PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 9:

Continue reading Chapter 4

Dipolar fields and dielectrics

A. Electric field due to a dipole

B. Electric polarization **P**

C. Electric displacement **D** and dielectric functions

Course schedule for Spring 2015

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	Assign.	Due date
1	Mon: 01/12/2015	Chap. 1	Introduction, units and Poisson equation	#1	01/23/2015
2	Wed: 01/14/2015	Chap. 1	Electrostatic energy calculations	#2	01/23/2015
	Fri: 01/16/2015	No class	NAWH out of town		
	Mon: 01/19/2015	No class	MLK Holiday		
3	Wed: 01/21/2015	Chap. 1	Poisson equation and Green's theorem	#3	01/23/2015
4	Fri: 01/23/2015	Chap. 1 & 2	Green's functions in Cartesian coordinates	#4	01/26/2015
5	Mon: 01/26/2015	Chap. 1 & 2	Brief introduction to grid solution methods	#5	01/28/2015
6	Wed: 01/28/2015	Chap. 2	Method of images	#6	01/30/2015
7	Fri: 01/30/2015	Chap. 3	Cylindrical and spherical geometries	#7	02/02/2015
8	Mon: 02/02/2015	Chap. 4	Multipole analysis	#8	02/04/2015
9	Wed: 02/04/2015	Chap. 4	Dipoles and dielectrics	#9	02/06/2015
10	Fri: 02/06/2015	Chap. 4	Dipoles and dielectrics	#10	02/09/2015

WFU Physics Colloquium

TITLE: Diagnosis and treatment of cancer with radiofrequency electromagnetic fields amplitude modulated at tumor-specific frequencies

SPEAKER: Dr. Boris Pasche,

Department of Cancer Biology
Wake Forest University

TIME: Wednesday February 4, 2015 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

In the past century, there have been many attempts to treat cancer with low levels of electric and magnetic fields. We have developed noninvasive biofeedback examination

Review: General results for a multipole analysis of the electrostatic potential due to an isolated charge distribution:

General form of electrostatic potential with boundary value $\Phi(r \to \infty) = 0$ for confined charge density $\rho(\mathbf{r})$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{1}{4\pi\varepsilon_0}\int d^3r' \rho(\mathbf{r}')\left(\sum_{lm} \frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y_{lm}(\theta,\varphi) Y_{lm}^*(\theta',\varphi')\right)$$

Suppose that $\rho(\mathbf{r}) = \sum_{lm} \rho_{lm}(r) Y_{lm}(\theta,\varphi)$

$$\Rightarrow \Phi(\mathbf{r}) = \frac{1}{\varepsilon_0}\sum_{lm}\frac{1}{2l+1} Y_{lm}(\theta,\varphi)\left(\frac{1}{r^{l+1}}\int_0^r r'^{2+l}\, dr' \rho_{lm}(r') + r^l \int_r^\infty r'^{1-l}\, dr' \rho_{lm}(r')\right)$$

For $r \to \infty$: $\Phi(\mathbf{r}) = \frac{1}{\varepsilon_0}\sum_{lm}\frac{1}{2l+1} Y_{lm}(\theta,\varphi)\frac{1}{r^{l+1}}\underbrace{\int_0^\infty r'^{2+l}\, dr' \rho_{lm}(r')}_{q_{lm}}$

Notion of multipole moment:

In the spherical harmonic representation - -

define the moment q_{lm} of the (confined) charge distribution $\rho(\mathbf{r})$:

$$q_{lm} \equiv \int d^3r' \, r'^l \, Y_{lm}^*(\theta', \phi') \rho(\mathbf{r}')$$

In the Cartesian representation - -

define the monopole moment q:

$$q \equiv \int d^3r' \, \rho(\mathbf{r}')$$

define the dipole moment $\mathbf{p}$:

$$\mathbf{p} \equiv \int d^3r' \, \mathbf{r}' \rho(\mathbf{r}')$$

define the quadrupole moment components Q_{ij} $(i,j \to x,y,z)$:

$$Q_{ij} \equiv \int d^3r' \left(3r'_i \, r'_j - r'^2 \delta_{ij}\right)\rho(\mathbf{r}')$$

General form of electrostatic potential in terms of multipole moments:

For r outside the extent of $\rho(\mathbf{r})$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{lm}\frac{4\pi}{2l+1}\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}\left(\int d^3r' \, r'^l \, Y_{lm}^*(\theta',\varphi')\rho(\mathbf{r}')\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{lm}\frac{4\pi \, q_{lm}}{2l+1}\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}$$

In terms of Cartesian expansion:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\mathbf{p}\cdot\mathbf{r}}{r^3} + \frac{1}{2}\sum_{i,j} Q_{ij}\frac{r_i r_j}{r^5}\cdots\right)$$

Focus on dipolar contributions:

For r outside the extent of $\rho(\mathbf{r})$:

Electrostatic potential:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}\right)$$

Electrostatic field:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3\mathbf{r}\,(\mathbf{p}\cdot\mathbf{r}) - r^2\mathbf{p}}{r^5} - \frac{4\pi}{3}\mathbf{p}\,\delta^3(\mathbf{r})\right)$$

Coarse grain representation of macroscopic distribution of dipoles:

Electric polarization $\mathbf{P}(\mathbf{r})$:

$$\mathbf{P}(\mathbf{r}) \equiv \sum_i \mathbf{p}_i \delta^3(\mathbf{r}-\mathbf{r}_i)$$

Mono electric charge density $\rho_{\text{mono}}(\mathbf{r})$:

$$\rho_{\text{mono}}(\mathbf{r}) \equiv \sum_i q_i \delta^3(\mathbf{r}-\mathbf{r}_i)$$

Electrostatic potential for a single monopole charge q and a single dipole $\mathbf{p}$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\mathbf{p}\cdot\mathbf{r}}{r^3}\right)$$

Coarse grain representation of macroscopic distribution of dipoles -- continued:

Electrostatic potential for a single monopole charge q and a single dipole $\mathbf{p}$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\mathbf{p}\cdot\mathbf{r}}{r^3}\right)$$

Electrostatic potential for collections of monopole charges q_i and dipoles $\mathbf{p}_i$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\int d^3r'\,\frac{\rho_{mono}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} + \int d^3r'\,\frac{\mathbf{P}(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\right)$$

Note: $\int d^3r'\,\frac{\mathbf{P}(\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} = \int d^3r'\,\mathbf{P}(\mathbf{r}')\cdot\nabla'\frac{1}{|\mathbf{r}-\mathbf{r}'|} = -\int d^3r'\,\frac{\nabla'\cdot\mathbf{P}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$

Coarse grain representation of macroscopic distribution of dipoles -- continued:

Electrostatic potential for collections of monopole charges q_i and dipoles $\mathbf{p}_i$:

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\int d^3r' \frac{\rho_{mono}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} - \int d^3r' \frac{\nabla'\cdot\mathbf{P}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\right)$$

$$-\nabla^2\Phi(\mathbf{r}) = \nabla\cdot\mathbf{E}(\mathbf{r}) = \frac{1}{\varepsilon_0}\left(\rho_{mono}(\mathbf{r}) - \nabla\cdot\mathbf{P}(\mathbf{r})\right)$$

$$\Rightarrow \nabla\cdot(\varepsilon_0\mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r})) = \rho_{mono}(\mathbf{r})$$

Define Displacement field : $\mathbf{D}(\mathbf{r}) \equiv \varepsilon_0\mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r})$

Macroscopic form of Gauss's law : $\nabla\cdot\mathbf{D}(\mathbf{r}) = \rho_{mono}(\mathbf{r})$

Coarse grain representation of macroscopic distribution of dipoles -- continued:

Many materials are polarizable and produce a polarization field in the presence of an electric field with a proportionality constant χ_e :

$\mathbf{P}(\mathbf{r}) = \varepsilon_0\chi_e\mathbf{E}(\mathbf{r})$

$\mathbf{D}(\mathbf{r}) \equiv \varepsilon_0\mathbf{E}(\mathbf{r}) + \mathbf{P}(\mathbf{r}) = \varepsilon_0(1+\chi_e)\mathbf{E}(\mathbf{r}) \equiv \varepsilon\mathbf{E}(\mathbf{r})$

ε represents the dielectric function of the material

Boundary value problems in the presence of dielectrics

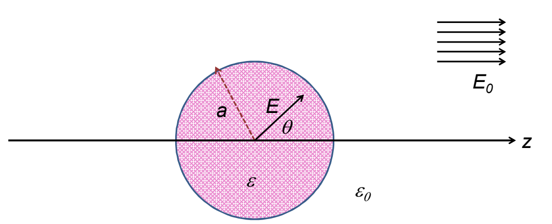
For $\rho_{mono}(\mathbf{r}) = 0$

$\nabla\cdot\mathbf{D}(\mathbf{r}) = 0$ and $\nabla\times\mathbf{E}(\mathbf{r}) = 0$

$\Rightarrow$ At a surface between two dielectrics, in terms of surface normal $\hat{\mathbf{r}}$:

$\hat{\mathbf{r}}\cdot\mathbf{D}(\mathbf{r}) = \text{continuous} = \hat{\mathbf{r}}\times\mathbf{E}(\mathbf{r})$

Boundary value problems in the presence of dielectrics – example:



$\nabla\cdot\mathbf{D}(\mathbf{r}) = 0$ and $\nabla\times\mathbf{E}(\mathbf{r}) = 0$ At $r = a$: $\varepsilon\frac{\partial\Phi_<(\mathbf{r})}{\partial r} = \varepsilon_0\frac{\partial\Phi_>(\mathbf{r})}{\partial r}$

For $r \le a$ $\mathbf{D}(\mathbf{r}) = -\varepsilon\nabla\Phi(\mathbf{r})$

For $r > a$ $\mathbf{D}(\mathbf{r}) = -\varepsilon_0\nabla\Phi(\mathbf{r})$

$$\frac{\partial\Phi_<(\mathbf{r})}{\partial\theta} = \frac{\partial\Phi_>(\mathbf{r})}{\partial\theta}$$

Boundary value problems in the presence of dielectrics – example -- continued:

$$\Phi_<(\mathbf{r}) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$\Phi_>(\mathbf{r}) = \sum_{l=0}^{\infty} \left(B_l r^l + \frac{C_l}{r^{l+1}} \right) P_l(\cos\theta)$$

At $r = a$:
$$\varepsilon \frac{\partial \Phi_<(\mathbf{r})}{\partial r} = \varepsilon_0 \frac{\partial \Phi_>(\mathbf{r})}{\partial r}$$
$$\frac{\partial \Phi_<(\mathbf{r})}{\partial \theta} = \frac{\partial \Phi_>(\mathbf{r})}{\partial \theta}$$

For $r \to \infty$ $\quad \Phi_>(\mathbf{r}) = -E_0 r \cos\theta$

Solution -- only $l = 1$ contributes

$$B_1 = -E_0$$

$$A_1 = -\left(\frac{3}{2 + \varepsilon/\varepsilon_0} \right) E_0 \qquad C_1 = \left(\frac{\varepsilon/\varepsilon_0 - 1}{2 + \varepsilon/\varepsilon_0} \right) a^3 E_0$$

Boundary value problems in the presence of dielectrics – example -- continued:

$$\Phi_<(\mathbf{r}) = -\left(\frac{3}{2 + \varepsilon/\varepsilon_0} \right) E_0 \, r \cos\theta$$

$$\Phi_>(\mathbf{r}) = -\left(r - \left(\frac{\varepsilon/\varepsilon_0 - 1}{2 + \varepsilon/\varepsilon_0} \right) \frac{a^3}{r^2} \right) E_0 \cos\theta$$