PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 29:

Continue reading Chap. 14 – Synchrotron radiation

1. Radiation from electron synchrotron devices
2. Radiation from astronomical objects in circular orbits

20	Mon: 03/16/2015	Chap. 8	Review Exam; Wave guides	#19	03/18/2015
21	Wed: 03/18/2015	Chap. 8	Wave guides	#20	03/20/2015
22	Fri: 03/20/2015	Chap. 9	Radiation sources	#21	03/23/2015
23	Mon: 03/23/2015	Chap. 9 & 10	Radiation and scattering	#22	03/25/2015
24	Wed: 03/25/2015	Chap. 9 & 10	Radiation and scattering		
25	Fri: 03/27/2015	Chap. 11	Special relativity	#23	03/30/2015
26	Mon: 03/30/2015	Chap. 11	Special relativity	#24	04/01/2015
27	Wed: 04/01/2015	Chap. 11	Special relativity	#25	04/06/2015
	Fri: 04/03/2015	Good Friday	No class		
28	Mon: 04/06/2015	Chap. 14	Radiation from moving charges	#26	04/08/2015
29	Wed: 04/08/2015	Chap. 14	Radiation from moving charges	#27	04/10/2015
30	Fri: 04/10/2015				04/13/2015
31	Mon: 04/13/2015				04/15/2015
32	Wed: 04/15/2015				04/17/2015
33	Fri: 04/17/2015				04/20/2015
34	Mon: 04/20/2015				
35	Wed: 04/22/2015				
36	Fri: 04/24/2015				
	Mon: 04/27/2015		Presentations I		
	Wed: 04/29/2015		Presentations II		
	Fri: 05/01/2015		Presentations III & Take home exam		

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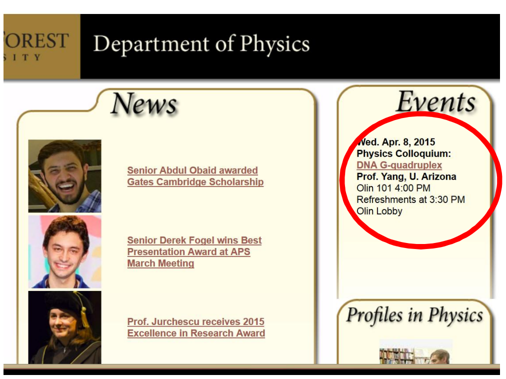


WFU Physics Colloquium

TITLE: Structure and Function of DNA G-quadruplex and its Potential as Anticancer Drug Target

SPEAKER: Dr. Danzhou Yang,

Department of Chemistry and Biochemistry
University of Arizona, Tucson

TIME: Wednesday April 8, 2015 at 4:00 PM

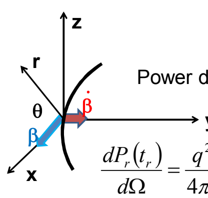
PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

G-quadruplex DNA secondary structures, formed in specific G-rich sequences, have recently emerged as a new class of cancer-specific molecular targets for cancer therapeutics. Specifically, the DNA G-quadruplex secondary structures formed in gene proximal promoter regions have been demonstrated as to function as transcriptional regulators. Significantly, DNA quadruplexes can readily form in solution under physiological conditions. In my talk, I will discuss our recent progress on structural studies of the promoter G-quadruplexes and their interactions with small molecule compounds and proteins.

Radiation from charged particle in circular path



Power distribution for circular acceleration

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^5}\Bigg|_{t_r=t-R/c}$$

$$= \frac{q^2}{4\pi c}\frac{\left|\dot{\boldsymbol{\beta}}\right|^2\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^2-\left(\hat{\mathbf{R}}\cdot\dot{\boldsymbol{\beta}}\right)^2\left(1-\beta^2\right)}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^5}\Bigg|_{t_r=t-R/c}$$

$$P_r(t_r) = \int d\Omega\,\frac{dP_r(t_r)}{d\Omega} = \frac{2}{3}\frac{q^2}{c^3}\left|\dot{\mathbf{v}}\right|^2\gamma^4$$

Spectral composition of electromagnetic radiation -- continued

When the dust clears, the spectral intensity depends on the following integral :

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right]e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$

Synchrotron radiation light source installations

Synchrotron radiation center in Madison, Wisconsin

E_c=0.5 GeV and 1 GeV; λ_c = 20 Å and 10 Å

http://www.src.wisc.edu/

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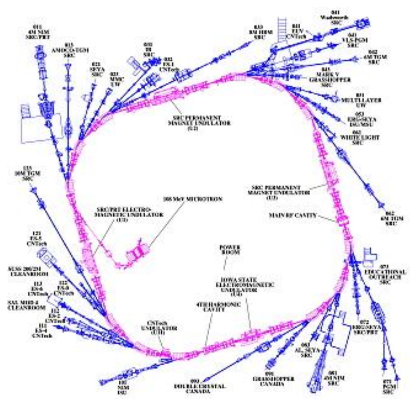
SRC – "Aladdin" -- Madison Wisconsin



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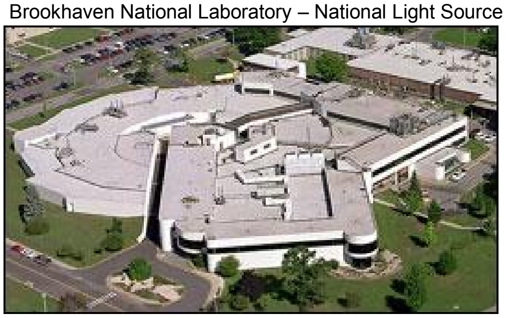
Brookhaven National Laboratory – National Light Source



E_c=0.6 GeV; λ_c = 20 Å

http://www.bnl.gov/ps/

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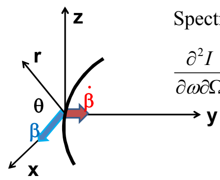
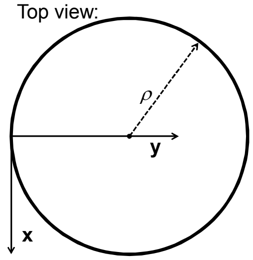


Spectral intensity relationship:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r [\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r))] e^{i\omega(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c)}\right|^2$$

Top view:

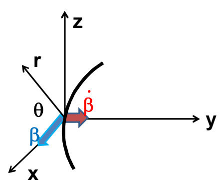


$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho) + \rho\hat{\mathbf{y}}(1-\cos(vt_r/\rho))$$

$$\boldsymbol{\beta}(t_r) = \beta(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho))$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



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$$\boldsymbol{\beta}(t_r) = \beta(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho))$$

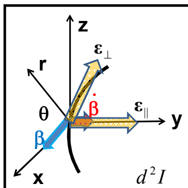
For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Note that we have previous shown that in the radiation zone, the Poynting vector is in the $\hat{\mathbf{r}}$ direction; we can then choose to analyze two orthogonal polarization directions:

$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}) = \beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt_r/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt_r/\rho)\right)$$



$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}) = \beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt_r/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt_r/\rho)\right)$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\beta)\mathrm{e}^{i\omega(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c)}dt\right|^2$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left\{|C_{\parallel}(\omega)|^2 + |C_{\perp}(\omega)|^2\right\}$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt\sin(vt/\rho)\mathrm{e}^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt\sin\theta\cos(vt/\rho)\mathrm{e}^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

We will analyze this expression for two different cases. The first case, is appropriate for man-made synchrotrons used as light sources. In this case, the light is produced by short bursts of electrons moving close to the speed of light ($v \approx c(1-1/(2\gamma^2))$) passing a beam line port. In addition, because of the design of the radiation ports, $\theta \approx 0$, and the relevant integration times t are close to $t \approx 0$. This results in the form shown in Eq. 14.79 of your text. It is convenient to rewrite this form in terms of a critical frequency $\omega_c \equiv \frac{3c\gamma^3}{2\rho}$.

$$\frac{d^2I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2\right.$$

Some details:

Modified Bessel functions

$$K_{1/3}(\xi) = \sqrt{3}\int_0^\infty dx \cos\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right] \qquad K_{2/3}(\xi) = \sqrt{3}\int_0^\infty dx\, x \sin\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right]$$

Exponential factor

$$\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)\right) = \omega\left(t_r - \frac{\rho}{c}\cos\theta\sin(vt_r/\rho)\right)$$

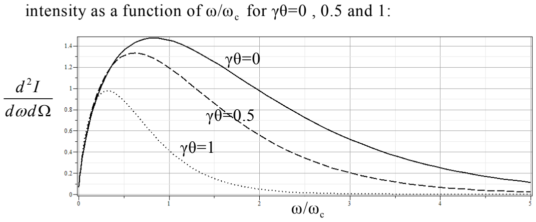
In the limit of $t_r \approx 0,\ \theta \approx 0,\ v \approx c\left(1-\frac{1}{2\gamma^2}\right)$

$$\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)\right) \approx \frac{\omega t_r}{2\gamma^2}\left(1+\gamma^2\theta^2\right) + \frac{\omega c^2 t_r^{\,3}}{6\rho^2} = \frac{3}{2}\xi\left(x+\frac{1}{3}x^3\right)$$

where $\xi = \frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2}$ and $x = \frac{c\gamma t_r}{\rho\left(1+\gamma^2\theta^2\right)^{1/2}}$

$$\frac{d^2I}{d\omega d\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2\theta^2)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{\frac{3}{2}}\right)\right]^2\right.$$

By plotting the intensity as a function of ω, we see that the intensity is largest near $\omega \approx \omega_c$. The plot below shows the intensity as a function of ω/ω_c for $\gamma\theta$=0 , 0.5 and 1:



The second example of synchroton radiation comes from a distant charged particle moving in a circular trajectory such that the spectrum represents a superposition of light generated over many complete circles. In this case, there is an interference effect which results in the spectrum consisting of discrete multiples of v/ρ. For this case we need to reconsider the analysis. There is a very convenient Bessel function identity of the form:

$$e^{-ia\sin\alpha} = \sum_{m=-\infty}^{\infty} J_m(a) e^{-im\alpha} \quad \text{Here } J_m(a) \text{ is a Bessel function of integer order } m.$$

In our case $a = \frac{\omega\rho}{c}\cos\theta$ and $\alpha = \frac{vt}{\rho}$.

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))} = \frac{c}{-i\omega\rho}\frac{\partial}{\partial\cos\theta}\int_{-\infty}^{\infty} dt e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= \frac{c}{-i\omega\rho}\frac{\partial}{\partial\cos\theta}\sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right) 2\pi\delta(\omega - m\frac{v}{\rho}).$$

Astronomical synchrotron radiation -- continued:

Note that:

$$\int_{-\infty}^{\infty} dt e^{i(\omega - m\frac{v}{\rho})t} = 2\pi\delta(\omega - m\frac{v}{\rho}).$$

$$\Rightarrow C_{\parallel}(\omega) = 2\pi i \sum_{m=-\infty}^{\infty} J_m'\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}),$$

where $J_m'(a) \equiv \frac{dJ_m(a)}{da}$

Similarly:

$$C_{\perp}(\omega) = \int_{-\infty}^{\infty} dt \sin\theta\cos(vt/\rho) e^{i\omega(t-\frac{\rho}{c}\cos\theta\sin(vt/\rho))}$$

$$= 2\pi\frac{\tan\theta}{v/c}\sum_{m=-\infty}^{\infty} J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\delta(\omega - m\frac{v}{\rho}).$$

Astronomical synchrotron radiation -- continued:

In both of the expressions, the sum over *m* includes both negative and positive values. However, only the positive values of ω and therefore positive values of *m* are of interest. Using the identity: $J_{-m}(a) = (-1)^m J_m(a)$, the result becomes:

$$\frac{d^2 I}{d\omega d\Omega} =$$

$$\frac{q^2\omega^2\beta^2}{c}\sum_{m=0}^{\infty}\delta(\omega - m\frac{v}{\rho})\left\{\left[J_m'\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2 + \frac{\tan^2\theta}{v^2/c^2}\left[J_m\left(\frac{\omega\rho}{c}\cos\theta\right)\right]^2\right\}.$$

These results were derived by Julian Schwinger (Phys. Rev. **75**, 1912-1925 (1949)). The discrete case is similar to the result quoted in Problem 14.15 in Jackson's text. For information on man-made synchrotron sources, the following web page is useful: http://www.als.lbl.gov/als/synchrotron_sources.html.