PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 30:

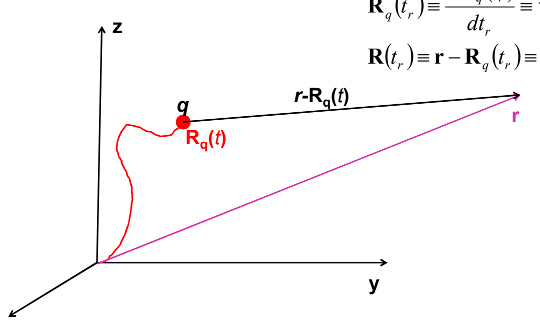
Finish reading Chap. 14 –

Radiation from charged particles

1. **Review of synchrotron radiation**
2. **Free electron laser**
3. **Thompson and Compton scattering**

21	Wed: 03/18/2015	Chap. 8	Wave guides	#20	03/20/2015
22	Fri: 03/20/2015	Chap. 9	Radiation sources	#21	03/23/2015
23	Mon: 03/23/2015	Chap. 9 & 10	Radiation and scattering	#22	03/25/2015
24	Wed: 03/25/2015	Chap. 9 & 10	Radiation and scattering		
25	Fri: 03/27/2015	Chap. 11	Special relativity	#23	03/30/2015
26	Mon: 03/30/2015	Chap. 11	Special relativity	#24	04/01/2015
27	Wed: 04/01/2015	Chap. 11	Special relativity	#25	04/06/2015
	Fri: 04/03/2015	Good Friday	No class		
28	Mon: 04/06/2015	Chap. 14	Radiation from moving charges	#26	04/08/2015
29	Wed: 04/08/2015	Chap. 14	Radiation from moving charges	#27	04/10/2015
30	Fri: 04/10/2015	Chap. 14	Radiation from moving charges	#28	04/13/2015
31	Mon: 04/13/2015	Chap. 15	Radiation due to scattering	#29	04/15/2015
32	Wed: 04/16/2015	Chap. 13	Cherenkov radiation	#30	04/17/2015
33	Fri: 04/17/2015		Special topics -- superconductivity		04/20/2015
34	Mon: 04/20/2015		Special topics -- superconductivity		
35	Wed: 04/22/2015		Review		
36	Fri: 04/24/2015		Review		
	Mon: 04/27/2015		Presentations I		
	Wed: 04/29/2015		Presentations II		
	Fri: 05/01/2015		Presentations III & Take home exam		

Radiation from a moving charged particle



Variables (notation) :

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

Liénard-Wiechert fields (cgs Gaussian units):

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\left(1 - \frac{v^2}{c^2}\right) + \left(\mathbf{R} \times \left\{\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \times \frac{\dot{\mathbf{v}}}{c^2}\right\}\right)\right]. \quad (19)$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c}\left[\frac{-\mathbf{R}\times\mathbf{v}}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}}\cdot\mathbf{R}}{c^2}\right) - \frac{\mathbf{R}\times\dot{\mathbf{v}}/c}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^2}\right]. \quad (20)$$

In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}. \quad (21)$$

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v} \qquad \mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R} \qquad \dot{\mathbf{v}} \equiv \frac{d^2\mathbf{R}_q(t_r)}{dt_r^{\,2}}$$

Electric and magnetic fields far from source:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left\{\mathbf{R}\times\left[\left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}$$

Let $\hat{\mathbf{R}} \equiv \frac{\mathbf{R}}{R} \qquad \boldsymbol{\beta} \equiv \frac{\mathbf{v}}{c} \qquad \dot{\boldsymbol{\beta}} \equiv \frac{\dot{\mathbf{v}}}{c}$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR\left(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\left\{\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

Poynting vector:

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\left(\mathbf{E}\times\mathbf{B}\right)$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR\left(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\left\{\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\hat{\mathbf{R}}\left|\mathbf{E}(\mathbf{r},t)\right|^2 = \frac{q^2}{4\pi c R^2}\hat{\mathbf{R}}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

$$\frac{dP}{d\Omega} = \mathbf{S}\cdot\hat{\mathbf{R}}R^2 = \frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

Spectral composition of electromagnetic radiation

Time integrated power per solid angle:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt \left|\boldsymbol{\mathcal{a}}(t)\right|^2 = \int_{-\infty}^{\infty} d\omega \left|\tilde{\boldsymbol{\mathcal{a}}}(\omega)\right|^2$$

Fourier amplitude:

$$\tilde{\boldsymbol{\mathcal{a}}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, \boldsymbol{\mathcal{a}}(t) e^{i\omega t} \qquad \boldsymbol{\mathcal{a}}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\, \tilde{\boldsymbol{\mathcal{a}}}(\omega) e^{-i\omega t}$$

Note that: $\tilde{\boldsymbol{\mathcal{a}}}(\omega) = \tilde{\boldsymbol{\mathcal{a}}}^*(-\omega)$

$$\Rightarrow \frac{dW}{d\Omega} = \int_0^{\infty} d\omega \left(\left|\tilde{\boldsymbol{\mathcal{a}}}(\omega)\right|^2 + \left|\tilde{\boldsymbol{\mathcal{a}}}(-\omega)\right|^2 \right) \equiv \int_0^{\infty} d\omega \frac{\partial^2 I}{\partial\Omega\partial\omega}$$

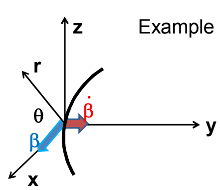
$$\frac{\partial^2 I}{\partial\Omega\partial\omega} \equiv 2\left|\tilde{\boldsymbol{\mathcal{a}}}(\omega)\right|^2$$

Spectral composition of electromagnetic radiation -- continued

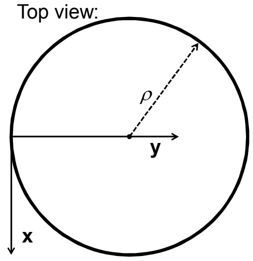
The spectral intensity therefore depends on the following integral:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\beta}(t_r) \right) \right] e^{i\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)} \right|^2$$

Example for circular motion:



Top view:

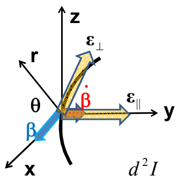


$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}} \sin(vt_r/\rho) + \rho\hat{\mathbf{y}}(1 - \cos(vt_r/\rho))$$

$$\boldsymbol{\beta}(t_r) = \beta(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho))$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



$$\boldsymbol{\varepsilon}_\parallel = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_\perp = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\beta}) =$$

$$\beta\left(-\boldsymbol{\varepsilon}_\parallel \sin(vt_r/\rho) + \boldsymbol{\varepsilon}_\perp \sin\theta\cos(vt_r/\rho)\right)$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \beta) e^{i\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)} dt\right|^2$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_\parallel(\omega)|^2 + |C_\perp(\omega)|^2\right\}$$

$$C_\parallel(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_\perp(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) e^{i\omega(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

Synchrotron radiation geometry – using modified Bessel functions

$$K_{1/3}(\xi) = \sqrt{3}\int_0^\infty dx \cos\left[\tfrac{3}{2}\xi\left(x + \tfrac{1}{3}x^3\right)\right] \qquad K_{2/3}(\xi) = \sqrt{3}\int_0^\infty dx\, x \sin\left[\tfrac{3}{2}\xi\left(x + \tfrac{1}{3}x^3\right)\right]$$

Exponential factor

$$\omega\left(t_r - \frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right) = \omega\left(t_r - \frac{\rho}{c}\cos\theta\sin(vt_r/\rho)\right)$$

$$\text{In the limit of } \quad t_r \approx 0, \quad \theta \approx 0, \; v \approx c\left(1 - \frac{1}{2\gamma^2}\right)$$

$$\omega\left(t_r - \frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right) \approx \frac{\omega t_r}{2\gamma^2}\left(1 + \gamma^2\theta^2\right) + \frac{\omega c^2 t_r^{\,3}}{6\rho^2} = \frac{3}{2}\xi\left(x + \frac{1}{3}x^3\right)$$

$$\text{where} \quad \xi = \frac{\omega\rho}{3c\gamma^3}\left(1 + \gamma^2\theta^2\right)^{3/2} \quad \text{and} \quad x = \frac{c\gamma t_r}{\rho\left(1 + \gamma^2\theta^2\right)^{1/2}}$$

Spectral form of synchrotron radiation in this case:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2\left(1 + \gamma^2\theta^2\right)^2\left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}\left(1 + \gamma^2\theta^2\right)^{3/2}\right)\right]^2 + \frac{\gamma^2\theta^2}{1 + \gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}\left(1 + \gamma^2\theta^2\right)^{3/2}\right)\right]^2\right\}$$

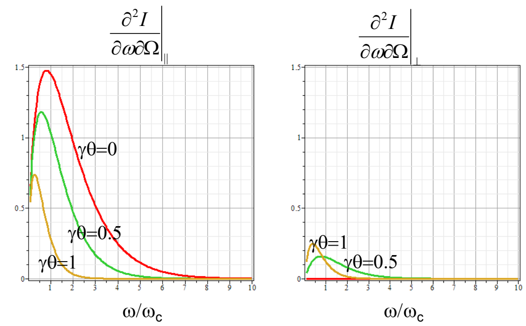
$$\omega_c \equiv \frac{3c\gamma^3}{2\rho}$$

At $\theta = 0$:

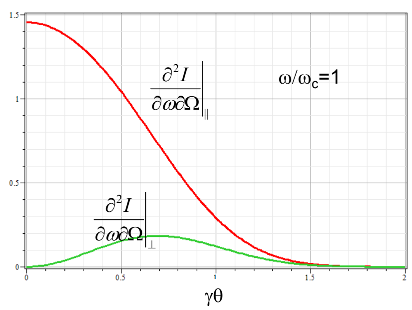
$$\text{note that for} \quad \omega << \omega_c \quad \Rightarrow \frac{\partial^2 I}{\partial\omega\partial\Omega} \approx \frac{q^2}{\pi^2 c}\left(\Gamma\left(\tfrac{2}{3}\right)\right)^2\left(\frac{3\omega^2\rho^2}{4c^2}\right)^{1/3}$$

$$\text{and for} \quad \omega >> \omega_c \quad \Rightarrow \frac{\partial^2 I}{\partial\omega\partial\Omega} \approx \frac{3q^2}{4\pi c}\gamma^2\left(\frac{\omega}{\omega_c}\right)^2 e^{-\omega/\omega_c}$$

Plots of the intensity function for synchrotron radiation



Plots of the intensity function for synchrotron radiation



Synchrotron facilities in USA

http://www.lightsources.org/regions

Advanced Light Source	USA	http://www-als.lbl.gov/
Advanced Photon Source	USA	http://www.aps.anl.gov
Center for Advanced Microstructures and Devices	USA	http://www.camd.lsu.edu/
Cornell High Energy Synchrotron Source	USA	http://www.chess.cornell.edu/
National Synchrotron Light Source II	USA	http://www.bnl.gov/ps/
Stanford Synchrotron Radiation Lightsource	USA	http://www-ssrl.slac.stanford.edu
Synchrotron Ultraviolet Radiation Facility	USA	http://physics.nist.gov/MajResFac/SURF/SURF/index.html

Free electron laser
Reference:

Classical Theory of Free-Electron Lasers

A text for students and researchers

Eric B Szarmes

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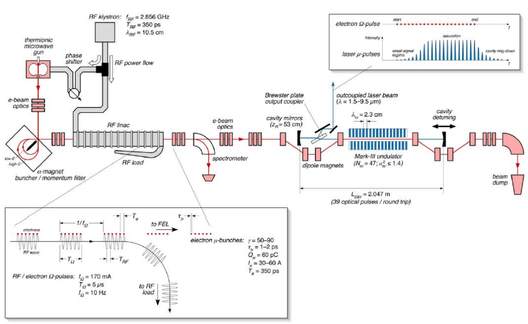


Figure 1.1. The MkIII RF-linac FEL.

Electron emission in periodic magnet

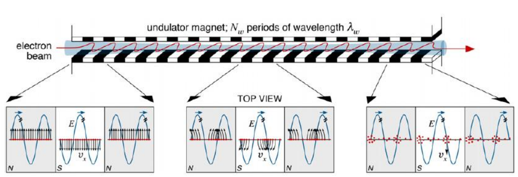


Figure 1.3. Conceptual illustration of the bunching mechanism in an FEL.

Because of Doppler shift, effective

wavelength is: $\lambda = \frac{\lambda_w}{2\gamma^2}$

frequency is: $\omega = 2\gamma^2 k_w c$

Electron trajectory near magnetic in undulator viewed in the electron rest frame (ERF)

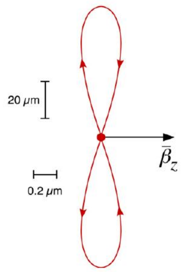
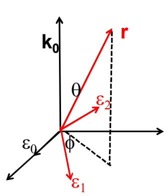


Figure 4.1. Electron motion in the ERF in a plane-polarized undulator for $\hat{K}^2 = 1.2$; $\gamma = 80$; $k_w = 2.73\,\mathrm{cm}^{-1}$. The maximum transverse and longitudinal displacements are ±71 μm and ±0.17 μm respectively.

Free-Electron Laser

Name	Country	Website
Institute for Terahertz Science and Technology	USA	http://www.itst.ucsb.edu/
Jefferson Lab FEL	USA	https://www.jlab.org/free-electron-laser
Linac Coherent Light Source	USA	http://lcls.slac.stanford.edu

Some details of scattering of electromagnetic waves incident on a particle of charge q and mass m_q



$$\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \dot{\mathbf{v}}\right)\right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q) is caused by an electric field : $\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$

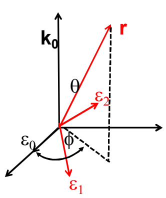
$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega t}\right)$$

Time averaged power : $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0\right)\right|^2$

Thompson scattering – non relativistic approximation -- continued

Time averaged power: $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \boldsymbol{\varepsilon}_0\right)\right|^2$

$$\hat{\mathbf{r}} = \sin\theta\left(\cos\phi\ \hat{\mathbf{x}} + \sin\phi\ \hat{\mathbf{y}}\right) + \cos\theta\hat{\mathbf{z}}$$



Polarization of incident light: $\varepsilon_0 = \hat{\mathbf{x}}$

Polarization of scattered light:

$$\varepsilon_1 = \cos\theta(\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi) - \hat{\mathbf{z}}\sin\theta$$

$$\varepsilon_2 = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$

Thompson scattering – non relativistic approximation -- continued

Time averaged power with polarization ε^*:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0\right|^2$$

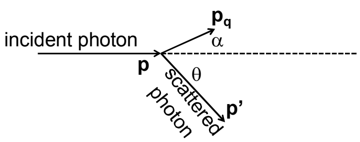
Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

$$\left\langle \left|\boldsymbol{\varepsilon}^* \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle_\phi = \left\langle \left|\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle_\phi + \left\langle \left|\boldsymbol{\varepsilon}_2 \cdot \boldsymbol{\varepsilon}_0\right|^2 \right\rangle_\phi = \frac{1}{2}\cos^2\theta + \frac{1}{2}$$

Averaged cross section: $\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \frac{1}{2}\left(1 + \cos^2\theta\right)$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy :

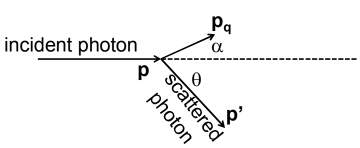
$$p = p'\cos\theta + p_q \cos\alpha \qquad pc = \hbar\omega$$

$$0 = p'\sin\theta - p_q \sin\alpha \qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p_q{}^2 c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$

Thompson scattering – relativistic and quantum modifications



Relativistic and quantum modifications to averaged cross section :

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \left(\frac{p'}{p}\right)^2 \frac{1}{2}\left(1 + \cos^2\theta\right)$$

$$\frac{p'}{p} = \frac{1}{1 + \frac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$