PHY 712 Electrodynamics
9-9:50 AM Olin 103

Plan for Lecture 19:

Review Chapter 1-7 in Jackson

1. **Brief review**
2. **Comments on some homework problems**
3. **Distribution of take home exam**

8	Mon: 02/02/2015	Chap. 4	Multipole analysis	#8	02/04/2015
9	Wed: 02/04/2015	Chap. 4	Dipoles and dielectrics	#9	02/06/2015
10	Fri: 02/06/2015	Chap. 4	Dipoles and dielectrics	#10	02/09/2015
11	Mon: 02/09/2015	Chap. 5	Magnetostatics	#11	02/11/2015
12	Wed: 02/11/2015	Chap. 5	Magnetostatics	#12	02/13/2015
13	Fri: 02/13/2015	Chap. 5	Magnetostatics	#13	02/16/2015
14	Mon: 02/16/2015	Chap. 6	Maxwell's equations	#14	02/18/2015
15	Wed: 02/18/2015	Chap. 6	Electromagnetic energy and force	#15	02/20/2015
16	Fri: 02/20/2015	Chap. 7	Electromagnetic plane waves	#16	02/23/2015
17	Mon: 02/23/2015	Chap. 7	Dielectric media	#17	02/25/2015
18	Wed: 02/25/2015	Chap. 7	Complex dielectrics	#18	02/27/2015
19	Fri: 02/27/2015	Chap. 1-7	Review -- Take home exam distributed		
	Mon. 03/02/2015	*APS Meeting*	Take-home exam (no class meeting)		
	Mon. 03/04/2015	*APS Meeting*	Take-home exam (no class meeting)		
	Wed. 03/06/2015	*APS Meeting*	Take-home exam (no class meeting)		
	Fri. 03/09/2015	*Spring Break*			
	Wed. 03/11/2015	*Spring Break*			
	Fri. 03/13/2015	*Spring Break*			
20	Fri: 03/16/2015	Chap. 8	Wave guides		

Review

Maxwell's equations

Coulomb's law: $\nabla \cdot \mathbf{D} = \rho_{free}$

Ampere-Maxwell's law: $\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_{free}$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

For linear isotropic media and no sources: $\mathbf{D} = \varepsilon \mathbf{E}; \quad \mathbf{B} = \mu \mathbf{H}$

Review -- continued

Maxwell's equations

Microscopic or vacuum form $(\mathbf{P}=0;\ \mathbf{M}=0)$:

Coulomb's law: $\nabla\cdot\mathbf{E}=\rho/\varepsilon_0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}=\mu_0\mathbf{J}$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

$$\Rightarrow c^2=\frac{1}{\varepsilon_0\mu_0}$$

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Review -- continued

$$\nabla\cdot\mathbf{B}=0 \qquad \Rightarrow \mathbf{B}=\nabla\times\mathbf{A}$$

$$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0 \quad \Rightarrow \nabla\times\left(\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t}\right)=0$$

$$\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t}=-\nabla\Phi$$

$$\text{or}\quad \mathbf{E}=-\nabla\Phi-\frac{\partial\mathbf{A}}{\partial t}$$

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Review -- continued

Analysis of the scalar and vector potential equations:

$$-\nabla^2\Phi-\frac{\partial(\nabla\cdot\mathbf{A})}{\partial t}=\rho/\varepsilon_0$$

$$\nabla\times(\nabla\times\mathbf{A})+\frac{1}{c^2}\left(\frac{\partial(\nabla\Phi)}{\partial t}+\frac{\partial^2\mathbf{A}}{\partial t^2}\right)=\mu_0\mathbf{J}$$

Lorentz gauge form -- require $\nabla\cdot\mathbf{A}_L+\frac{1}{c^2}\frac{\partial\Phi_L}{\partial t}=0$

$$-\nabla^2\Phi_L+\frac{1}{c^2}\frac{\partial^2\Phi_L}{\partial t^2}=\rho/\varepsilon_0$$

$$-\nabla^2\mathbf{A}_L+\frac{1}{c^2}\frac{\partial^2\mathbf{A}_L}{\partial t^2}=\mu_0\mathbf{J}$$

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Review -- continued

When to solve equations using integral form versus differential form?

Examples from electrostatic and magnetostatic cases:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Useful for spatially confined sources.

Review -- continued

Useful identity:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{lm} \frac{4\pi}{2l+1} \frac{r_<^{\,l}}{r_>^{\,l+1}} Y_{lm}(\theta,\varphi) Y_{lm}^*(\theta',\varphi')$$

Review -- continued

Some spherical harmonic functions:

$$Y_{00}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{4\pi}}$$

$$Y_{1(\pm1)}(\hat{\mathbf{r}}) = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\phi}$$

$$Y_{10}(\hat{\mathbf{r}}) = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_{2(\pm2)}(\hat{\mathbf{r}}) = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{\pm 2i\phi}$$

$$Y_{2(\pm1)}(\hat{\mathbf{r}}) = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{\pm i\phi}$$

$$Y_{20}(\hat{\mathbf{r}}) = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Review -- continued

General form of electrostatic potential with boundary value $r \to \infty$, for isolated charge density $\rho(\mathbf{r})$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{1}{4\pi\varepsilon_0}\int d^3r' \rho(\mathbf{r}')\left(\sum_{lm}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}(\theta,\varphi)Y_{lm}^*(\theta',\varphi')\right)$$

Suppose that $\rho(\mathbf{r}) = \sum_{lm}\rho_{lm}(r)Y_{lm}(\theta,\varphi)$

$$\Rightarrow \Phi(\mathbf{r}) = \frac{1}{\varepsilon_0}\sum_{lm}\frac{1}{2l+1}Y_{lm}(\theta,\varphi)\left(\frac{1}{r^{l+1}}\int_0^r r'^{2+l}\,dr'\,\rho_{lm}(r') + r^l\int_r^\infty r'^{1-l}\,dr'\rho_{lm}(r')\right)$$

Review -- continued

Hyperfine interaction energy:

$$E_{int} \equiv H_{HF} = -\mu_e \cdot \mathbf{B}_{\mu_N} - \mu_N \cdot \mathbf{B}_{\mathbf{J}_e}(0)$$

Putting all of the terms together:

$$H_{\mathrm{HF}} = -\frac{\mu_0}{4\pi}\left(\left\langle \frac{3(\boldsymbol{\mu}_\mathrm{N}\cdot\hat{\mathbf{r}})(\boldsymbol{\mu}_\mathrm{e}\cdot\hat{\mathbf{r}}) - \boldsymbol{\mu}_\mathrm{N}\cdot\boldsymbol{\mu}_\mathrm{e}}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu}_\mathrm{N}\cdot\boldsymbol{\mu}_\mathrm{e}\delta^3(\mathbf{r})\right\rangle + \frac{e}{m_e}\left\langle\frac{\mathbf{L}\cdot\boldsymbol{\mu}_\mathrm{N}}{r^3}\right\rangle\right).$$

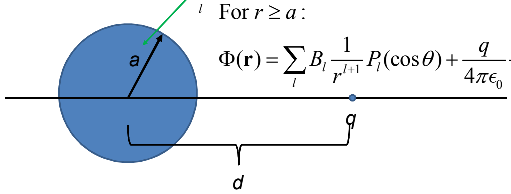
In this expression the brackets $\langle\ \rangle$ indicate evaluating the expectation value relative to the electronic state.

Review of HW problem:
4.9 in Jackson
A point charge *q* is located in free space a distance d from the center of a dielectric sphere of radius *a* (*a*<*d*) and dielectric constant $\varepsilon/\varepsilon_0$. Find the electrostatic potential. For $r \le a$:

$$\Phi(\mathbf{r}) = \sum_l A_l r^l P_l(\cos\theta)$$

For $r \ge a$:

$$\Phi(\mathbf{r}) = \sum_l B_l \frac{1}{r^{l+1}}P_l(\cos\theta) + \frac{q}{4\pi\epsilon_0}\frac{1}{|\mathbf{r}-d\hat{\mathbf{z}}|}$$



Review of HW -- continued

For $r \le a$:

$$\Phi(\mathbf{r}) = \sum_l A_l r^l P_l(\cos\theta)$$

For $r \ge a$:

$$\Phi(\mathbf{r}) = \sum_l B_l \frac{1}{r^{l+1}} P_l(\cos\theta) + \frac{q}{4\pi\epsilon_0}\frac{1}{|\mathbf{r} - d\hat{\mathbf{z}}|}$$

In order to match BC's at $r = a$:

$$\frac{1}{|\mathbf{r} - d\hat{\mathbf{z}}|} = \sum_{l=0} \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta) = \sum_{l=0} \frac{a^l}{d^{l+1}} P_l(\cos\theta)$$

Comment about HW 17 (7.4 in Jackson)

$$\frac{E_0''}{E_0} = \frac{n'-1}{n'+1} \qquad n' = \sqrt{\frac{\mu}{\mu_0}\left(\frac{\epsilon_b}{\epsilon_0} + \frac{i\sigma}{\epsilon_0\omega}\right)}$$

Let $re^{i\phi} = \frac{\mu}{\mu_0}\left(\frac{\epsilon_b}{\epsilon_0} + \frac{i\sigma}{\epsilon_0\omega}\right)$

$$r = \frac{\mu}{\mu_0}\left(\left(\frac{\epsilon_b}{\epsilon_0}\right)^2 + \left(\frac{\sigma}{\epsilon_0\omega}\right)^2\right)^{1/2}$$

$$\tan\phi = \frac{\sigma}{\omega\epsilon_b}$$

$$\frac{E_0''}{E_0} = \frac{\sqrt{r}e^{i\phi/2} - 1}{\sqrt{r}e^{i\phi/2} + 1}$$

$$\left|\frac{E_0''}{E_0}\right|^2 = \frac{r + 1 - 2\sqrt{r}\cos\phi/2}{r + 1 + 2\sqrt{r}\cos\phi/2}$$

Skin depth: $\delta = \sqrt{\frac{2}{\mu\epsilon_b\omega}}$