathbf{p}(t)| \approx \gamma Mc^2|\Delta\boldsymbol{\beta}|$$

mass of particle having charge *q*

$$\frac{dI}{d\omega}=\frac{2}{3\pi}\frac{q^2}{c}\gamma^2|\Delta\boldsymbol{\beta}|^2 \approx \frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2$$

Estimation of Δβ -- for the case of Rutherford scattering



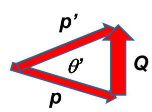
Assume that target nucleus (charge *Ze*) has mass >>M;

Rutherford scattering cross-section:

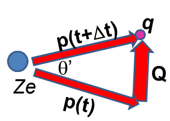
$$\frac{d\sigma}{d\Omega}=\left(\frac{2Zeq}{pv}\right)^2\frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

Assuming elastic scattering:

$$Q^2=\left(2p\sin(\theta'/2)\right)^2=2p^2(1-\cos\theta')$$



Case of Rutherford scattering -- continued



Rutherford scattering cross-section:

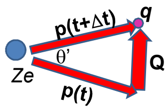
$$\frac{d\sigma}{d\Omega}=\left(\frac{2Zeq}{pv}\right)^2\frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

$$\frac{d\sigma}{dQ}=\int d\phi'\frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ}$$

$$Q^2=\left(2p\sin(\theta'/2)\right)^2=2p^2(1-\cos\theta')$$

$$\Rightarrow \quad \frac{d\sigma}{dQ}=8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}$$

Case of Rutherford scattering -- continued



Differential radiation cross section :

$$\frac{d^2\chi}{d\omega dQ}=\frac{dI}{d\omega}\frac{d\sigma}{dQ}=\left(\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2\right)\left(8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}\right)$$

$$=\frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\frac{1}{Q}$$

Differential radiation cross section - - continued

Integrating over momentum transfer

$$\frac{d\chi}{d\omega}=\int_{Q_{\min}}^{Q_{\max}}dQ\frac{d^2\chi}{d\omega dQ}=\frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Comment on frequency dependence --

Original expression for radiation intensity :

$$\frac{d^2I}{d\omega d\Omega}=\frac{q^2}{4\pi^2c}\left|\int dt\, e^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)}\frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

In the previous derivations, we have assumed that

$\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)<<1.$

$$\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)=\omega\left(t-\hat{\mathbf{r}}\cdot\int_0^t dt'\boldsymbol{\beta}(t')\right)\approx\omega\tau(1-\hat{\mathbf{r}}\cdot\langle\boldsymbol{\beta}\rangle)$$

Differential radiation cross section - - continued

Radiation cross section in terms of momentum transfer

$$\frac{d\chi}{d\omega}=\int_{Q_{\min}}^{Q_{\max}}dQ\frac{d^2\chi}{d\omega dQ}=\frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Note that: $Q^2=2p^2(1-\cos\theta')\quad\Rightarrow Q_{\max}=2p$

In general, $Q_{\min}$ is determined by the collision time

condition $\omega\tau<1\quad\Rightarrow Q_{\min}\approx\frac{2Zeq\omega}{v^2}$

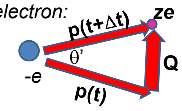
Radiation cross section for classical non - relativistic process

$$\frac{d\chi}{d\omega}=\frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{\lambda Mv^3}{Zeq\omega}\right)$$

λ= “fudge factor” of order unity

Electromagnetic effects in energy loss processes
(see Chap. 13 of Jackson)

Again consider Rutherford scattering – now of a neucleus (or alpha particle *ze* incident on an electron *–e in rest frame of electron:*



Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{ze^2}{2pv}\right)^2 \frac{1}{\left(\sin(\theta'/2)\right)^4}$$

$$\frac{d\sigma}{dQ^2} = \int d\varphi' \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ^2}$$

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2(1-\cos\theta')$$

$$\Rightarrow \quad \frac{d\sigma}{dQ^2} = 4\pi\left(\frac{ze^2}{\beta c Q^2}\right)^2$$

Energy loss continued

Let T represent energy loss due to electron of mass m:

$$T = Q^2/2m$$

$$\frac{d\sigma}{dT} = \frac{2\pi z^2 e^4}{mc^2\beta^2 T^2}$$

Estimate of energy loss per unit distance
in the presence of NZ electrons per unit volume

$$\frac{dE}{dx} \approx NZ \int_{\varepsilon}^{T_{max}} dT\, T \frac{d\sigma}{dT}$$

minimum energy transfer

$$= 2\pi NZ \frac{z^2 e^4}{mc^2\beta^2} \ln\left(\frac{2\gamma^2\beta^2 mc^2}{\varepsilon}\right) + \text{(quantum effects)}$$

Energy loss continued

Refining this result, Bethe and Fermi noticed that the analysis lacked consideration of the effects of electromagnetic fields. Representing the colliding electrons in terms of a dielectric function $\varepsilon(\omega)$ and the energetic partical of charge *ze* in terms of the charge and current density:

In Fourier space:

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\Phi(\mathbf{k},\omega) = \frac{4\pi}{\varepsilon(\omega)}\rho(\mathbf{k},\omega)$$

$$\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\mathbf{A}(\mathbf{k},\omega) = \frac{4\pi}{c}\mathbf{J}(\mathbf{k},\omega)$$

$$\rho(\mathbf{k},\omega) = \frac{ze}{2\pi}\delta(\omega - \mathbf{v}\cdot\mathbf{k})$$

$$\mathbf{J}(\mathbf{k},\omega) = \mathbf{v}\rho(\mathbf{k},\omega)$$

Energy loss continued

$$\Phi(\mathbf{k},\omega) = \frac{2ze}{\varepsilon(\omega)} \frac{\delta(\omega - \mathbf{v}\cdot\mathbf{k})}{k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)}$$

$$\mathbf{A}(\mathbf{k},\omega) = \varepsilon(\omega)\frac{\mathbf{v}}{c}\Phi(\mathbf{k},\omega)$$

The energy loss will be calculated from the work on the electron by the field:

$$\Delta E = -e\int_{-\infty}^{\infty} dt\ \mathbf{v}\cdot\mathbf{E}(t) = 2e\Re\left(\int_0^{\infty} d\omega\ i\omega\mathbf{r}(\omega)\cdot\mathbf{E}^*(\omega)\right)$$

The resultant loss estimate is

$$\frac{dE}{dx} \approx \frac{z^2e^2\omega_P^2}{2c^2}\ln\left(\frac{2mc^2\varepsilon}{\hbar^2\omega_P^2}\right) \quad \text{where } \omega_P^2 \equiv \frac{4\pi NZe^2}{m}$$

04/13/2015 PHY 712 Spring 2015 -- Lecture 31 19