

**PHY 712 Electrodynamics**  
**9-9:50 AM MWF Olin 103**

**Plan for Lecture 27:**

**Finish Chap. 11 and begin Chap. 14**

**A. Electromagnetic field transformations & corresponding analysis using Liénard-Wiechert potentials for constant velocity sources**

**B. Radiation by moving charged particles**

04/1/2015 PHY 712 Spring 2015 – Lecture 27 1

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22	Fri	03/20/2015	Chap. 9	Radiation sources	#21	03/23/2015
23	Mon	03/23/2015	Chap. 9 & 10	Radiation and scattering	#22	03/25/2015
24	Wed	03/25/2015	Chap. 9 & 10	Radiation and scattering		
25	Fri	03/27/2015	Chap. 11	Special relativity	#23	03/30/2015
26	Mon	03/30/2015	Chap. 11	Special relativity	#24	04/01/2015
27	Wed	04/01/2015	Chap. 11	Special relativity	#25	04/06/2015
	Fri	04/03/2015	Good Friday	No class		
28	Mon	04/06/2015				04/08/2015
29	Wed	04/08/2015				04/10/2015
30	Fri	04/10/2015				04/13/2015
31	Mon	04/13/2015				04/15/2015
32	Wed	04/15/2015				04/17/2015
33	Fri	04/17/2015				04/20/2015
34	Mon	04/20/2015				
35	Wed	04/22/2015				
36	Fri	04/24/2015				
	Mon	04/27/2015		Presentations I		
	Wed	04/29/2015		Presentations II		
	Fri	05/01/2015		Presentations III & Take home exam		

04/1/2015 PHY 712 Spring 2015 – Lecture 27 2

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**Department of Physics**

**News**

 [Senior Abdul Obaid awarded Gates Cambridge Scholarship](#)

 [Senior Derek Foell wins Best Presentation Award at APS March Meeting](#)

 [Prof. Iurcescu receives 2015 Excellence in Research Award](#)

**Events**

**Mar 27-28, 2015**  
**NCS-AAPT Spring Meeting**

**Wed. Apr. 1, 2015**  
**Physics Colloquium:**  
**Molecular Simulation of Nanomaterials**  
**Prof. Garofalini, Rutgers**  
 Olin 101 4:00 PM  
 Refreshments at 3:30 PM  
 Olin Lobby

**Wed. Apr. 8, 2015**  
**Physics Colloquium:**  
**DNA G-quadruplex**  
**Prof. Yang, U. Arizona**  
 Olin 101 4:00 PM  
 Refreshments at 3:30 PM  
 Olin Lobby

04/1/2015 PHY 712 Spring 2015 – Lecture 27 3

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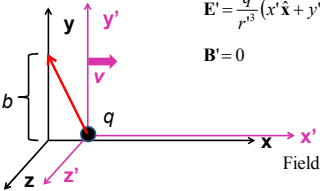
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Example:



Fields in moving frame:

$$\mathbf{E}' = \frac{q}{r'^3} (x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}) = \frac{q(-vt' \hat{\mathbf{x}} + b \hat{\mathbf{y}})}{((-vt')^2 + b^2)^{3/2}}$$

$$\mathbf{B}' = 0$$

Fields in stationary frame:

$$E_x = E'_x = \frac{q(-vt')}{((-vt')^2 + b^2)^{3/2}}$$

$$E_y = \gamma_v(E'_y) = \frac{q(\gamma_v b)}{((-vt')^2 + b^2)^{3/2}}$$

$$B_z = \gamma_v(\beta_v E'_y) = \frac{q(\gamma_v \beta_v b)}{((-vt')^2 + b^2)^{3/2}}$$

04/1/2015 PHY 712 Spring 2015 -- Lecture 27 7

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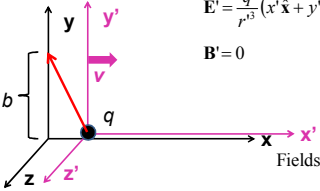
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Example:



Fields in moving frame:

$$\mathbf{E}' = \frac{q}{r'^3} (x' \hat{\mathbf{x}} + y' \hat{\mathbf{y}}) = \frac{q(-vt' \hat{\mathbf{x}} + b \hat{\mathbf{y}})}{((-vt')^2 + b^2)^{3/2}}$$

$$\mathbf{B}' = 0$$

Fields in stationary frame:

$$E_x = E'_x = \frac{q(-v\gamma_v t)}{((-v\gamma_v t)^2 + b^2)^{3/2}}$$

Expression in terms of consistent coordinates

$$E_y = \gamma_v(E'_y) = \frac{q(\gamma_v b)}{((-v\gamma_v t)^2 + b^2)^{3/2}}$$

$$B_z = \gamma_v(\beta_v E'_y) = \frac{q(\gamma_v \beta_v b)}{((-v\gamma_v t)^2 + b^2)^{3/2}}$$

04/1/2015 PHY 712 Spring 2015 -- Lecture 27 8

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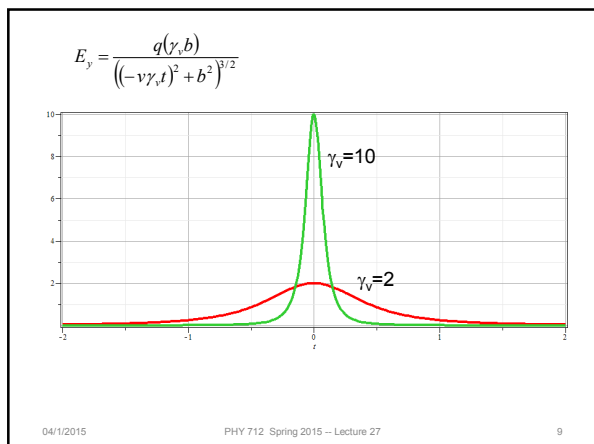
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Examination of this system from the viewpoint of the Liénard-Wiechert potentials –(Gaussian units)

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \left(1 - \frac{v^2}{c^2}\right) + \mathbf{R} \times \left\{ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \times \frac{\dot{\mathbf{v}}}{c^2} \right\} \right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{q}{c} \left[ \frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}} \cdot \mathbf{R}}{c^2}\right) - \frac{\mathbf{R} \times \dot{\mathbf{v}} / c}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^2} \right]$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r},t)}{R}$$

04/1/2015 PHY 712 Spring 2015 – Lecture 27 10

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Examination of this system from the viewpoint of the Liénard-Wiechert potentials –(Gaussian units)

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \left(1 - \frac{v^2}{c^2}\right) \right]$$

For our example:  
 $\mathbf{R}_q(t_r) = vt_r \hat{\mathbf{x}} \quad \mathbf{r} = b\hat{\mathbf{y}}$   
 $\mathbf{R} = b\hat{\mathbf{y}} - vt_r \hat{\mathbf{x}} \quad R = \sqrt{v^2 t_r^2 + b^2}$   
 $\mathbf{v} = v\hat{\mathbf{x}} \quad t_r = t - \frac{R}{c}$

This should be equivalent to the result given in Jackson (11.152):

$$\mathbf{E}(x, y, z, t) = \mathbf{E}(0, b, 0, t) = q \frac{-v\gamma t \hat{\mathbf{x}} + \gamma b \hat{\mathbf{y}}}{\left(b^2 + (v\gamma t)^2\right)^{3/2}}$$

$$\mathbf{B}(x, y, z, t) = \mathbf{B}(0, b, 0, t) = q \frac{\gamma \beta b \hat{\mathbf{z}}}{\left(b^2 + (v\gamma t)^2\right)^{3/2}}$$

04/1/2015 PHY 712 Spring 2015 – Lecture 27 11

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Some details

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \left(1 - \frac{v^2}{c^2}\right) \right]$$

For our example:  
 $\mathbf{R}_q(t_r) = vt_r \hat{\mathbf{x}} \quad \mathbf{r} = b\hat{\mathbf{y}}$   
 $\mathbf{R} = b\hat{\mathbf{y}} - vt_r \hat{\mathbf{x}} \quad R = \sqrt{v^2 t_r^2 + b^2}$   
 $\mathbf{v} = v\hat{\mathbf{x}} \quad t_r = t - \frac{R}{c}$

$t_r$  must be a solution to a quadratic equation:  
 $t_r^2 - 2\gamma^2 t_r + \gamma^2 t^2 - \gamma^2 b^2 / c^2 = 0$   
 with the physical solution:  
 $t_r = \gamma \left( \gamma t - \frac{\sqrt{(v\gamma t)^2 + b^2}}{c} \right)$

04/1/2015 PHY 712 Spring 2015 – Lecture 27 12

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Some details continued: Now we can express  $R$  as:

$$R = \gamma(-\beta v \gamma t + \sqrt{(v \gamma t)^2 + b^2})$$

and the related quantities:

$$\mathbf{R} - \mathbf{v}R/c = -v \hat{x} + b \hat{y}$$

$$R - \mathbf{v} \cdot \mathbf{R} / c = \frac{\sqrt{(v \gamma t)^2 + b^2}}{\gamma}$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \left(1 - \frac{v^2}{c^2}\right) \right] = q \frac{-v \gamma t \hat{x} + \gamma b \hat{y}}{(b^2 + (v \gamma t)^2)^{3/2}}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[ \frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2}\right) \right] = q \frac{\gamma \beta b \hat{z}}{(b^2 + (v \gamma t)^2)^{3/2}}$$

04/1/2015 PHY 712 Spring 2015 – Lecture 27 13

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Radiation from a moving charged particle Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

04/1/2015 PHY 712 Spring 2015 – Lecture 27 14

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**Liénard-Wiechert fields (cgs Gaussian units):**

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left[ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \left(1 - \frac{v^2}{c^2}\right) + \left(\mathbf{R} \times \left\{ \left(\mathbf{R} - \frac{\mathbf{v}R}{c}\right) \times \frac{\dot{\mathbf{v}}}{c^2} \right\} \right) \right]. \quad (19)$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{q}{c} \left[ \frac{-\mathbf{R} \times \mathbf{v}}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left(1 - \frac{v^2}{c^2} + \frac{\dot{\mathbf{v}} \cdot \mathbf{R}}{c^2}\right) - \frac{\mathbf{R} \times \dot{\mathbf{v}}/c}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^2} \right]. \quad (20)$$

In this case, the electric and magnetic fields are related according to

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}. \quad (21)$$

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v} \quad \mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R} \quad \dot{\mathbf{v}} \equiv \frac{d^2\mathbf{R}_q(t_r)}{dt_r^2}$$

04/1/2015 PHY 712 Spring 2015 – Lecture 27 15

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Electric field far from source:

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{\left(R - \frac{\mathbf{v} \cdot \mathbf{R}}{c}\right)^3} \left\{ \mathbf{R} \times \left[ \left( \mathbf{R} - \frac{\mathbf{v}R}{c} \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right] \right\}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{R} \times \mathbf{E}(\mathbf{r}, t)}{R}$$

Let  $\hat{\mathbf{R}} \equiv \frac{\mathbf{R}}{R}$      $\boldsymbol{\beta} \equiv \frac{\mathbf{v}}{c}$      $\dot{\boldsymbol{\beta}} \equiv \frac{\dot{\mathbf{v}}}{c}$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{cR(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \left\{ \hat{\mathbf{R}} \times \left[ (\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right\}$$

$$\mathbf{B}(\mathbf{r}, t) = \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r}, t)$$

04/1/2015

PHY 712 Spring 2015 -- Lecture 27

16

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Poynting vector:

$$\mathbf{S}(\mathbf{r}, t) = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{B})$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{cR(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \left\{ \hat{\mathbf{R}} \times \left[ (\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right\}$$

$$\mathbf{B}(\mathbf{r}, t) = \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r}, t)$$

$$\mathbf{S}(\mathbf{r}, t) = \frac{c}{4\pi} \hat{\mathbf{R}} |\mathbf{E}(\mathbf{r}, t)|^2 = \frac{q^2}{4\pi c R^2} \hat{\mathbf{R}} \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|^2}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^6}$$

Note: We have assumed that

$$\hat{\mathbf{R}} \cdot \mathbf{E}(\mathbf{r}, t) = 0$$

04/1/2015

PHY 712 Spring 2015 -- Lecture 27

17

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Power radiated

$$\mathbf{S}(\mathbf{r}, t) = \frac{c}{4\pi} \hat{\mathbf{R}} |\mathbf{E}(\mathbf{r}, t)|^2 = \frac{q^2}{4\pi c R^2} \hat{\mathbf{R}} \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|^2}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^6}$$

$$\frac{dP}{d\Omega} = \mathbf{S} \cdot \hat{\mathbf{R}} R^2 = \frac{q^2}{4\pi c} \frac{|\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|^2}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^6}$$

In the non-relativistic limit:  $\beta \ll 1$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} |\hat{\mathbf{R}} \times [\hat{\mathbf{R}} \times \dot{\boldsymbol{\beta}}]|^2 = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2 \Theta$$

04/1/2015

PHY 712 Spring 2015 -- Lecture 27

18

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Radiation from a moving charged particle

Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2 \Theta$$

04/1/2015 PHY 712 Spring 2015 -- Lecture 27 19

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Radiation power in non-relativistic case -- continued

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2 \Theta$$

$$P = \int d\Omega \frac{dP}{d\Omega} = \frac{2}{3} \frac{q^2}{c^3} |\dot{\mathbf{v}}|^2$$

04/1/2015 PHY 712 Spring 2015 -- Lecture 27 20

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